Strange quark star properties from the vector meson extended linear sigma model

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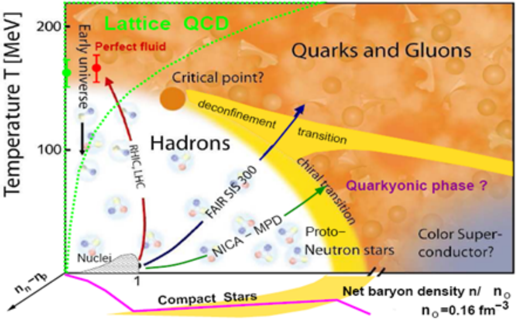
Envisaged phase diagram of QCD

Phase diagram in the $T - \mu_\mathrm{B} - \mu_\mathrm{I}$ space

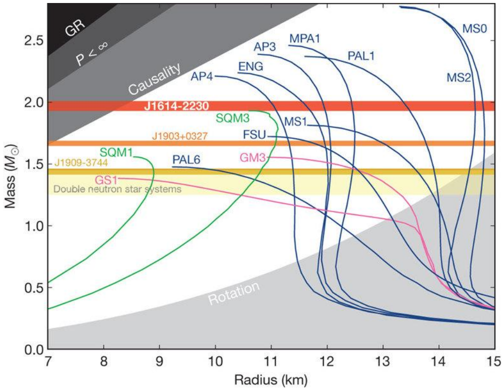


- At $\mu_\mathrm{B} = 0$ $T_c = 153(3)$ MeV Y. Aoki, *et al.*, PLB **643**, 46 (2006)
- Is there a CEP?
- The T-dependence of thermodynamical quantities like pressure, interaction measure, quark density is known from lattice only at $\mu_B = 0$.
- At which μ_B is there the phase boundary for $T = 0$?
- In medium changes of masses and widths

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

Compact star EOSs

- QCD directly unsolvable at finite density and temperature
- One can use effective models in the zero temperature finite density region
- Neutron star observations restrict such models [1,2]



$M - R$ relations from various models [1]

[1] Demorest P., et al. (2010), Nature, 467, 1081

[2] Antoniadis J., et al. (2013), Science, 340, 6131

Lagrangian

$\mathcal{L}$ constructed based on linearly realized global $U(3)_L \times U(3)_R$ symmetry and its explicit breaking

$$\mathcal{L} = \mathrm{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \mathrm{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi)^2$$
$$+ c_1(\det \Phi + \det \Phi^\dagger) + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2)$$
$$+ \mathrm{Tr}\left[\left(\frac{m_1^2}{2}\mathbb{1} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})$$
$$+ \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger \Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2 \mathrm{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \mathrm{Tr}(L_\mu \Phi R^\mu \Phi^\dagger)$$
$$+ \bar{\Psi} i \gamma_\mu D^\mu \Psi - g_F \bar{\Psi}(\Phi_S + i\gamma_5 \Phi_{PS})\Psi,$$

$$\begin{aligned}
D^\mu \Phi &= \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu[T_3, \Phi], \\
L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}, \\
R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\}, \\
D^\mu \Psi &= \partial^\mu \Psi - iG^\mu \Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.
\end{aligned}$$

+ Polyakov loop potential

Determination of the parameters

14 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F) \longrightarrow$ determined by the min. of χ^2:

$$\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1, \ldots, x_N) - Q_i^{\exp}}{\delta Q_i} \right]^2 ,$$

$(x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1, \ldots, x_N) \longrightarrow$ from the model, $Q_i^{\exp} \longrightarrow$ PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$
multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Curvature masses $\rightarrow$ 16 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- Pseudocritical temperature T_c at $\mu_B = 0$

Features of our approach

- D.O.F's: – scalar, pseudoscalar, vector, and axial-vector nonets
  – u, d, s constituent quarks ($m_u = m_d$)
  – Polyakov loop variables $\Phi, \bar{\Phi}$ with $\mathcal{U}_{\log}^{\mathrm{YM}}$ or $\mathcal{U}_{\log}^{\mathrm{glue}}$
- no mesonic fluctuations, only fermionic ones
  $$\mathcal{Z}=e^{-\beta V\Omega(T,\mu_q)}=\int_{\mathrm{PBC}}\prod_a \mathcal{D}\xi_a \int_{\mathrm{APBC}}\prod_f \mathcal{D}q_f\mathcal{D}q_f^\dagger \exp\left[-\int_0^\beta d\tau \int_V d^3x\left(\mathcal{L}+\mu_q\sum_f q_f^\dagger q_f\right)\right]$$
  approximated as $\Omega(T,\mu_q) = U_{\mathrm{meson}}^{\mathrm{tree}}(\langle M\rangle) + \Omega_{\bar{q}q}^{(0)}(T,\mu_q) + \mathcal{U}_{\log}(\Phi,\bar{\Phi})$, $\tilde{\mu}_q=\mu_q-iG_4$
  $$e^{-\beta V\Omega_{\bar{q}q}^{(0)}} = \int_{\mathrm{APBC}}\prod_{f,g}\mathcal{D}q_g\mathcal{D}q_f^\dagger \exp\left\{\int_0^\beta d\tau\int_x q_f^\dagger\left[\left(i\gamma_0\vec{\gamma}\cdot\vec{\nabla}-\frac{\partial}{\partial\tau}+\tilde{\mu}_q\right)\delta_{fg}-\gamma_0\mathcal{M}_{fg}|_{\xi_a=0}\right]q_g\right\}$$
- tree-level (axial)vector masses
- fermionic vacuum and thermal fluctuations included in the (pseudo)scalar curvature masses used to parameterize the model
- 4 coupled T/μ_B-dependent field equations for the condensates $\phi_N, \phi_S, \Phi, \bar{\Phi}$
- thermal contribution of π, K, f_0^L included in the pressure, however their curvature mass contains no mesonic fluctuations

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Inclusion of vector meson Yukawa term

$$\mathcal{L}_{\text{Yukawa-vec}} = -g_v\sqrt{6}\bar{\Psi}\gamma_\mu V_0^\mu\Psi$$
$$V_0^\mu = \frac{1}{\sqrt{6}}\mathrm{diag}\left(v_0 + \frac{v_8}{\sqrt{2}}, v_0 + \frac{v_8}{\sqrt{2}}, v_0 - \sqrt{2}v_8\right)$$

mean-field treatment

$$< v_0^\mu >= v_0\delta^{0\mu},\ < v_8^\mu >= 0^\mu$$

Modification of the grand canonical potential

$$\Omega(T=0,\mu_q) \to \Omega(T=0,\tilde{\mu}_q) - \frac{1}{2}m_v^2 v_0^2,\ \text{with } \tilde{\mu}_q = \mu_q - g_v v_0$$

While the field equations

$$\left.\frac{\partial\Omega}{\partial\phi_{\mathrm{N}}}\right|_{\phi_{\mathrm{N}}=\bar{\phi}_{\mathrm{N}}} = \left.\frac{\partial\Omega}{\partial\phi_{\mathrm{S}}}\right|_{\phi_{\mathrm{S}}=\bar{\phi}_{\mathrm{S}}} = 0 \quad \text{and} \quad \left.\frac{\partial\Omega}{\partial v_0}\right|_{v_0=\bar{v}_0} = 0\,,$$

Equation of state

The pressure and the energy density at $g_v = 0$

$$p(\mu_q) = \Omega(\mu_q = 0) - \Omega(\mu_q)$$
$$\varepsilon(\mu_q) = -p - \mu_q \frac{\partial \Omega(\mu_q)}{\partial \mu_q} \equiv -p + \mu_q \rho_q(\mu_q)$$

modifications for $g_v \neq 0$

$$p(\mu_q) \to p(\tilde{\mu}_q)|_{g_v=0} + \frac{g_v^2}{2m_v^2}\rho_q^2(\tilde{\mu}_q)$$
$$\varepsilon(\mu_q) \to \varepsilon(\tilde{\mu}_q)|_{g_v=0} + \frac{g_v^2}{2m_v^2}\rho_q^2(\tilde{\mu}_q) + \tilde{\mu}_q \rho_q(\tilde{\mu}_q)$$

Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein's equation for spherically symmetric case and homogeneous matter $\longrightarrow$ TOV eqs.:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{[p(r)+\varepsilon(r)]\,[M(r)+4\pi r^3 p(r)]}{r[r-2M(r)]} \tag{1}$$

with

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \varepsilon(r)$$

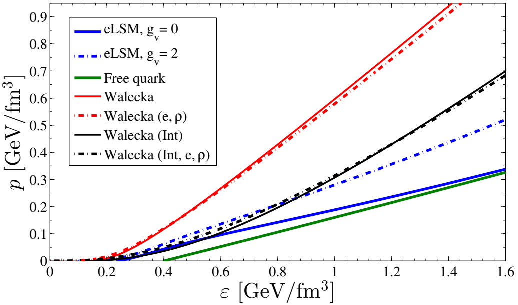
These are integrated numerically for a specific $p(\varepsilon)$

- For a fixed ε_c central energy density Eq. (1) is integrated until $p = 0$
- Varying ε_c a series of compact stars is obtained (with given M and R)
- Once the maximal mass is reached, the stable series of compact stars ends

The EOS

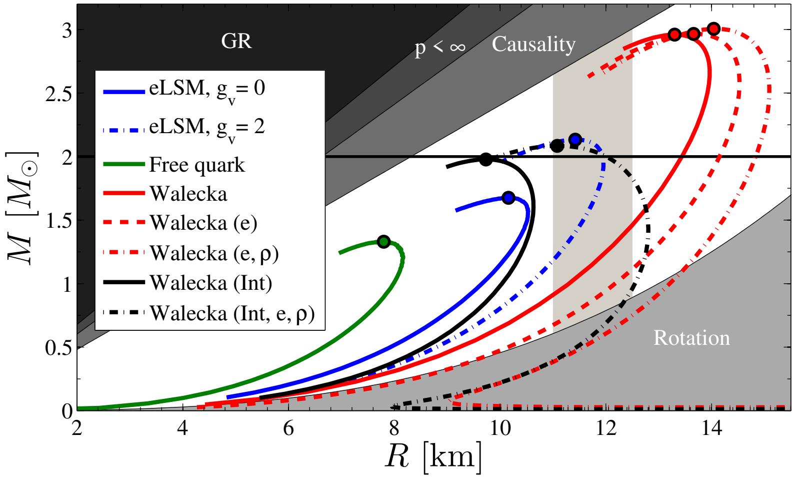
- Our result compared to results from other models
- At low energies the EoS of the eLSM is close to the EoS of the Walecka - model
- At higher energies it tends to the EoS of the free quark model
- note: Walecka Int means

$\mathcal{L}_{W,Int} = -\frac{b}{3} m_n (g_\sigma \sigma)^3 - \frac{c}{4} (g_\sigma \sigma)^4$



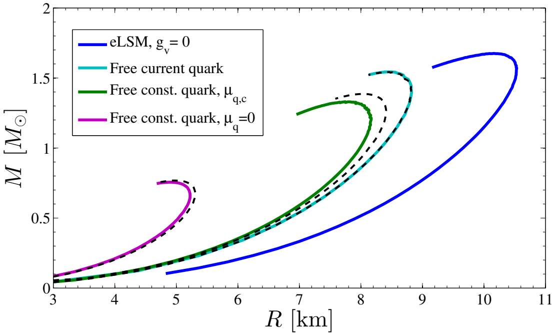
- In case of Walecka models (with/without Interaction) the effects of electrons and the ρ mesons are also included (dashed-dotted lines)

Mass-radius relations



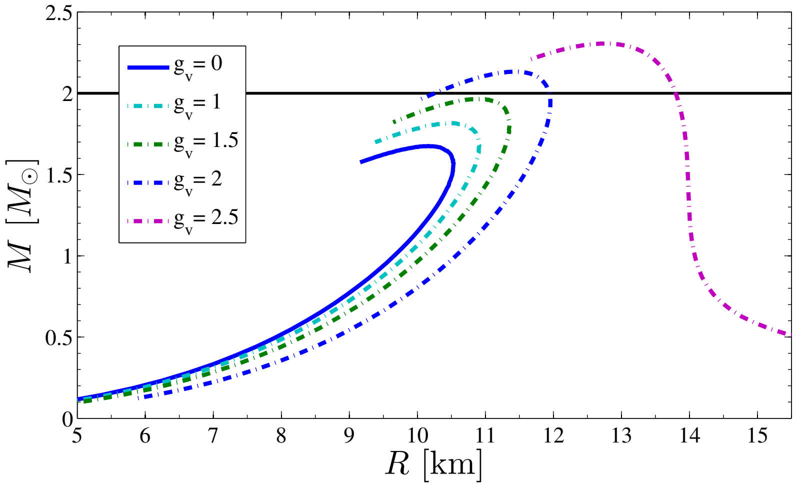
M-R relations of various free quark models

- We used free quark models with different quark masses and compared to the eLSM
- $m_{\text{free}} < m_{\text{eLSM}} < m_{\mu_q=0}$
- Interactions of the eLSM has a relevant effect on the M-R relation



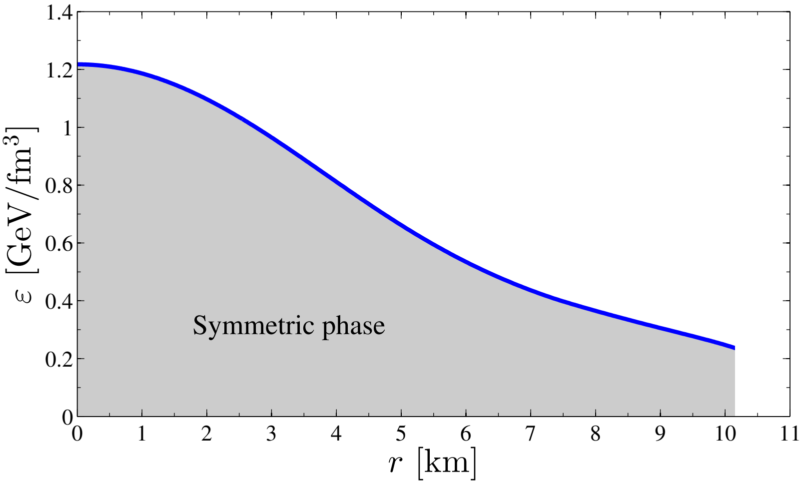
- current quark: $m_{u/d} = 0$ MeV, $m_s = 90$ MeV
- const. quark $\mu_{q,c}$: $m_{u/d} = 75$ MeV, $m_s = 365$ MeV
- const. quark $\mu_q = 0$: $m_{u/d} = 322$ MeV, $m_s = 458$ MeV

M-R relations with different g_v vector couplings



For $g_v \gtrsim 2.5$ $p(\varepsilon)$ becomes zero at $\varepsilon = 0 \Rightarrow$ small M, large R

Phase of the compact star for M_{max}



This is a strange quark star with a very thin crust

Conclusion

Conclusion

- With appropriate g_v value the resulting $M - R$ curve is consistent with restrictions coming from $2M_\odot$ neutron stars observed in 2010
- The quark–meson Yukawa interactions have a very important role in the EoS and consequently in the $M - R$ curves
- Pure quark stars are not excluded by current observations

Outlook

- Consistent treatment of one-loop mesonic corrections, expansion of fermion determinants in mesonic fields
- Inclusion of the total vector-quark Yukawa term, consistent treatment
- Constriction of a hybrid model: baryons at low density and constituent quarks at high density

Thank you for your attention!

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Lagrangian II.

the matter and external fields are

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i, \quad \Delta = \sum_{i=0}^{8}\delta_i T_i$$

$$\Psi = (u, d, s)^{\mathrm{T}}$$

non strange – strange base:

$$\xi_N = \sqrt{2/3}\xi_0 + \sqrt{1/3}\xi_8,$$
$$\xi_S = \sqrt{1/3}\xi_0 - \sqrt{2/3}\xi_8, \qquad \xi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle\sigma_{N/S}\rangle \equiv \bar{\sigma}_{N/S}$

Particle content

- Vector and Axial-vector meson nonets

$$V^{\mu}=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star +}\\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0}\\ K^{\star -} & \bar{K}^{\star 0} & \omega_S\end{pmatrix}^{\mu} \quad A^{\mu}=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+\\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0\\ K_1^- & \bar{K}_1^0 & f_{1S}\end{pmatrix}^{\mu}$$

$\rho \to \rho(770), K^{\star} \to K^{\star}(894)$
$\omega_N \to \omega(782), \omega_S \to \phi(1020)$

$a_1 \to a_1(1230), K_1 \to K_1(1270)$
$f_{1N} \to f_1(1280), f_{1S} \to f_1(1426)$

- Scalar $(\sim \bar{q}_i q_j)$ and pseudoscalar $(\sim \bar{q}_i \gamma_5 q_j)$ meson nonets

$$\Phi_S=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_0^{\star +}\\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_0^{\star 0}\\ K_0^{\star -} & \bar{K}_0^{\star 0} & \sigma_S\end{pmatrix} \quad \Phi_{PS}=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+\\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0\\ K^- & \bar{K}^0 & \eta_S\end{pmatrix}$$

unknown assignment
mixing in the $\sigma_N - \sigma_S$ sector

$\pi \to \pi(138), K \to K(495)$
mixing: $\eta_N, \eta_S \to \eta(548),\ \eta'(958)$

Spontaneous symmetry breaking: $\sigma_{N/S}$ acquire nonzero expectation values $\phi_{N/S}$

fields shifted by their expectation value: $\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S}$

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp\left[i \int_0^\beta d\tau\, G_4(\vec{x},\tau)\right]$$

$\hookrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement transition

low T: confined phase, $\langle\Phi(\vec{x})\rangle, \langle\bar{\Phi}(\vec{x})\rangle = 0$
high T: deconfined phase, $\langle\Phi(\vec{x})\rangle, \langle\bar{\Phi}(\vec{x})\rangle \neq 0$

- Polyakov gauge: $G_4(\vec{x},\tau) = G_4(\vec{x})$, plus gauge rotation to diagonal form in color space
- further simplification: $\vec{x}$-independence

$$\hookrightarrow \quad L = e^{i\beta G_4} = \mathrm{diag}(a,b,c) \left(\overset{!}{\in} SU(3)^{\mathrm{color}}\right); \quad a,b,c \in \mathbb{Z}$$

$\hookrightarrow$ use this to calculate partition function of free quarks

Form of the potential

I.) Simple polynomial potential invariant under $\mathbb{Z}_3$ and charge conjugation: R.D.Pisarski, PRD 62, 111501

$$\frac{\mathcal{U}^{\mathrm{YM}}_{\mathrm{poly}}(\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2$$

with $\qquad b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2} + a_3\frac{T_0^3}{T^3}$

II.) Logarithmic potential coming from the $SU(3)$ Haar measure of group integration $\qquad$ K. Fukushima, Phys. Lett. **B591**, 277 (2004)

$$\frac{\mathcal{U}^{\mathrm{YM}}_{\mathrm{log}}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi\bar{\Phi} + b(T)\ln\left[1-6\Phi\bar{\Phi}+4\left(\Phi^3+\bar{\Phi}^3\right)-3\left(\Phi\bar{\Phi}\right)^2\right]$$

with $\qquad a(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2}, \qquad b(T) = b_3\frac{T_0^3}{T^3}$

$\mathcal{U}^{\mathrm{YM}}\left(\Phi,\bar{\Phi}\right)$ models the free energy of a pure gauge theory

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Improved Polyakov loop potential

Previous potentials describe successfully the first order phase transition of the pure $SU(3)$ Yang – Mills

$\hookrightarrow$ taking into account the gluon dynamics (quark polarization of gluon propagator) $\longrightarrow$ QCD glue potential

$\hookrightarrow$ can be implemented by changing the reduced temperature

$$t_{\text{glue}} \equiv \frac{T - T_c^{\text{glue}}}{T_c^{\text{glue}}}, \quad t_{\text{YM}} \equiv \frac{T^{\text{YM}} - T_c^{\text{YM}}}{T_c^{\text{YM}}}$$

$$t_{\text{YM}}(t_{\text{glue}}) \approx 0.57 t_{\text{glue}}$$

$$\frac{\mathcal{U}^{\text{glue}}}{T^4}(\Phi, \bar{\Phi}, t_{\text{glue}}) = \frac{\mathcal{U}^{\text{YM}}}{(T^{\text{YM}})^4}(\Phi, \bar{\Phi}, t_{\text{YM}}(t_{\text{glue}}))$$

L. M. Haas *et al.*, PRD 87, 076004 (2013)

Result of the parameterization

- 40 possible assignments of scalar mesons to the scalar nonet states
- 3 values of M_0 are used $\Longrightarrow$ 120 cases to investigate
  for each case $5 \cdot 10^4 - 10^5$ configurations are used for the χ^2 minimization
- lowest χ^2 obtained for $M_0 = 0.3$ GeV $\chi^2 = 18.57$ and $\chi^2_{\rm red} \equiv \frac{\chi^2}{N_{\rm dof}} = 1.16$
  assignment: $a_0^{\bar{q}q} \to a_0(980)$, $K_0^{\star,\bar{q}q} \to K_0^\star(800)$, $f_0^{L,\bar{q}q} \to f_0(500)$, $f_0^{H,\bar{q}q} \to f_0(980)$

  problems: $m_{a_0} < m_{K_0^\star}$, $m_{f_0^{H/L}}$ too light
- by minimizing also for M_0 we obtain using $\mathcal{U}^{\rm YM}_{\rm log}(\Phi, \bar{\Phi})$ with $T_0 = 182$ MeV:

Parameter	Value	Parameter	Value
ϕ_N [GeV]	0.1411	g_1	5.6156
ϕ_S [GeV]	0.1416	g_2	3.0467
m_0^2 [GeV^2]	2.3925_{E-4}	h_1	27.4617
m_1^2 [GeV^2]	6.3298_{E-8}	h_2	4.2281
λ_1	-1.6738	h_3	5.9839
λ_2	23.5078	g_F	4.5708
c_1 [GeV]	1.3086	M_0 [GeV]	0.3511
δ_S [GeV^2]	0.1133		