arXiv:1811.09990

Lifetime estimation from RHIC Au+Au data

T. Csörgő and G. Kasza

XIV. WPCF and Balaton Workshop

June 2019, Dubna and Tihany

Outline

- New, exact solution of relativistic perfect fluid hydro (Csörgő-Kasza-Csanád-Jiang: CKCJ)
  - Observables – dN/dη, dN/dy, R_L
  - Beauty of hydro: useful formulae
- Fit results
  - Observables Inverse slope fits
  - dN/dη fits
  - R_L fits
- Power of analytic hydro: initial energy densities (ε_0)

Perfect fluid hydrodynamics

- Conservation of four-momentum:

$$\partial_\nu T^{\mu\nu} = 0$$

- Energy-momentum tensor (perfect fluid):

$$T^{\mu\nu} = (\varepsilon + p)\, u^\mu u^\nu - p g^{\mu\nu}$$

- Continuity equation:

$$\partial_\mu \left(\sigma u^\mu\right) = 0$$

- Equation of state (EoS): closes the equation system
- Typically: $\varepsilon = \kappa(T)\, p$, T dependent speed of sound
- In this work $\kappa = const(T)$: the speed of sound (c_s) is replaced by its average value

$$\varepsilon = \kappa p \qquad \kappa = c_s^{-2}$$

New, exact solutions of perfect fluid

Csörgő, Kasza, Csanád, Jiang:
arXiv:1805.01427
arXiv:1806.06794

- Rindler coordinates
- Velocity field: $u^\mu = (\cosh(\Omega), \sinh(\Omega))$
- 1+1d, parametric exact solution
- If $\kappa \to 1$ limit: CNC):

$$(\tau, \eta_x) = \left(\sqrt{t^2 - r_z^2}\,, \frac{1}{2}\ln\left[\frac{t+r_z}{t-r_z}\right]\right)$$

Csörgő, Nagy, Csanád solution
arXiv:0709.3677

λ: acceleration parameter
accelerating solution

$$\begin{aligned}
\eta_x(H) &= \Omega(H) - H, \\
\Omega(H) &= \frac{\lambda}{\sqrt{\lambda-1}\sqrt{\kappa-\lambda}}\arctan\left(\sqrt{\frac{\kappa-\lambda}{\lambda-1}}\tanh(H)\right) \\
\sigma(\tau, H) &= \sigma_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda} \mathcal{V}_\sigma(s)\left[1 + \frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\frac{\lambda}{2}}, \\
T(\tau, H) &= T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{\kappa}} \mathcal{T}(s)\left[1 + \frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\frac{\lambda}{2\kappa}}, \\
\mathcal{T}(s) &= \frac{1}{\mathcal{V}_\sigma(s)}, \\
s(\tau, H) &= \left(\frac{\tau_0}{\tau}\right)^{\lambda-1}\sinh(H)\left[1 + \frac{\kappa-1}{\lambda-1}\sinh^2(H)\right]^{-\lambda/2}
\end{aligned}$$

arXiv:1805.01427

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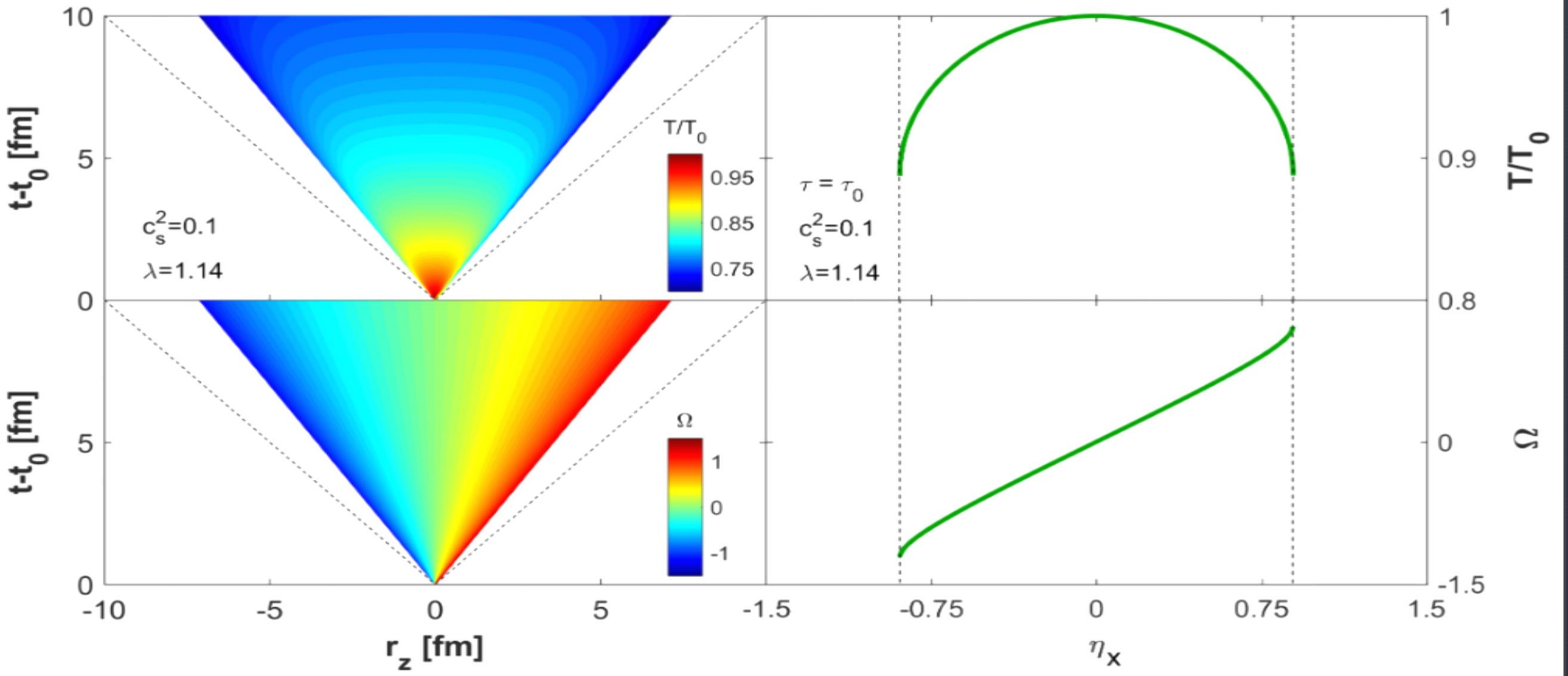
New, exact solutions of perfect fluid



Pseudorapidity density

- Rapidity density (embedded to 1+3 d space):

$$\frac{dN}{dy} \approx \left.\frac{dN}{dy}\right|_{y=0} \cosh^{-\frac{1}{2}\alpha(\kappa,\lambda)-1}\left(\frac{y}{\alpha(1,\lambda)}\right) \exp\left(-\frac{m}{T_{\rm eff}}\left[\cosh^{\alpha(\kappa,\lambda)}\left(\frac{y}{\alpha(1,\lambda)}\right)-1\right]\right)$$

- Lorentzian average p_T as a function of y:

$$\alpha(\kappa)=\frac{2\lambda-\kappa}{\lambda-\kappa}$$

$$\bar{p}_T(y) \approx \sqrt{T_{\rm eff}^2 + 2mT_{\rm eff}}\left(1+\frac{\alpha(\kappa)}{2\alpha(1)^2}\frac{T_{\rm eff}+m}{T_{\rm eff}+2m}y^2\right)^{-1}$$

T_{eff}: effective temperature

- Pseudorapidity density: parametric curve

$$\left(\eta_p(y), \frac{dN}{d\eta_p}(y)\right) = \left(\frac{1}{2}\log\left[\frac{\bar{p}(y)+\bar{p}_z(y)}{\bar{p}(y)-\bar{p}_z(y)}\right], \frac{\bar{p}(y)}{\bar{E}(y)}\frac{dN}{dy}\right)$$

- Four fit parameters: κ, λ, T_{eff} , $dN/dy|_{y=0}$

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Rapidity density

$$\frac{dN}{dy} \approx \left.\frac{dN}{dy}\right|_{y=0} \cosh^{-\frac{1}{2}\alpha(\kappa,\lambda)-1}\left(\frac{y}{\alpha(1,\lambda)}\right) \exp\left(-\frac{m}{T_{\rm eff}}\left[\cosh^{\alpha(\kappa,\lambda)}\left(\frac{y}{\alpha(1,\lambda)}\right)-1\right]\right)$$

- Rapidity density: depends on 4 hydro parameters (κ, λ, T_{eff} , $dN/dy|_{y=0}$)
- Near midrapidity, in leading order: a Gaussian

$$\frac{dN}{dy} \approx \frac{\langle N\rangle}{(2\pi\Delta^2 y)^{1/2}} \exp\left(-\frac{y^2}{2\Delta^2 y}\right)$$

From 4 hydro parameters (κ, λ, <N>, T_{eff})
→ only 2 fit parameters (<N>, Δy)
Beautiful example of hydrodynamical scaling behaviour!

Gaussian width: $\frac{1}{\Delta^2 y} = (\lambda-1)^2\left[1+\left(1-\frac{1}{\kappa}\right)\left(\frac{1}{2}+\frac{m}{T_{\rm eff}}\right)\right]$

Midrapidity density: $\left.\frac{dN}{dy}\right|_{y=0} = \frac{\langle N\rangle}{(2\pi\Delta^2 y)^{\frac{1}{2}}}$

Pseudorapidity density

$$\left(\eta_p(y)\,,\frac{dN}{d\eta_p}(y)\right) = \left(\frac{1}{2}\log\left[\frac{\bar{p}(y)+\bar{p}_z(y)}{\bar{p}(y)-\bar{p}_z(y)}\right],\frac{\bar{p}(y)}{\bar{E}(y)}\frac{dN}{dy}\right)$$

- Pseudorapidity density: parametric curve (κ, λ, T_{eff} , $dN/dy|_{y=0}$)
- Near midrapidity, up to leading order

$$\frac{dN}{d\eta_p} \approx \frac{\langle N\rangle}{(2\pi\Delta^2 y)^{1/2}}\frac{\cosh(\eta_p)}{(D^2+\cosh^2(\eta_p))^{1/2}}\exp\left(-\frac{y^2}{2\Delta^2 y}\right)\Bigg|_{y=y(\eta_p)}$$

From 4 hydro parameters (κ, λ, <N>, T_{eff})
→ 3 fit parameters (D, <N>, Δy)
Beautiful example of hydrodynamical scaling behaviour!

Midrapidity density:

$$\frac{dN}{dy}\bigg|_{y=0} = \frac{\langle N\rangle}{(2\pi\Delta^2 y)^{\frac{1}{2}}}$$

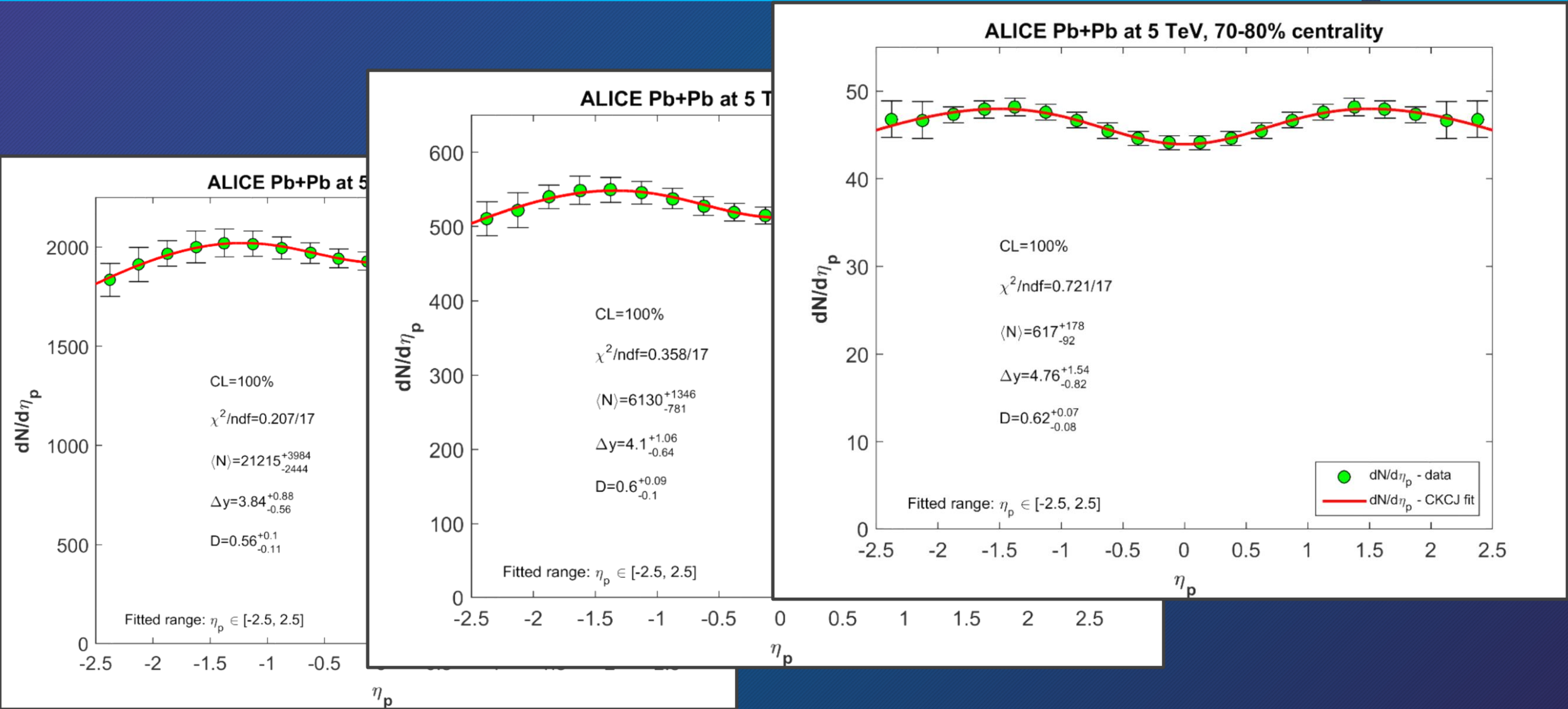
Depression parameter:

$$D^2 = \frac{m^2}{M^2}$$

Gaussian width:

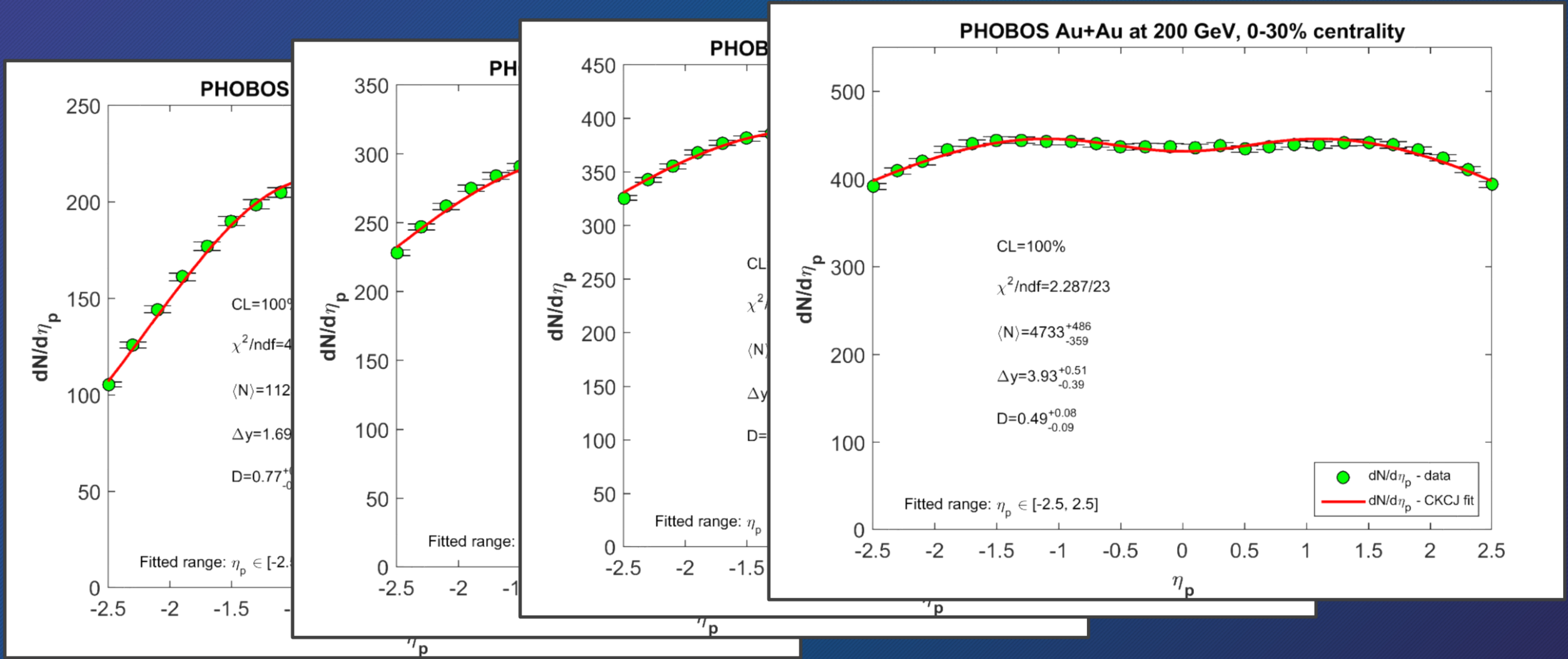
$$\frac{1}{\Delta^2 y} = (\lambda-1)^2\left[1+\left(1-\frac{1}{\kappa}\right)\left(\frac{1}{2}+\frac{m}{T_{\rm eff}}\right)\right]$$

ALICE Pb+Pb data at 5 TeV

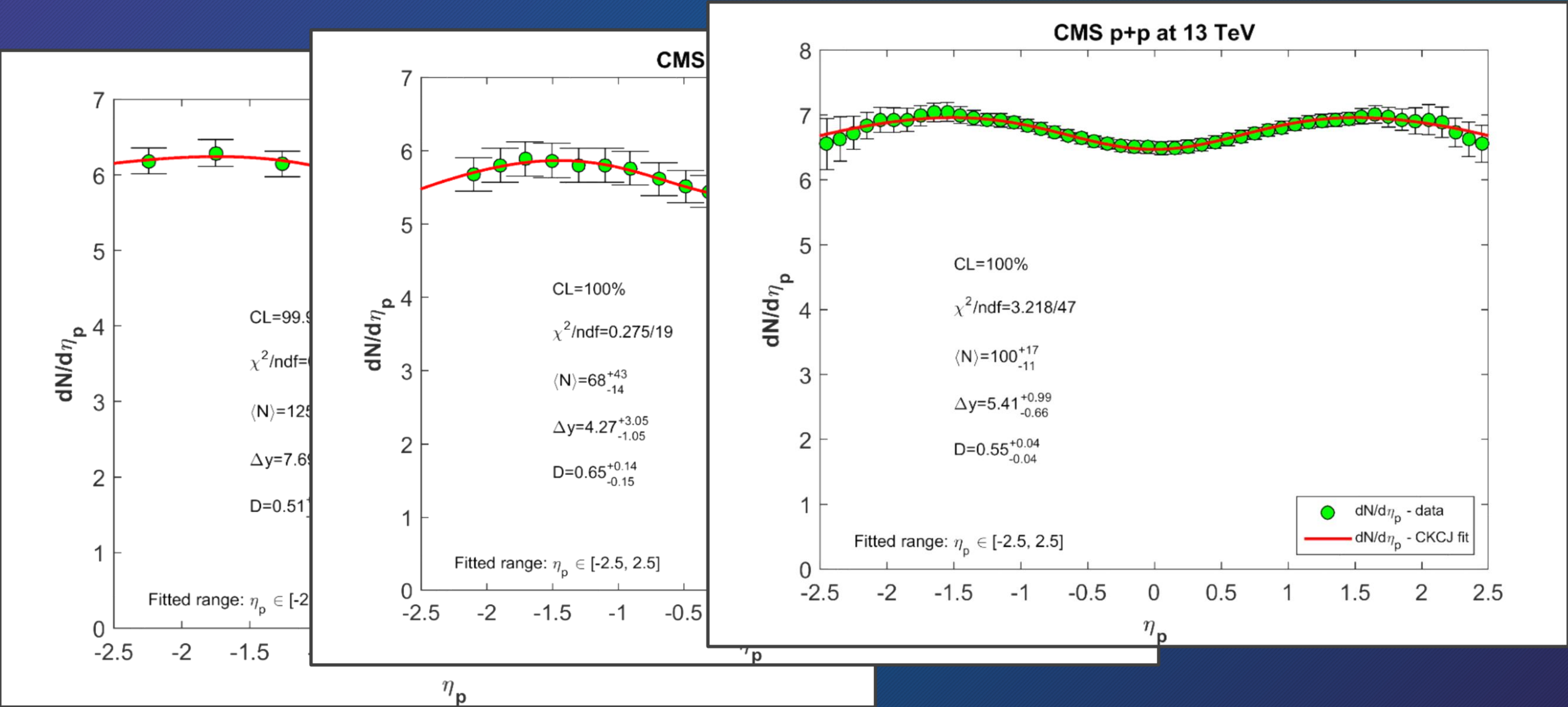


Fits at RHIC, $\sqrt{s_{NN}}$= 20-200 GeV

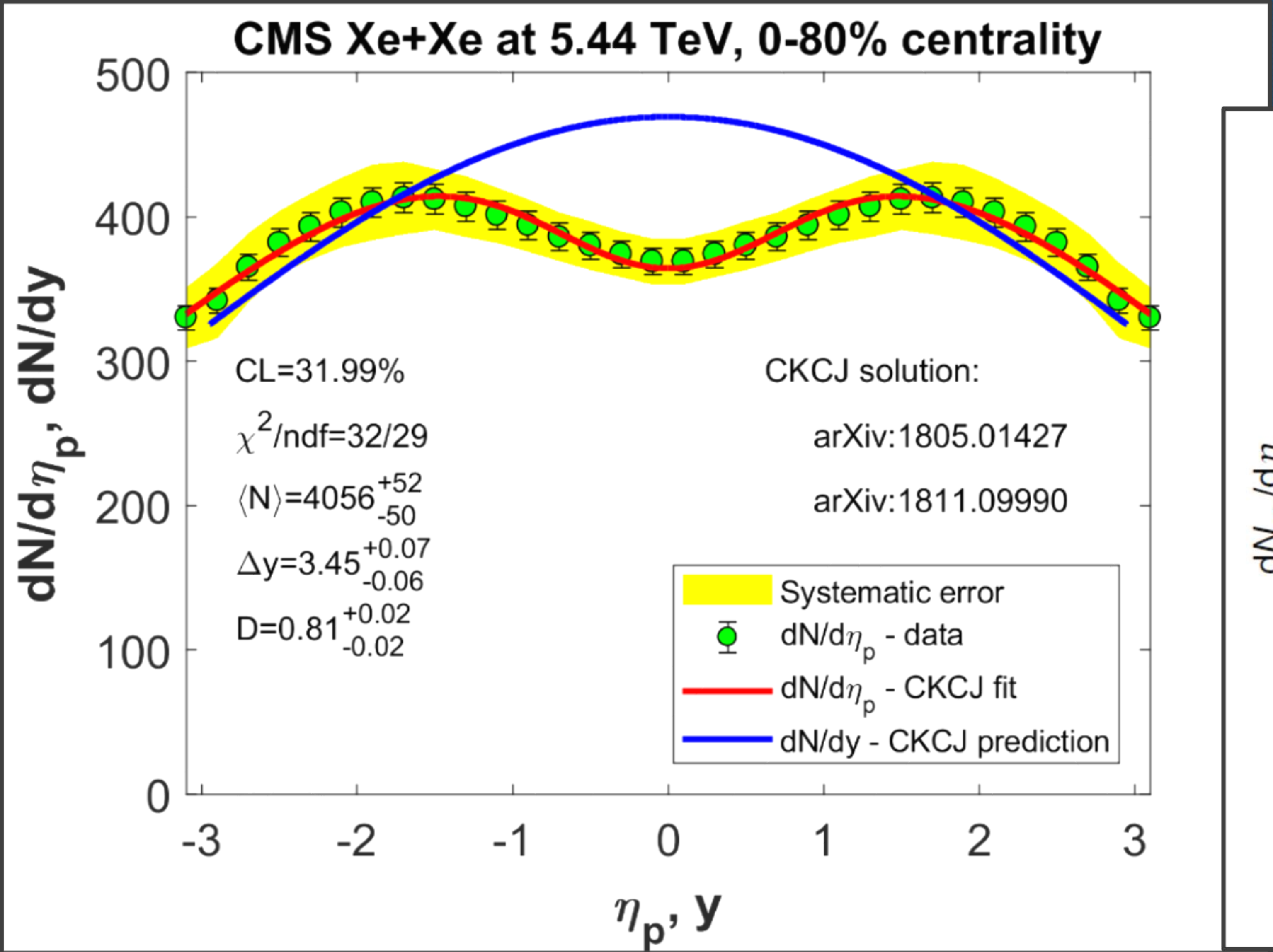
Powerful 3-parameter hydro formula. CL „too good”



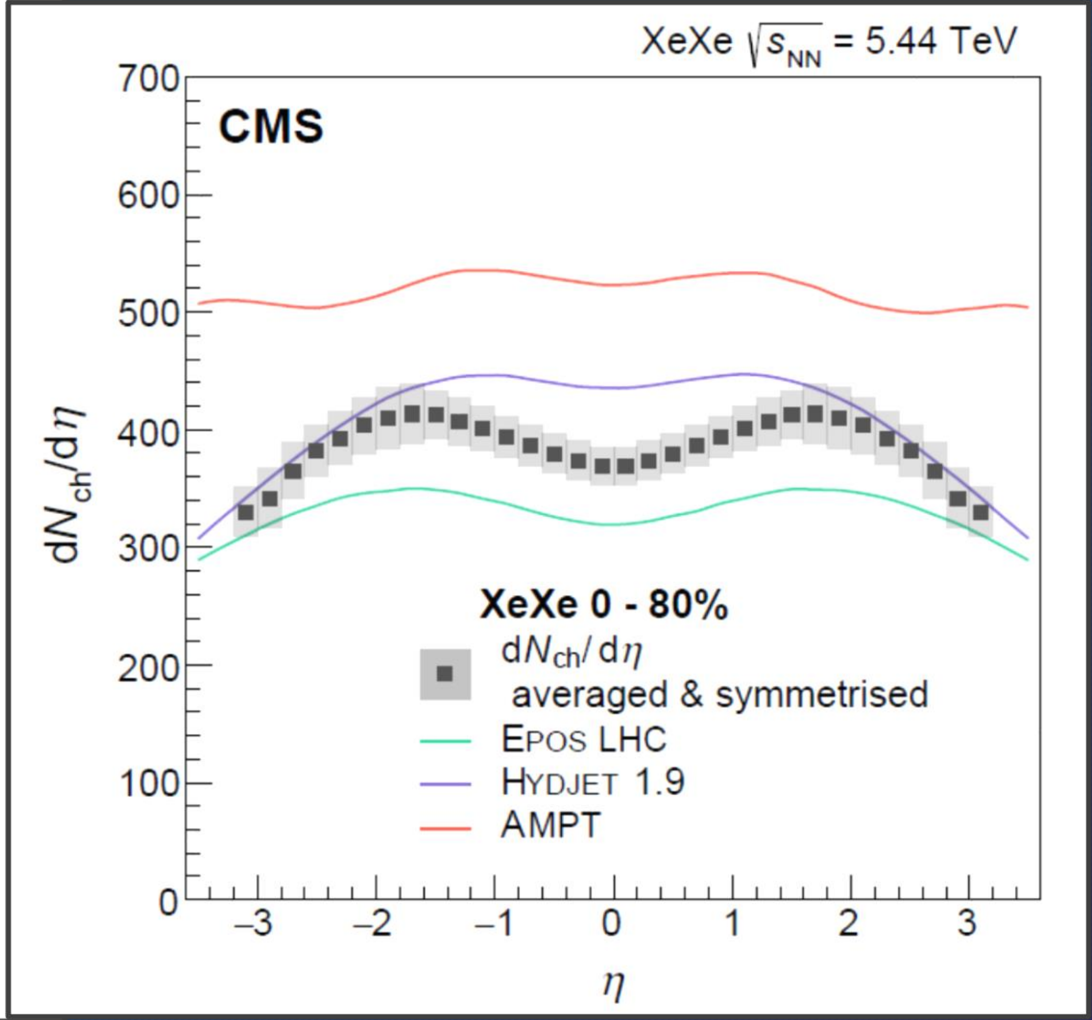
CMS p+p dn/dη fits, 7, 8, 13 TeV



CMS Xe+Xe data, 5.44 TeV



arXiv:1902.03603

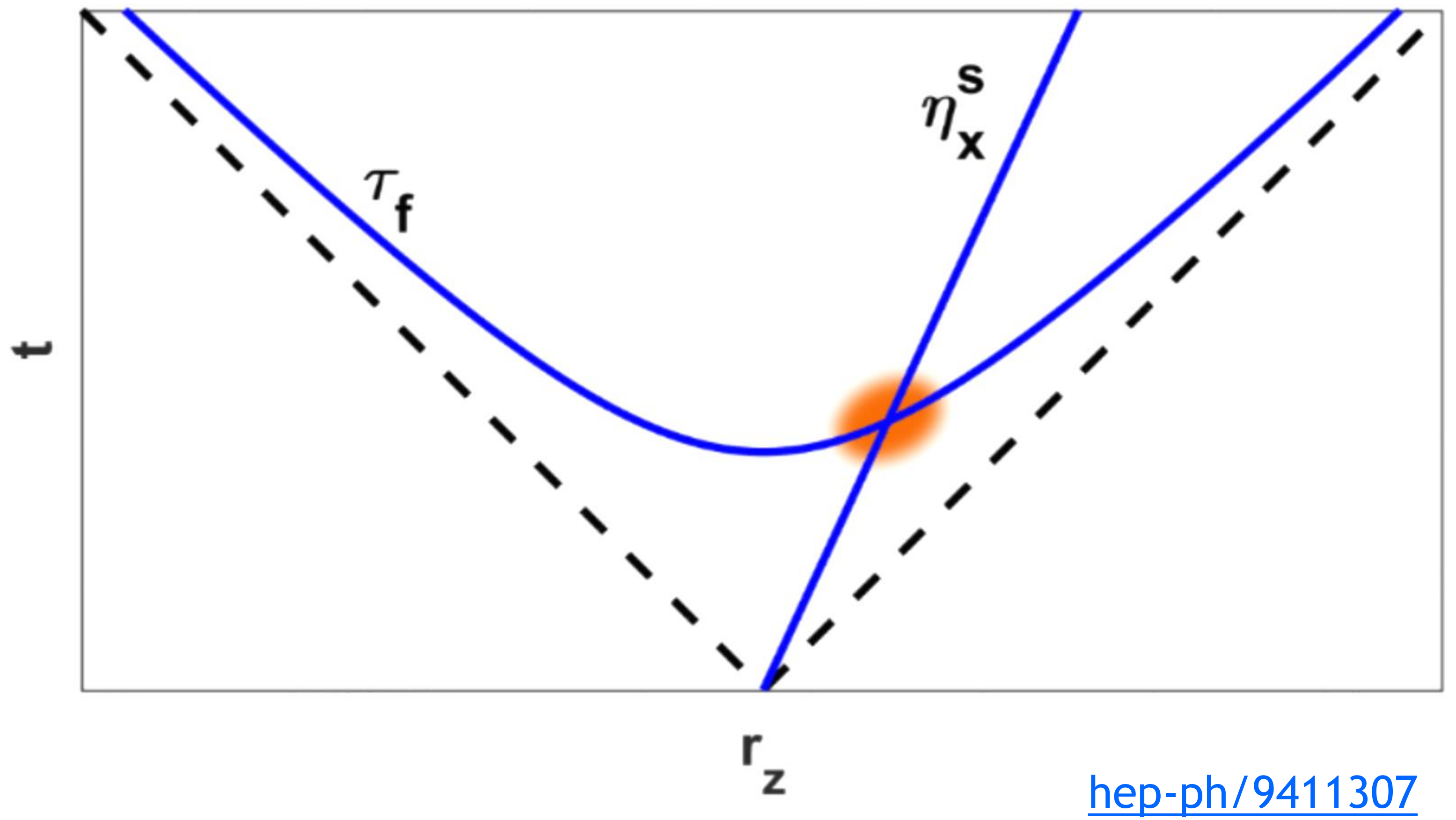


Observables – HBT R_L

- For a 1+1 d relativistic source (LCMS):

$$R_L^2 = \cosh^2(\eta_x^s)\tau_f^2\Delta\eta_x^2 + \sinh^2(\eta_x^s)\Delta\tau^2$$

- (η_x^s, τ_f) saddle point:



hep-ph/9411307

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$$R_L^2 = \cosh^2(\eta_x^s)\tau_f^2\Delta\eta_x^2 + \sinh^2(\eta_x^s)\Delta\tau^2$$

- (η_x^s, τ_f) saddle point
- CKCJ solutions are limited to a narrow rapidity interval around $\eta_x \approx 0$
- At midrapidity:

$$R_L = \tau_f \Delta\eta_x \longrightarrow R_L = \tau_f \Delta\eta_x \approx \frac{\tau_f}{\sqrt{\lambda(2\lambda - 1)}}\sqrt{\frac{T_f}{m_T}}$$

- $\Delta\eta_x$: gaussian width of rapidity distribution

hep-ph/9411307

HBT R_L fits

- Fitted to PHENIX and STAR R_L data
- From fits to dN/dη: λ, T_{eff} are known
- τ_f is the only fit parameter

$$R_L = \tau_f \Delta\eta_x \approx \frac{\tau_f}{\sqrt{\lambda(2\lambda-1)}}\sqrt{\frac{T_f}{m_T}}$$

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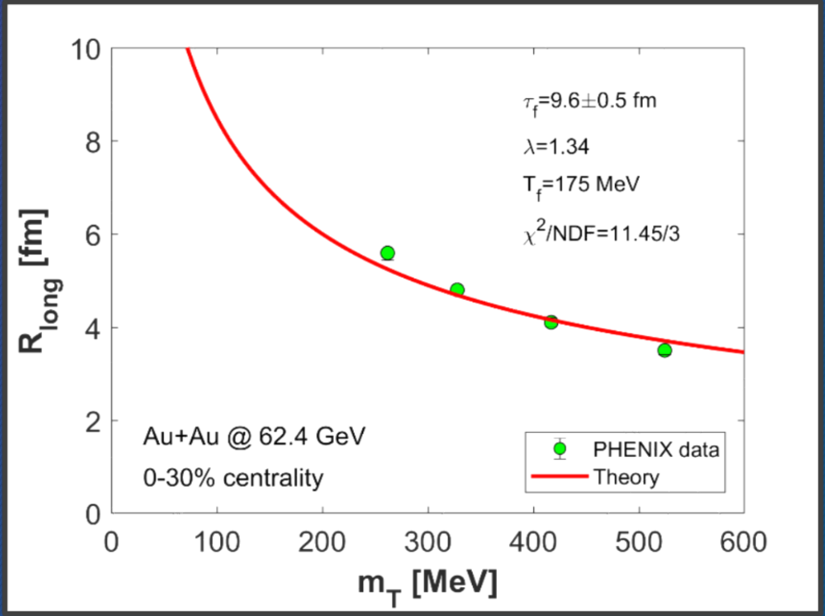
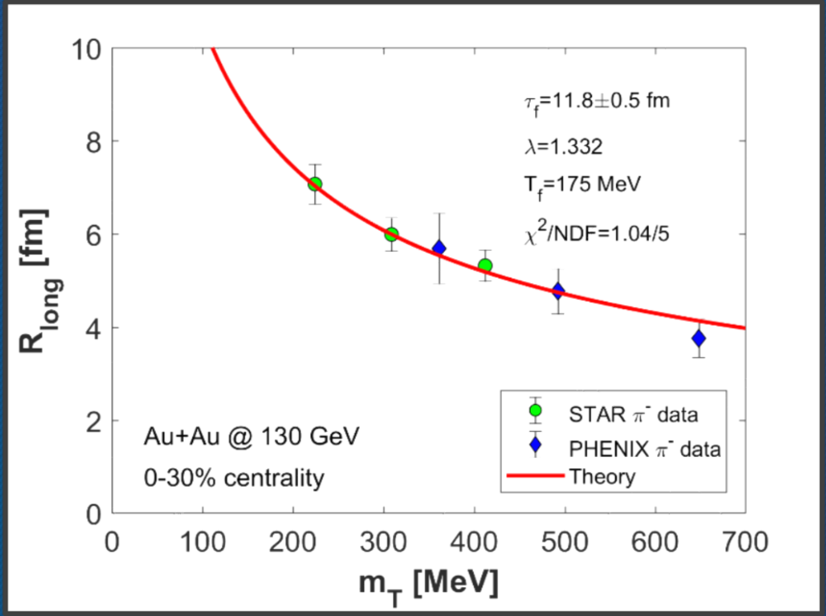
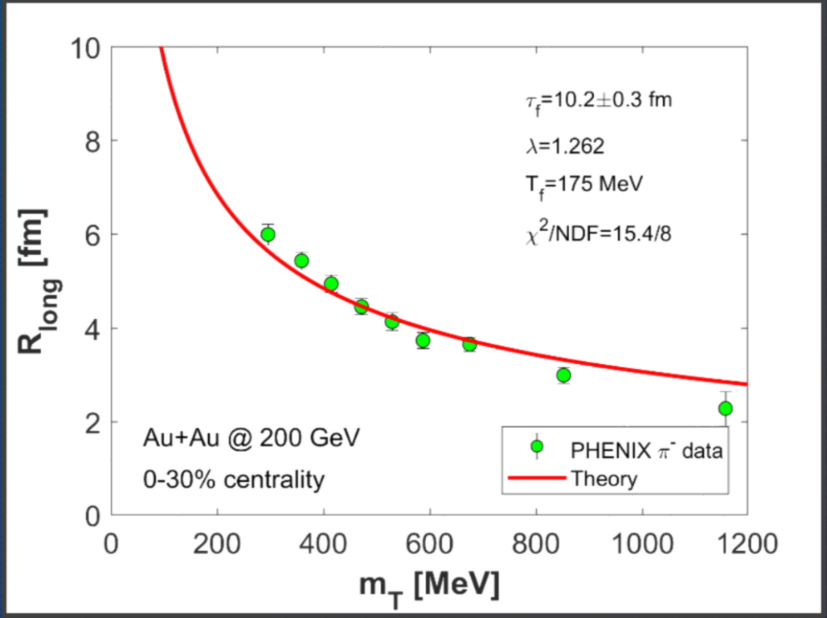


arXiv:1403.4972



arXiv:nucl-ex/0201008

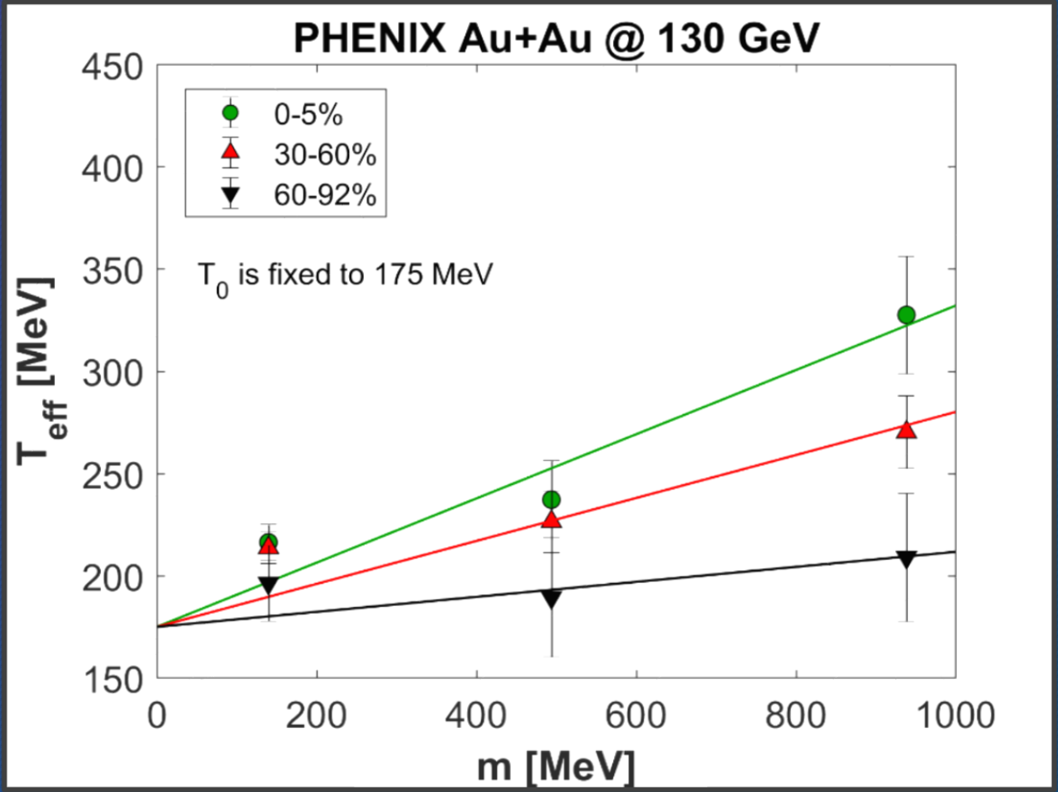


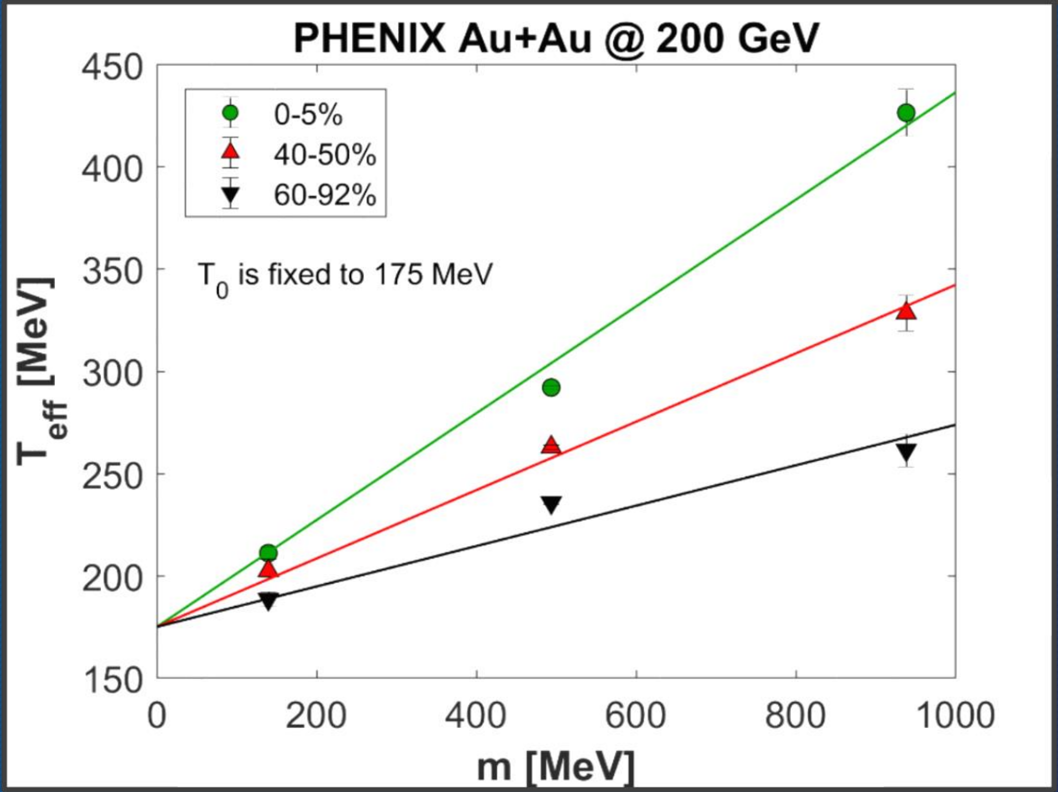
arXiv:nucl-ex/0401003

Inverse slope fits

- Effective temperature: need centrality dependence
- Linear model is fitted to m_0 vs T_{eff} PHENIX data (130 and 200 GeV)
- T_f is fixed, independent from centrality

$$T_{eff}(m) = T_f + m\langle u_T \rangle^2$$





arXiv:1801.05716

arXiv:1811.09990

Inverse slope fits

- Effective temperature: need centrality dependence
- Linear model is fitted to m_0 vs T_{eff} PHENIX data (130 and 200 GeV)
- If m_0 vs T_{eff} data is not available (62.4 GeV):
  - fit to p_T spectra or… → we chose this
  - T_{eff} can be obtained by dN/dη fits

Inverse slope fits

- Effective temperature: need centrality dependence
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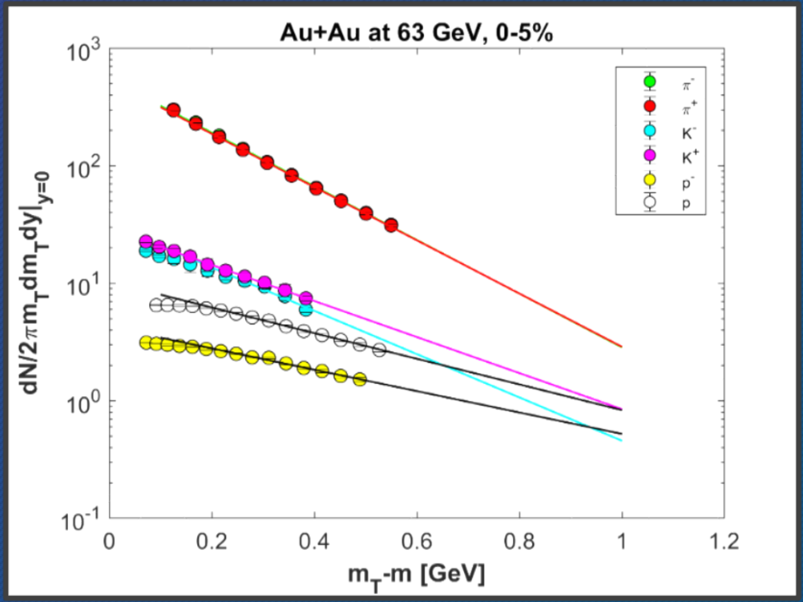
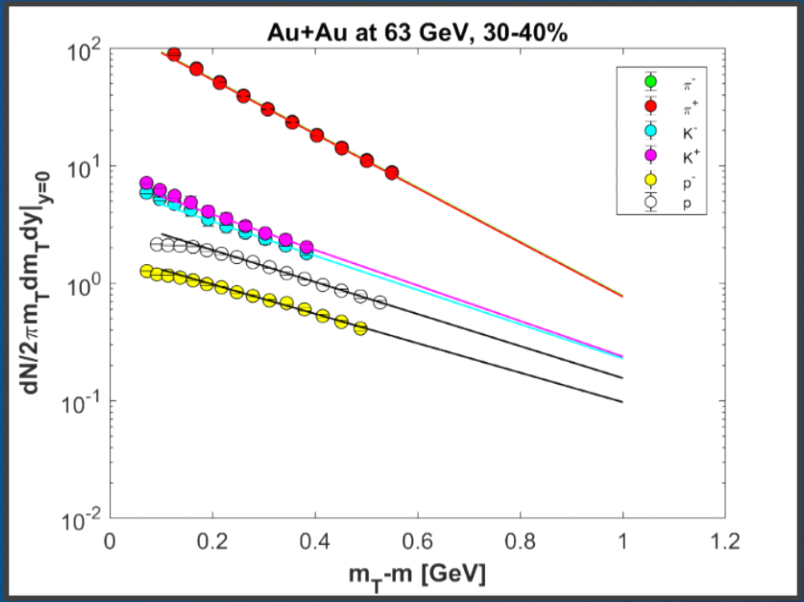
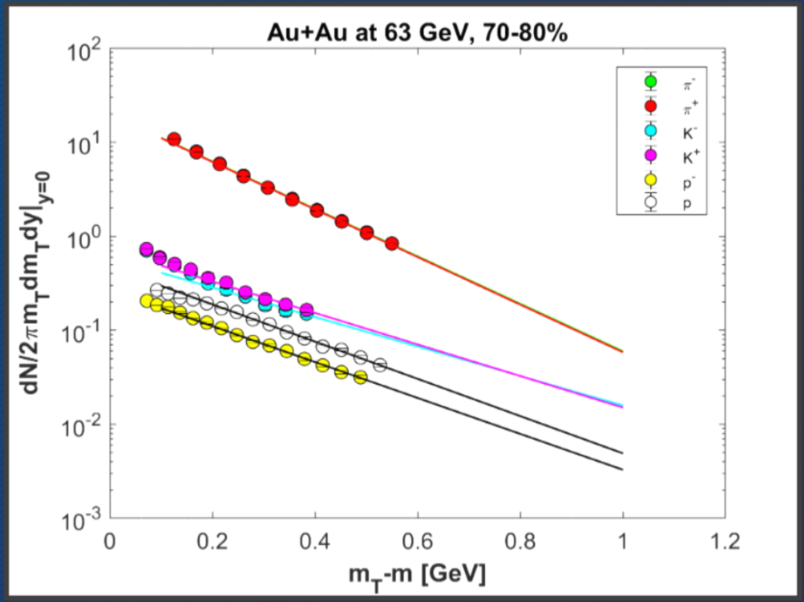
- If m_0 vs T_{eff} data is not available (62.4 GeV):

Fit function was from: arXiv:nucl-ex/0307022







arXiv:0808.2041

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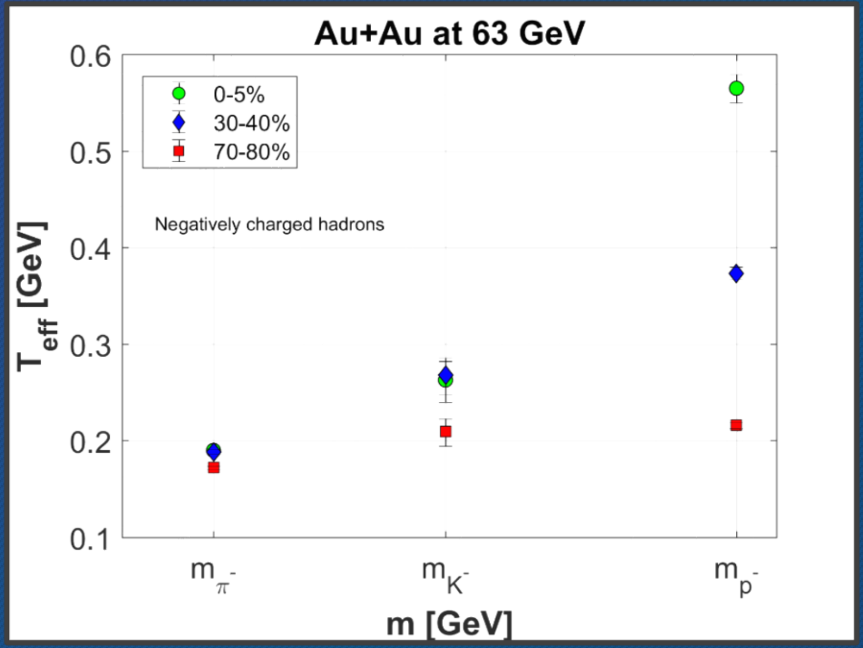
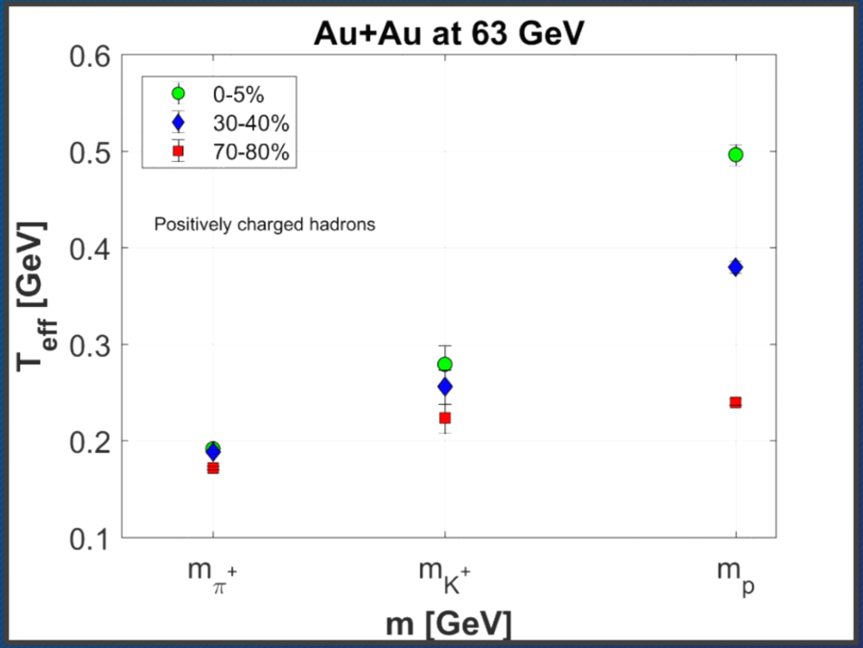
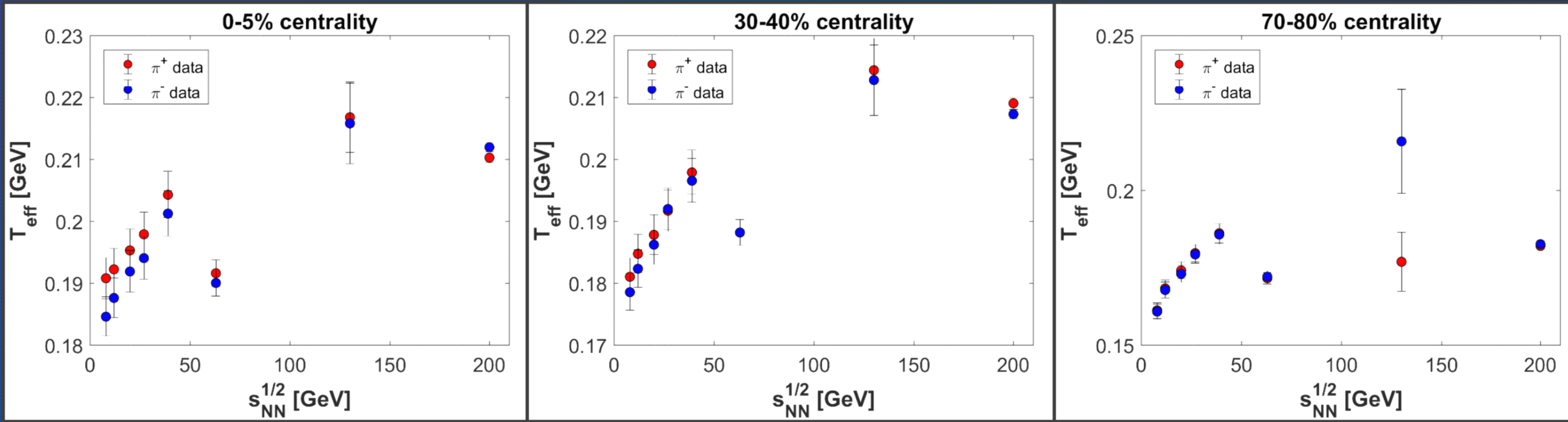




A new feature of STAR p_T spectras

- The p_T spectras of pions are fitted at
  7.7, 11.5, 19.6, 27, 39, 62.4, 130 and 200 GeV
- Three different centrality class: 0-5%, 30-40%, 70-80%
- Interesting behaviour of T_{eff}: *local minimum at 62.4 GeV*



Initial energy density

- From the CKCJ solution ε_0 is exactly calculated:

arXiv:1806.11309

$$\varepsilon_0(\kappa, \lambda) = \varepsilon_0^{Bj} (2\lambda - 1) \left(\frac{\tau_f}{\tau_0} \right)^{\lambda\left(1+\frac{1}{\kappa}\right)-1}$$

- *$\lambda \to 1$ limit: EoS dependence is not vanishing*, Bjorken's formula can be reobtained only in the case of dust
- The EoS dependent correction takes into account the *work of the pressure*
- In λ=1 case, the EoS dependence was found first by Gorenstein, Sinyukov and Zhdanov (before Bjorken!) Phys.Lett. 71B (1977) 199-202
- Now, the fundamental correction to ε_0^{Bj} is also known
- λ and τ_f are obtained by fits → τ_0 dependence

Exact result of initial energy density

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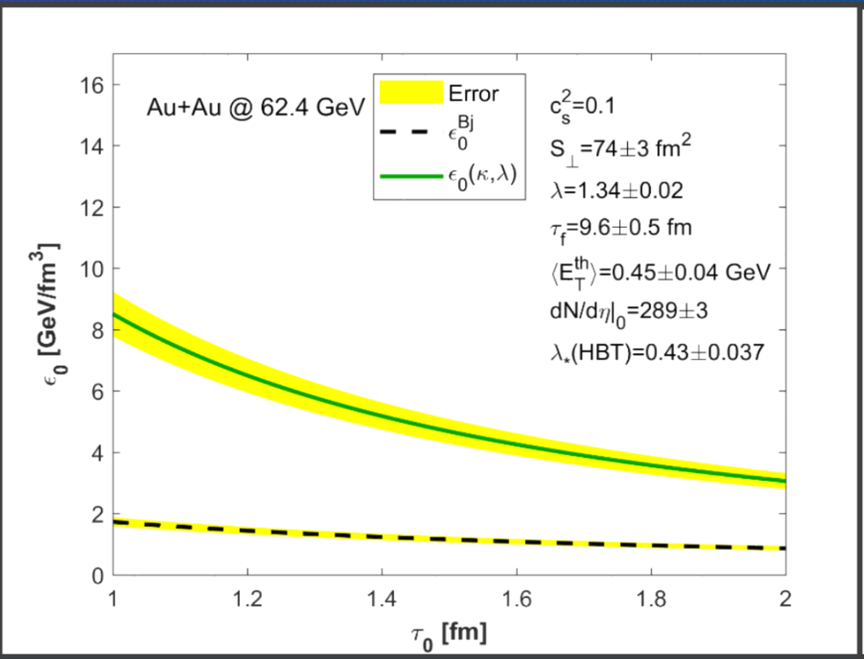
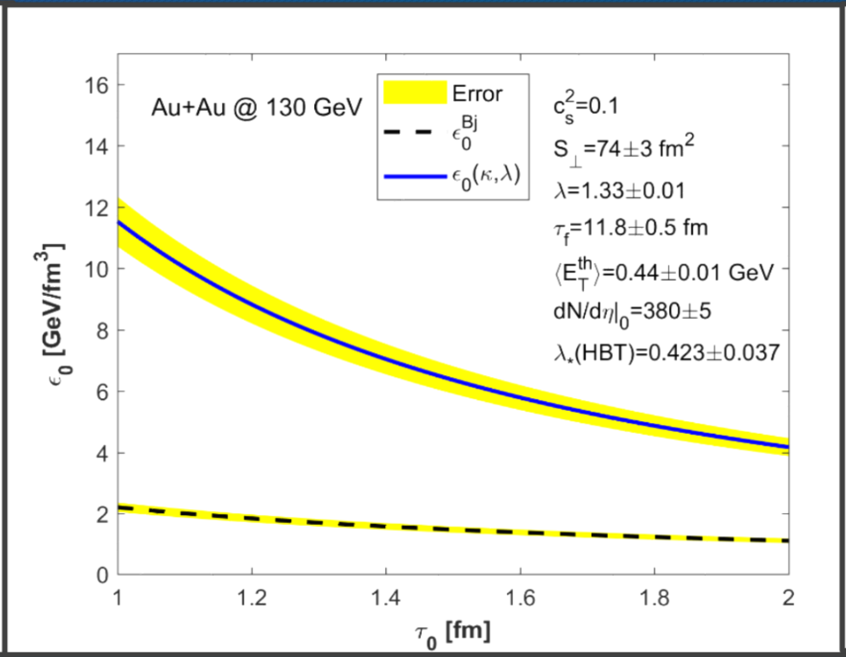
- λ and τ_f are obtained by fits → τ_0 dependence

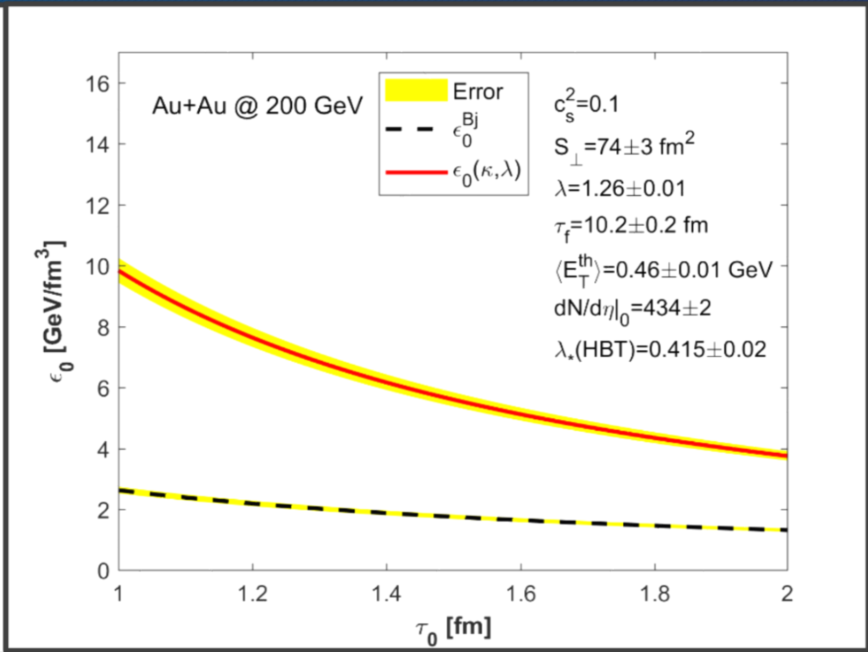
Bjorken's formula can be applied only for order of magnitude estimations!

Initial energy densities

- Only the thermalized energy is included
- Indication for an increasing initial energy density with decreasing colliding energy?
- Suggest a non-monotonic behaviour of the initial energy density with $\sqrt{s}$







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Exact result of initial energy density

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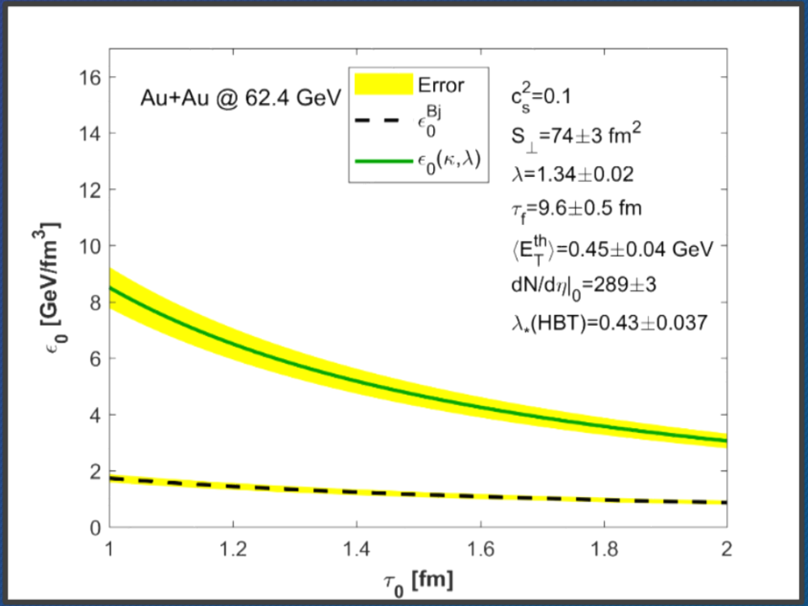

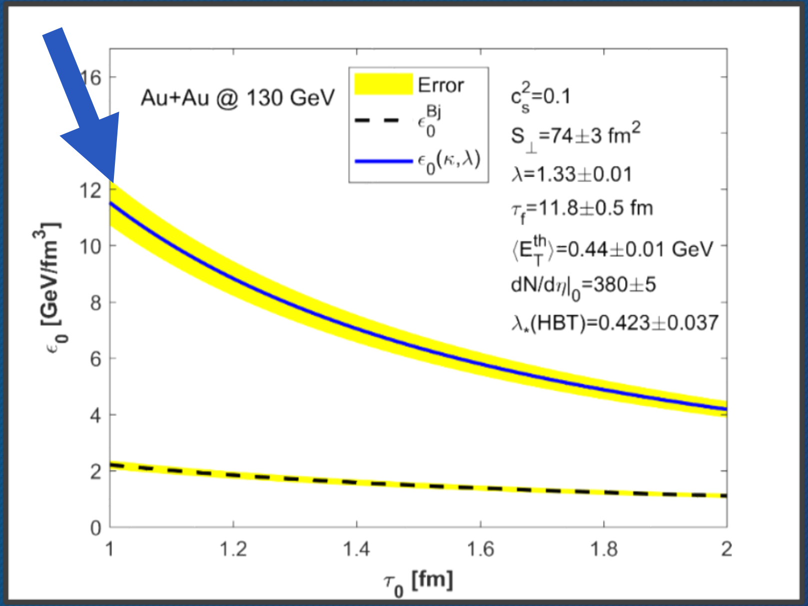

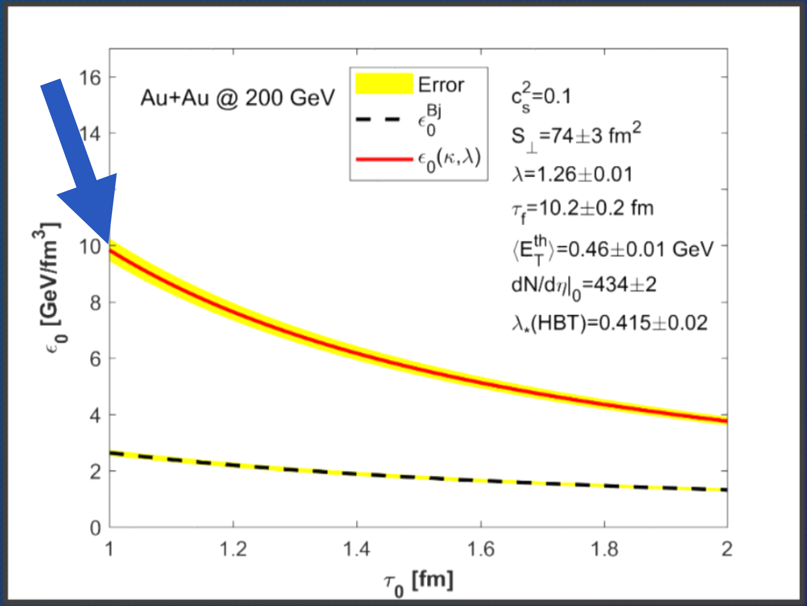

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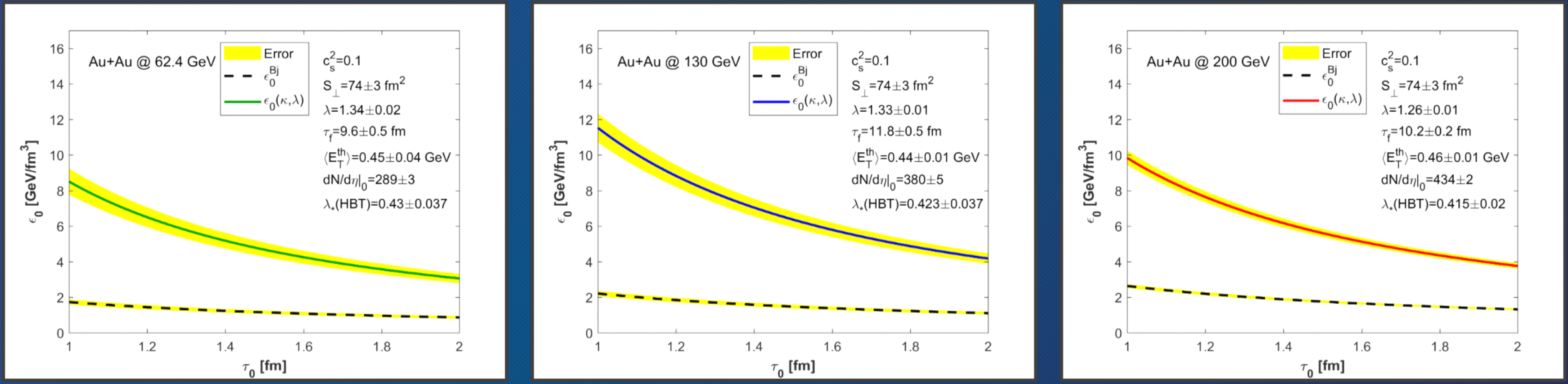

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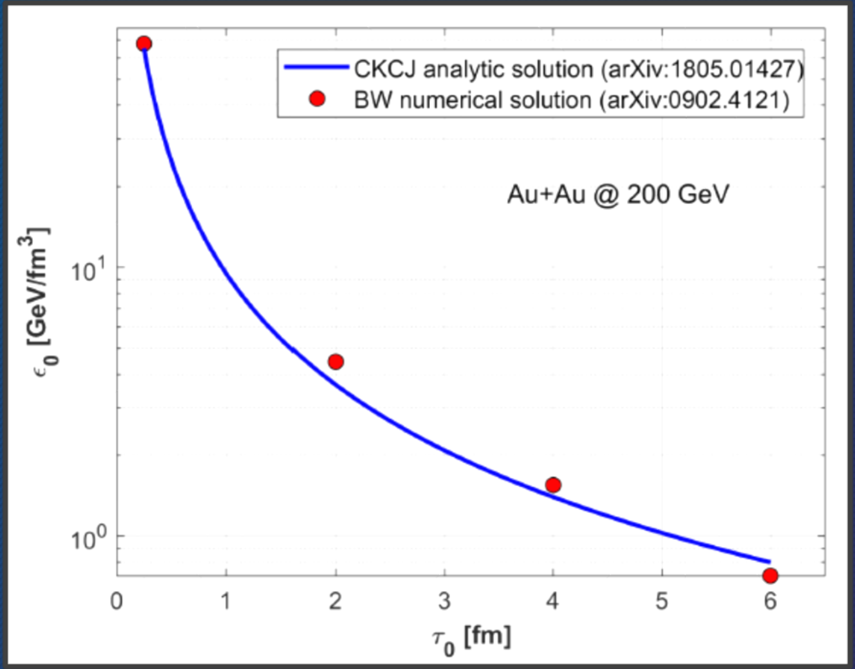
Cross-checks, initial energy density

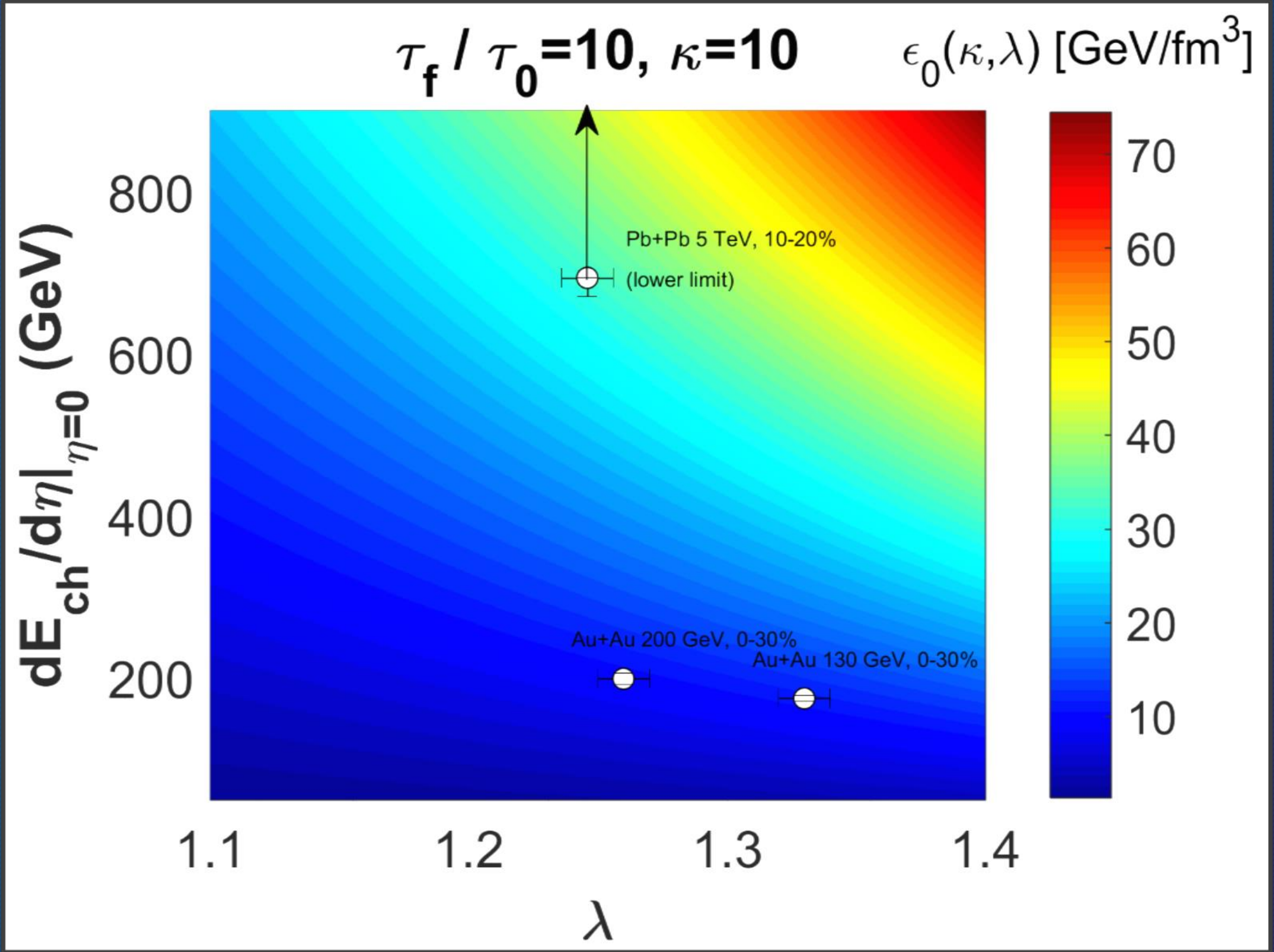
- Only the thermalized energy is included
- Indication for an increasing initial energy density with decreasing colliding energy?
- Suggest a *non-monotonic behaviour* of the initial energy density with $\sqrt{s}$
- At LHC energies, our method predicts greater ε_0
- Cross-check with 1+3 d numerical hydro, lQCD EoS:

Surprisingly good agreement in Au+Au 200 GeV!



Bivariate function: non-monotonic, with monotonic projections

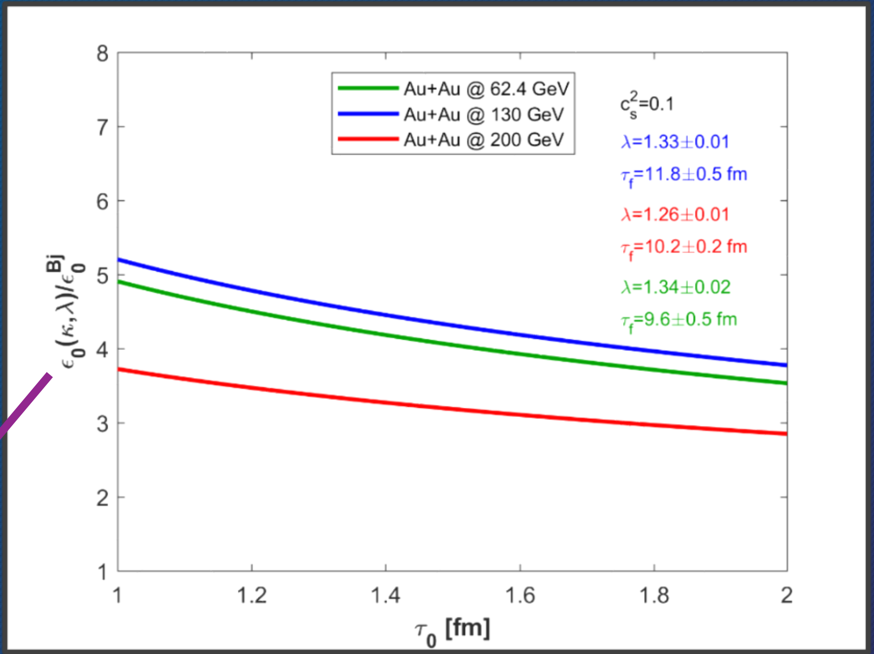
- Initial energy density



Exact result of initial energy density

- Only the thermalized energy is included
- Indication for an increasing initial energy density with decreasing colliding energy?
- Suggest a ***non-monotonic behaviour*** of the initial energy density with $\sqrt{s}$

❖ Need to extend the studies to ***lower energies*** (RHIC BES I, BES II, SPS and LHC data)
❖ Need to investigate the possibilities of ***shock waves***
❖ Need to extend the CKCJ solution to ***1+3 dimensions***
❖ Need to investigate the ***fit range iteration***



the corrections to Bjorken's initial energy estimate

Summary

- New, exact solutions of hydrodynamics: CKCJ solutions
- Powerful Gaussian formula for $dN/d\eta$ (more general than we think?)
- Initial energy density is calculated exactly

  Bjorken's estimate ignores the first law of thermodynamics
- Novel method to obtain the lifetime-parameter τ_f

  evaluation of T_{eff}, $dN/d\eta$ fits, R_L fits

  viscous hydro?
- τ_0 dependence of ε_0 can be calculated:

  surprising result: non-monotonic behaviour of $\varepsilon_0(s_{NN})$?
- Further investigations are needed:

  lower energies, extensions of CKCJ model, fit range iteration

Thank you for your attention!

Backup slide

Table 2. Initial energy density estimation from [48] by Bjorken's formula ε_0^{Bj}, and the conjectured values of $\varepsilon_0^{conj}(\kappa,\lambda)$, evaluated for $\tau_0 = 1$ fm/c . These values also indicate the non-monotonic behaviour of the initial energy density as a function of $\sqrt{s_{NN}}$, the center of mass energy of colliding nucleon pairs.

ε_0 [GeV/fm^3]	ε_0^{Bj}	$\varepsilon_0^{CNC}(\lambda)$	$\varepsilon_0^{conj}(\kappa,\lambda)$	$\varepsilon_0(\kappa,\lambda)$
Au+Au at 130 GeV, 6-15 %	4.1 ± 0.4	14.8 ± 2.2	11.2 ± 1.8	11.9 ± 0.5
Au+Au at 200 GeV, 6-15 %	4.7 ± 0.5	12.2 ± 2.3	9.9 ± 1.6	9.8 ± 0.4
Pb+Pb at 2.76 TeV, 10-20 %	10.1 ± 0.3	14.1 ± 0.5	13.3 ± 0.6	