On efforts to improve the classical particle dynamics in an external EM field

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Balaton Workshop 2019

June 20, 2019

"The magnetic gang"

Eur. Phys. J. C (2018) 78:6
https://doi.org/10.1140/epjc/s10052-017-5493-2

THE EUROPEAN PHYSICAL JOURNAL C

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Regular Article - Theoretical Physics

Relativistic dynamics of point magnetic moment

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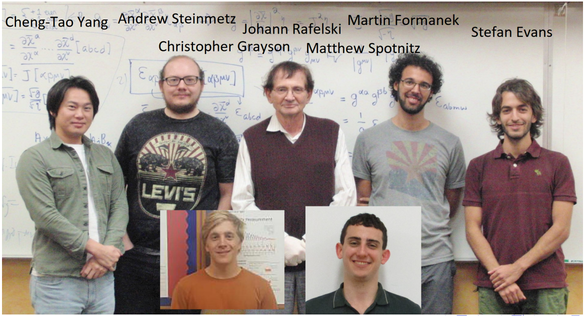
Received: 1 December 2017 / Accepted: 19 December 2017



Two models for magnetic dipole Stern-Gerlach force

All agree: magnetic potential $U = -\boldsymbol{\mu} \cdot \boldsymbol{\mathcal{B}}$

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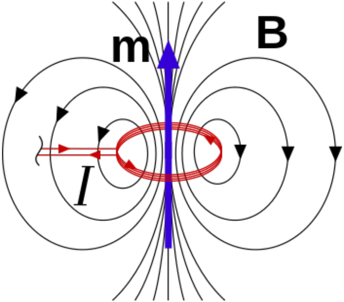
Amperian - current loop



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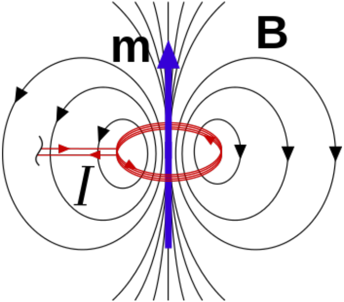

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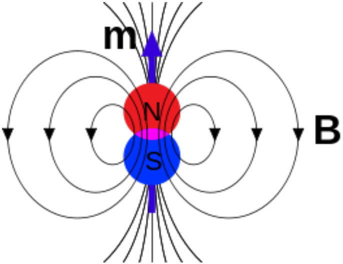

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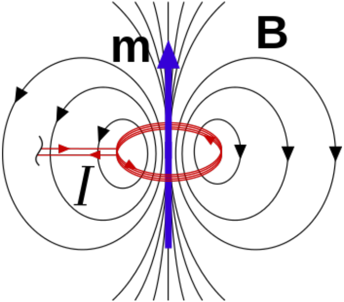

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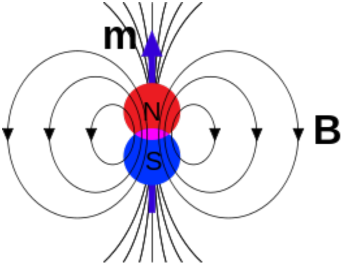

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There are no observed magnetic monopoles. Point particles have no current loops. We need a better model!

The new model should

- Apply to magnetic moment of point-spinning (classical) particle;
- Lead to one force-type only unifying Amperian and Gilbertian forms as equivalent;
- Be consistent in form with torque and spin dynamics:

Definition of torque:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{\mathcal{B}}$$

- We want forces to be in covariant relativistic format, that is we seek an extension of the 'Lorentz-Force'

Lorentz Force: EM-Fields $F^{\mu\nu}$, 4-velocity u_ν

$$\frac{du^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} u_\nu,$$

Spin dynamics in textbooks: Thomas-Bergmann-Michel-Telegdi

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Covariant model of spin dynamics

$$\frac{du^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} u_\nu,$$
$$\frac{ds^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} s_\nu + a\frac{e}{m}\left(F^{\mu\nu} s_\nu - \frac{u^\mu}{c^2} u \cdot F \cdot s\right).$$

where a is the $g \neq 2$ anomaly

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- Best way to see the problem: we need a force term valid for neutral particles to account for the Stern-Gerlach force and a torque equation that agrees with form of force.

What is s^ν: classical Spin of point particle

Non-rotating 'spin' natural in quantum Dirac equation; this doesn't mean that spin is a quantum property! Spin arises in the context of Minkowski space-time symmetry transformations: **Poincaré group.** There are two **Casimir operators** commuting with all 10 symmetry generators

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$$\bar{C}_1 \equiv \bar{p}_\mu \bar{p}^\mu; \quad C_1 = m^2 c^2$$

$$\bar{C}_2 \equiv \bar{w}_\mu \bar{w}^\mu; \quad \bar{s}^\mu \equiv \frac{\bar{w}^\mu}{\sqrt{C_1}}$$

$\bar{w}^\mu$ is axial Pauli-Lubanski 4-vector made out of generators of rotations $\bar{\boldsymbol{J}}$ and boosts $\bar{\boldsymbol{K}}$

$$\bar{w}_\mu = \bar{M}^*_{\mu\nu}\bar{p}^\nu, \quad \bar{M}^*_{\mu\nu} = \begin{pmatrix} 0 & -\bar{J}_1 & -\bar{J}_2 & -\bar{J}_3 \\ \bar{J}_1 & 0 & -\bar{K}_3 & \bar{K}_2 \\ \bar{J}_2 & \bar{K}_3 & 0 & -\bar{K}_1 \\ \bar{J}_3 & -\bar{K}_2 & \bar{K}_1 & 0 \end{pmatrix} \Rightarrow \boxed{\bar{u}_\mu \bar{s}^\mu = \frac{\bar{p}_\mu}{m}\bar{s}^\mu = 0}$$

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Any and each point particle belongs to an irreducible representation of the Poincare group described by the eigenvalues C_1 and C_2 of the Casimir operators. $\sqrt{C_1}$ relates to mass and $\sqrt{C_2/C_1} \equiv |s^\mu|$ to spin.

Relativistic 'magnetic potential'

Analogical to electric energy $E_{\text{el}} = eV = ecA^0$. Since $E_{\text{mag}} = -\boldsymbol{\mu} \cdot \boldsymbol{\mathcal{B}}$ In the rest frame of the particle

Need magnetic 'charge' d

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We look at a magnetic 4-potential B^μ akin to e-4-potential A^μ

$$B_\mu \equiv F^*_{\mu\nu} s^\nu, \quad F^*_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}, \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$$

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B^μ generates additional magnetic force

$$m\frac{du^\mu}{d\tau} \equiv F^\mu_{\text{ASG}} = (eF^{\mu\nu} + G^{\mu\nu} d)u_\nu, \quad G^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu.$$

Covariant Amperian and Gilbertian Stern-Gerlach force

The magnetic force will be now identified to be the Amperian form:

ASG force and the rest frame of a particle

$$F^\mu_{\mathrm{ASG}} = eF^{\mu\nu}u_\nu - u\cdot\partial\, F^{\star\,\mu\nu}s_\nu\, d + \partial^\mu(u\cdot\, F^\star\cdot s\, d)$$

$$F^\mu_{\mathrm{ASG}}|_{\mathrm{RF}} = \left\{0,\ e\boldsymbol{\mathcal{E}} + \boldsymbol{\nabla}(\boldsymbol{\mu}\cdot\boldsymbol{\mathcal{B}}) - \frac{1}{c^2}\,\boldsymbol{\mu}\times\frac{\partial\boldsymbol{\mathcal{E}}}{\partial t}\right\}$$

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Another approach that allows us to find the Gilbertian force:

We try to modify the fields

$$eF^{\mu\nu} \to \boxed{\widetilde{F}^{\mu\nu} = eF^{\mu\nu} - s\cdot\partial\, F^{\star\,\mu\nu}\, d}\ ,$$

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$$F^{\mu}_{\text{ASG}} = F^{\mu}_{\text{GSG}} = (eF^{\mu\nu} - s\cdot\partial\, F^{\star\,\mu\nu}\,d)\,u_{\nu} - \mu_0 j^{\gamma}\epsilon_{\gamma\alpha\beta\nu}u^{\alpha}s^{\beta}g^{\nu\mu}\,d$$

$$F^{\mu}_{\text{GSG}}|_{\text{RF}} = \{0,\ e\boldsymbol{\mathcal{E}} + (\boldsymbol{\mu}\cdot\boldsymbol{\nabla})\boldsymbol{\mathcal{B}} + \mu_0\boldsymbol{\mu}\times\boldsymbol{j}\}$$

Equivalence of point particle magnetic moment forces

Based on this we can write two equivalent generalizations of the Lorentz force

ASG, GSG: two ways to write one and the same thing

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In rest frame

$$[\boldsymbol{F}_{\rm ASG} - \boldsymbol{F}_{\rm GSG}]_{\rm RF} = \boldsymbol{\mu}\times\left(-\frac{1}{c^2}\frac{\partial\boldsymbol{E}}{\partial t} + \boldsymbol{\nabla}\times\boldsymbol{B} - \mu_0\boldsymbol{j}\right) = 0\,.$$

We recognize Maxwell equation in parenthesis

J. S. Schwinger, *"Spin precession: A dynamical discussion"*, American Journal of Physics **42**, (1974) 510,

Schwinger shows how the TMBT spin dynamics relates to EM force: given $u \cdot s = 0$ he takes proper time τ derivative $\dot{u} \cdot s + u \cdot \dot{s} = 0$ and substituting force for $\dot{u}$ for the case of Lorentz dynamics he argues:

$$u_\mu \left(\frac{ds^\mu}{d\tau} - \frac{e}{m} F^{\mu\nu} s_\nu \right) = 0 \; .$$

How the modified force generates new spin dynamics (torque)

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The general solution satisfying this equation is

$$\frac{ds^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} s_\nu + \widetilde{a} \left(g^{\mu\rho} - \frac{u^\mu u^\rho}{c^2} \right) F_{\rho\nu} s^\nu$$

We repeat the same for our generalized Lorentz force.

Covariant dynamical equations

From now on we use the Gilbertian form of the Lorentz force F^{μ}_{GSG} in vacuum $j^{\mu} = 0$.
The dynamical 'Schwinger' spin equation is obtained as described above

Coupled covariant motion of particle 4-velocity u^{μ} and spin s^{μ}

$$\frac{du^{\mu}}{d\tau} = \frac{1}{m}(eF^{\mu\nu} - s \cdot \partial F^{*\mu\nu} d)u_{\nu}$$

$$\frac{ds^{\mu}}{d\tau} = \frac{e}{m}F^{\mu\nu}s_{\nu} - \frac{d}{m}s \cdot \partial F^{*\mu\nu}s_{\nu} + \widetilde{a}\left(g^{\mu\rho} - \frac{u^{\mu}u^{\rho}}{c^2}\right)F_{\rho\nu}s^{\nu}$$

- Reduces to TBMT equations for $d = 0$ with $\widetilde{a} \to a$
- $dc = e/m + \widetilde{a}$
- Dynamics of a neutral particle depends only on d

Dynamical equations for neutral particles

For a neutral particle $e = 0$ and $d \neq 0$ the equations of motion reduce to

$$\frac{du^\mu}{d\tau} = -\, s \cdot \partial F^{*\mu\nu} u_\nu \, \frac{d}{m} \; ,$$

$$\frac{ds^\mu}{d\tau} = -\, s \cdot \partial F^{*\mu\nu} s_\nu \, \frac{d}{m} + cd \left(g^{\mu\nu} - \frac{u^\mu u^\nu}{c^2} \right) F_{\nu\rho} s^\rho \; .$$

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In the instantaneous co-moving frame

$$\frac{d}{dt}\boldsymbol{v} = \frac{1}{m}(\boldsymbol{\mu} \cdot \nabla)\boldsymbol{\mathcal{B}} \ ,$$
$$\frac{d}{dt}\boldsymbol{s} = \boldsymbol{\mu} \times \left(\boldsymbol{\mathcal{B}} - \frac{1}{mc}(\boldsymbol{s} \cdot \nabla)\frac{\boldsymbol{\mathcal{E}}}{c} \right) \ .$$

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Non-uniqueness

$$\frac{ds^\mu}{d\tau} = \ldots + b \left(g^{\mu\nu} - \frac{u^\mu u^\nu}{c^2} \right) T_{\nu\rho} s^\rho \quad \text{for example} \quad T_{\mu\nu} = (s \cdot \partial) F^{*}_{\mu\nu}$$

Neutral particle in an external plane wave (laser) field

Or: is it possible using lasers to guide neutral particles?

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Plasma Phys. Control. Fusion **00** (2019) 000000 (9pp)

Classical neutral point particle in linearly polarized EM plane wave field

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Plane wave field with profile function f has the 4-potential

$$A^\mu(\xi) = \mathcal{A}_0 \varepsilon^\mu f(\xi), \quad \xi = \frac{\omega}{c}\hat{k}\cdot x, \quad \hat{k}\cdot\varepsilon = 0, \quad \hat{k}^2 = 0\,.$$

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Integrals of motion:

$$\hat{k}\cdot u(\tau) = \hat{k}\cdot u(0), \quad \varepsilon\cdot u(\tau) = \varepsilon\cdot u(0)\,.$$

$\hat{k}\cdot u(0) = \gamma_0 c(1 - \boldsymbol{\beta_0}\cdot\hat{\boldsymbol{k}})$, a fancy way to write the initial Doppler factor.

Neutral particle in an external plane wave (laser) field

Squaring the generalized Lorentz force equation gives us a formula for invariant acceleration

$$\dot{u}^2(\tau) = -(\hat{k}\cdot s(\tau))^2(\hat{k}\cdot u(0))^2(f''(\xi(\tau)))^2\frac{\mathcal{A}_0^2 d^2\omega^4}{m^2c^4}, \quad f'(\xi) \equiv \frac{df}{d\xi}\,.$$

Particle acceleration depends on initial Doppler shifted laser frequency it sees and on alignment of the spin and the wave vector.

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Solution of the dynamical equations

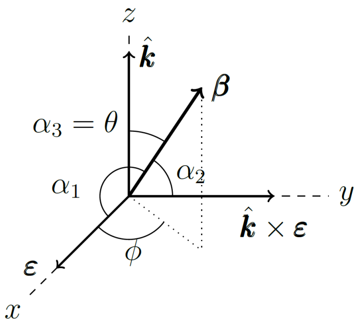
$$\hat{k}\cdot s(\tau) = \hat{k}\cdot s(0)\cos\left[\mathcal{A}_0 d(f(\xi(\tau)) - f(\xi(\tau_0)))\right] - \frac{W(0)}{c}\sin\left[\mathcal{A}_0 d(f(\xi(\tau)) - f(\xi(\tau_0)))\right]$$

$$W(0) = (\hat{k}\cdot u(0))(\varepsilon\cdot s(0)) - (\hat{k}\cdot s(0))(\varepsilon\cdot u(0))$$

$$u^\mu(\tau) = u^\mu(0) + \frac{1}{2}h^2(\tau)\hat{k}^\mu(k\cdot u(0)) + h(\tau)\epsilon^{\mu\nu\alpha\beta}u_\nu(0)\hat{k}_\alpha\varepsilon_\beta$$

$$h(\tau) = \frac{\mathcal{A}_0 d\omega^2}{mc^2}\int_{\tau_0}^{\tau}(\hat{k}\cdot s(\tilde{\tau}))f''(\xi(\tilde{\tau}))d\tilde{\tau}$$

Geometry of the problem



Initial laboratory frame quantities

$$\hat{k}^\mu = (1, \hat{\boldsymbol{k}})$$
$$\varepsilon^\mu = (0, \boldsymbol{\varepsilon})$$
$$u^\mu(0) = \gamma_0 c(1, \boldsymbol{\beta}_0)$$
$$s^\mu(0) = (\boldsymbol{\beta}_0 \cdot \boldsymbol{s}_{0L}, \boldsymbol{s}_{0L})$$

where $s^\mu(0)$ is Lorentz boosted $(0, \boldsymbol{s}_0)$.

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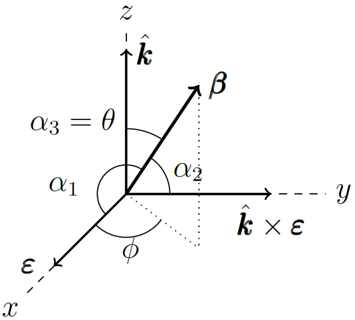

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$$s^\mu(0) = (\boldsymbol{\beta}_0 \cdot \boldsymbol{s}_{0L}, \boldsymbol{s}_{0L})$$

where $s^\mu(0)$ is Lorentz boosted $(0, \boldsymbol{s}_0)$.

Conserved quantity $\hat{k} \cdot u(\tau) = \gamma c(1 - \hat{k} \cdot \boldsymbol{\beta}) =$ const means that particle can lower it's velocity by increasing the angle θ and vice versa. The mechanism is the spin interaction!

$$\cos\theta(\tau) = \frac{G(\tau) + \beta_0 \cos\theta_0}{\sqrt{\beta_0^2 + G^2(\tau) + 2G(\tau)}}, \quad \beta^2(\tau) = 1 - \frac{1 - \beta_0^2}{(1 + G(\tau))^2},$$

$$G(\tau) = \frac{1}{2} h^2(\tau)(1 - \hat{\boldsymbol{k}} \cdot \boldsymbol{\beta}_0) + h(\tau)\boldsymbol{\beta}_0 \cdot (\hat{\boldsymbol{k}} \times \boldsymbol{\varepsilon})$$

Accelerating relativistic neutrinos

Neutrinos don't have any electric charge but they can have magnetic moment (Dirac vs Majorana neutrinos). The current range for the Dirac neutrino magnetic moment is

$$\mu_\nu = 10^{-11} - 10^{-20} \mu_B$$

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The square root of invariant acceleration can be estimated in the units of critical acceleration $a_c = m_\nu c^3/\hbar$ and using normalized dimensionless laser amplitude $\hat{a}_0 = \frac{ea_0}{m_e c}$ as

$$\sqrt{\dot{u}^2}[a_c] \propto \hat{a}_0 f''(\xi)\frac{E_\nu[eV](E_\gamma[eV])^2}{(m_\nu[eV])^3}\mu_\nu[\mu_B] \propto \hat{a}_0 f''(\xi)(10^2 - 10^{-8})$$

for 20 GeV neutrino (produced at CERN) with mass .2 eV, and 1 eV laser source. So in order to reach critical acceleration of unity we need laser parameters in range

$$\hat{a}_0 f''(\xi) \in (10^{-2} - 10^8)$$

Accelerating relativistic neutrinos

- State of the art laser systems $\hat{a}_0 \propto 10^2$
- Classical regime $\lambda_\gamma/\lambda_\nu \sim 10^{11}$
- How high can be $f''(\xi)$?
- Would be useful for determining neutrino properties - magnetic moment, mass.
- the product controlling the precession:

$$\mathcal{A}_0 d \approx 10^{-9} - 10^{-18}$$

  no precession for neutrinos.
- The function controlling the direction of the beam $G(\tau)$

$$G(\tau) \approx (10^{-8} - 10^{-17}) f'(\xi(\tau))$$

- Neutrons have much higher magnetic moment - spin precession happens, but they are much heavier - harder to change the trajectory.

Quantum considerations

Motivation: Comparing the relativistic quantum mechanics models in order to find the best one which could serve as a basis for our classical model.

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Eur. Phys. J. A (2019) **55**: 40
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THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Magnetic dipole moment in relativistic quantum mechanics

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Two often-used models

DP: $$\left(\gamma^\mu(i\hbar c\partial_\mu - eA_\mu) - mc^2 - a\frac{e\hbar}{4mc}\sigma_{\mu\nu}F^{\mu\nu}\right)\psi = 0$$

KGP: $$\left((i\hbar c\partial_\mu - eA_\mu)^2 - m^2c^4 - \frac{g}{2}\frac{e\hbar c}{2}\sigma_{\mu\nu}F^{\mu\nu}\right)\Psi = 0$$

Test system: Homogeneous magnetic field

Landau levels in the $\boldsymbol{B} = B\hat{z}$ field

$$\text{DP:} \quad E_n = \pm\sqrt{\left(\sqrt{m^2c^4 + 2e\hbar cB\left(n + \frac{1}{2} - s\right)} - \frac{eB\hbar}{2mc}(g-2)s\right)^2 + p_z^2c^2}$$

$$\text{KGP:} \quad E_n = \pm\sqrt{m^2c^4 + p_z^2c^2 + 2e\hbar cB\left(n + \frac{1}{2} - \frac{g}{2}s\right)}$$

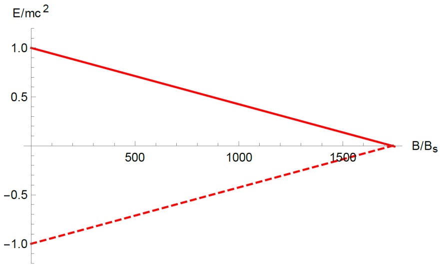
Notice that DP energy levels explicitly dependend on the anomaly $g-2$. Both reduce have the correct non-relativistic limit for particle states

$$E_n - mc^2 = \frac{p_z^2}{2m} + \frac{e\hbar B}{mc}\left(n + \frac{1}{2} - \frac{g}{2}s\right) \ ,$$

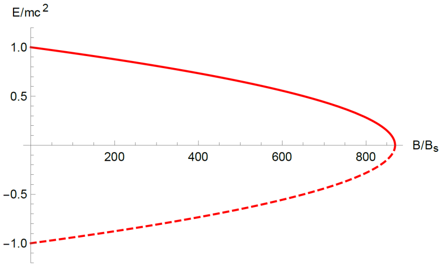
but their high field behavior is different!

High field behavior

Ground state of electron with $n = 0$, spin $s = 1/2$, no momentum in the field direction $p_z = 0$. Magnetic anomaly $g/2 - 1 = \alpha/2\pi$



Dirac Pauli (DP)



Klein Gordon Pauli (KGP)

Merging of the particle (solid) and antiparticle (dashed) states at different values of magnetic fields! Plotted in units of Schwinger critical field

$$B_s = \frac{m_e^2 c^2}{e\hbar} = 4.4141 \times 10^9 \text{ T} .$$

The Coulomb problem

KGP can be solved analytically (Niederle & Nikitin, 2006), DP only numerically (Thaller, 1992). In the non-relativistic limit we can compare the transition lines difference proportional to α^6.

- Lamb shift $j = $ const, l changes (line has 4.4×10^{-6} eV)

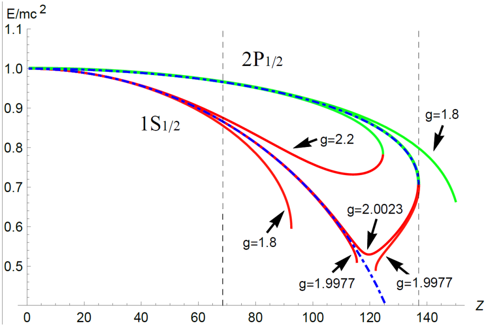
$\Delta E_{\text{KGP}}^{2S_{1/2}-2P_{1/2}}$	$\Delta E_{\text{DP}}^{2S_{1/2}-2P_{1/2}}$	Difference
$(a + a^2/2)\frac{Z^4\alpha^4}{6}$	$a\frac{Z^4\alpha^4}{6}$	$\frac{\alpha^6 mc^2}{48\pi^2} = 1.6 \times 10^{-10}$ eV

- Fine structure $l = $ const, j changes (line has 4.5×10^{-5} eV)

$\Delta E_{\text{KGP}}^{2P_{3/2}-2P_{1/2}}$	$\Delta E_{\text{DP}}^{2P_{3/2}-2P_{1/2}}$	Difference
$(1/2 + a + a^2/2)\frac{Z^4\alpha^4}{16}$	$(1/2 + a)\frac{Z^4\alpha^4}{16}$	$\frac{\alpha^6 mc^2}{128\pi^2} = 6.1 \times 10^{-11}$ eV

Both discrepancies get more enhanced in muonic hydrogen or proton - antiproton systems because of the higher mass.

High Z behavior for KGP



- $|g| < 2$ singularity similar to DP
- $|g| > 2$ joining of particle states with the same j and opposite spin. These states are problematic even in the DP numerical solutions, but they cross instead of merging.
- $g = 2$ a cusp point - character of the solution strongly modified even for a small anomaly.

Future developments

- Correspondence principle between relativistic quantum theory and our classical formulation for a suitable RQM model.
- Explore other RQM models - for example Improved KGP from Andrew's paper.
- RQM explanation of the extra gradient terms in the spin dynamics, inclusion of others if necessary.
- Is there a physics principle constraining dynamics of spin? (Establishing uniquely the torque dynamical equation).
- Extension of Pauli Lubanski definition of spin to be consistent with SU(2) symmetry of space. (Now it is R^3).
- Developping theoretical framework for experiments with neutrinos and neutrons.
- Classical charged particle behavior

Acknowledgments

My sincere thanks go to:

- Prof. Rafelski and our group members
- Fanfare and GPSC travel awards for offsetting cost of flight tickets
- Everyone at the Wigner institute for their hospitality

Thank you for your attention!

Any questions?