QED vacuum structure in quasi constant extreme EM fields

Balaton Workshop
June 20, 2019

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Physics

WIGNER

Euler-Heisenberg effective action

Euler-Heisenberg (EH) effective potential as a modification to the Maxwell Lagrangian:

$$\mathcal{L}_{\mathrm{EH}}(\mathcal{E},\mathcal{B}) = S + \frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-i(m^2-i\epsilon)s} \times \left(\frac{(eas)\cosh[eas](ebs)\cos[ebs]}{\sinh[eas]\sin[ebs]} - 1 - e^2 s^2 \frac{a^2-b^2}{3} \right) ,$$

$$a^2 - b^2 = \mathcal{E}^2 - \mathcal{B}^2 = 2S \,, \qquad a^2 b^2 = (\mathcal{E}\cdot\mathcal{B})^2 = P^2 \,.$$

$$\mathcal{E}_{EH} = \frac{m^2c^2}{e\hbar} = 1.32 \cdot 10^{18} \frac{\mathrm{V}}{\mathrm{m}} = 4.41 \cdot 10^9 c\mathrm{T} \,.$$

This computation precedes renormalization procedure and goes beyond capability of perturbative QED.

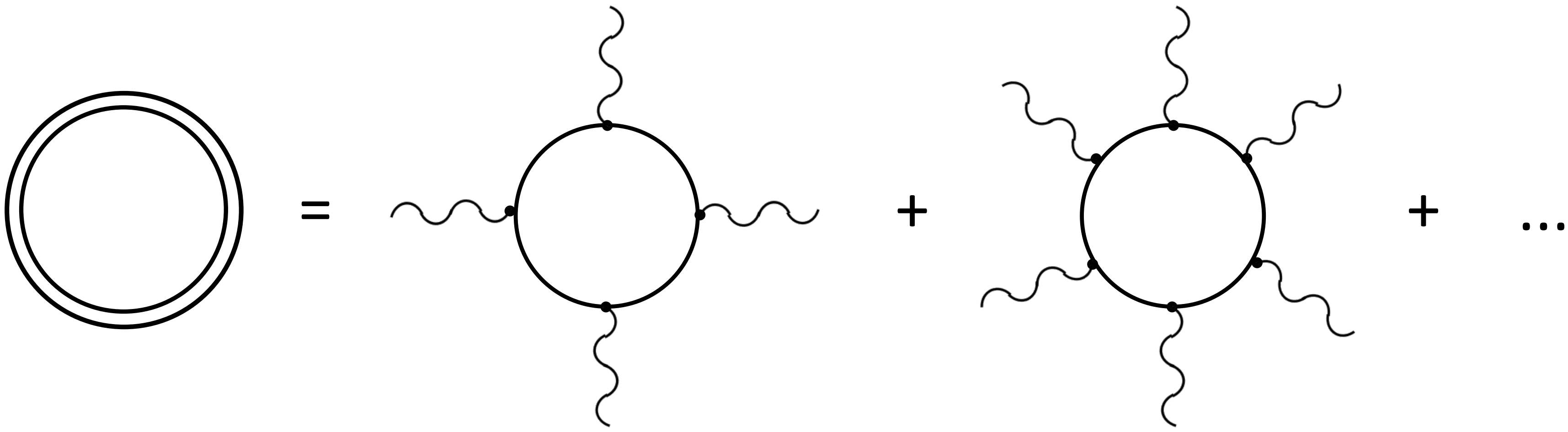
- W. Heisenberg and H. Euler, Zeitschrift für Physik **98**, 714 (1936).
- V. Weisskopf, Kong. Dan. Vid. Sel. Mat. Fys. Med. **14**, N6, 1 (1936).

Vacuum Non-Persistence

$$\boxed{\langle 0(t)|0(t+T)\rangle = e^{i(W^{(0)}+W^{(1)})}} \quad W^{(1)} = -\int d^4x \mathcal{L}^{(1)}(x).$$

Schwinger in 1951 re-derives EH result: imaginary part of effective potential causes decay of the vacuum state.

Association of this effective action with diagrams in perturbative QED is only permissible in weak field semi-convergent expansion: the contribution by infinite IR photons cannot be reproduced by QED treatment.



- J. Schwinger, Phys. Rev. **82**, 664 (1951).

Possibility of probing EH action after 80 years

While the laser intensity frontier draws closer to critical value at ELI (10^{24} W/cm^2 vs Euler Heisenberg 10^{29} W/cm^2), we comment on currently available probes of nonlinear vacuum:

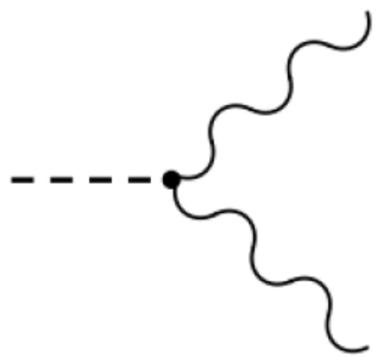
Laboratory experiment: magnetic field induced vacuum birefringence at PVLAS and BMV (~2.5 T<<EH but finesse ~10^6): search for signals which may be attributed to fields studied outside domain of QED, requiring addenda to EH action.

Magnetized neutron stars suggested to contain EM fields on order 10^2-10^4 EH critical field, and even greater in heavy ion collisions, strong enough to probe radiative corrections to EH action.

Magnetic Vacuum Birefringence

$$\mathcal{L}_{int} = \pm G_{\mathrm{A}} \varphi P \ ,$$

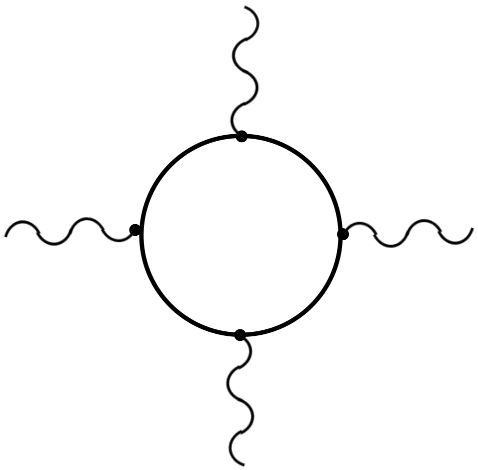
Most experiments search for two-photon-decay (real, free-streaming):



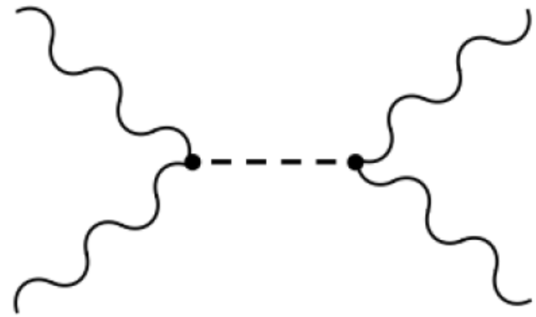
However PVLAS measurement of magnetic birefringence places constraints on ALP coupling and mass via virtual excitation effects, prompting a closer look to the 4-photon diagram supplement to EH action:

Magnetic Vacuum Birefringence

How to compare contributions



vs



$$\mathcal{L}_{EHS}^{(1)} = \frac{e^4}{(4\pi)^2}\frac{2}{45m_e^4}\left(4S^2 + 7P^2\right),$$

Objective to derive effective ALP action valid for external fields slowly varying over ALP Compton wavelength.

Magnetic Vacuum Birefringence

Total action

$$\mathcal{L}^{(1)} = S - \frac{m_{\mathrm{A}}^2}{2}\varphi^2 + G_{\mathrm{A}}\varphi P + \frac{e^4}{(4\pi)^2}\frac{2}{45m_e^4}\left(4S^2 + 7P^2\right).$$

The ALP contribution is configuration mixed with EM part

$$\tilde{\varphi} = \varphi - G_{\mathrm{A}}\frac{P}{m_{\mathrm{A}}^2}$$

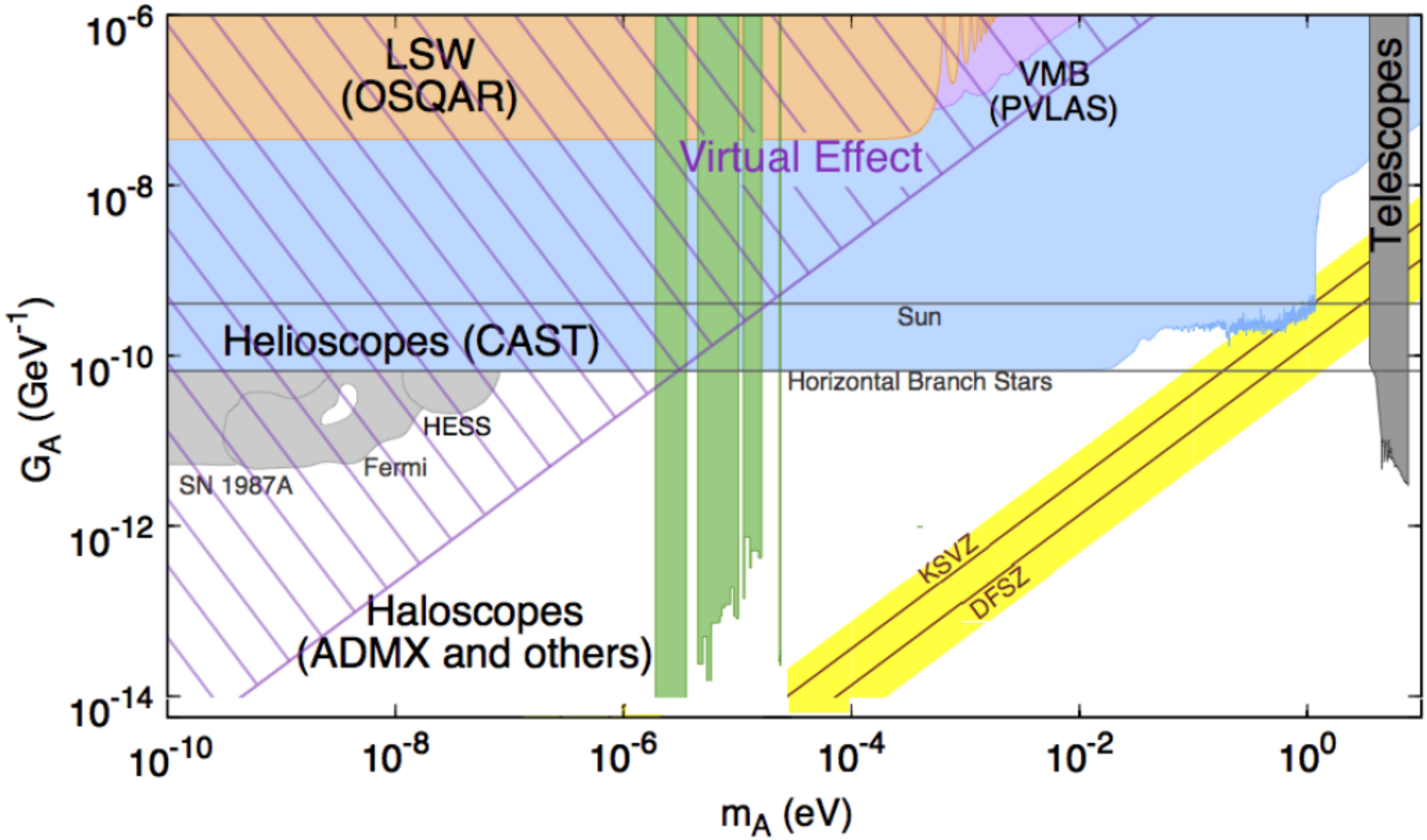
Giving supplement to EH P^2 coefficient:

$$\begin{aligned} -\frac{m_{\mathrm{A}}^2}{2}\varphi^2 + G_{\mathrm{A}}\varphi P &= -\frac{m_{\mathrm{A}}^2}{2}\left(\varphi - G_{\mathrm{A}}\frac{P}{m_{\mathrm{A}}^2}\right)^2 + \frac{G_{\mathrm{A}}^2}{2m_{\mathrm{A}}^2}P^2 \\ &= -\frac{m_{\mathrm{A}}^2}{2}\tilde{\varphi}^2 + \frac{G_{\mathrm{A}}^2}{2m_{\mathrm{A}}^2}P^2, \end{aligned}$$

$$P^2 \to P^2\left(1 + \frac{45m_e^4}{28\alpha^2}\frac{G_{\mathrm{A}}^2}{m_{\mathrm{A}}^2}\right)$$

Magnetic Vacuum Birefringence

Regime of experimental constraints:

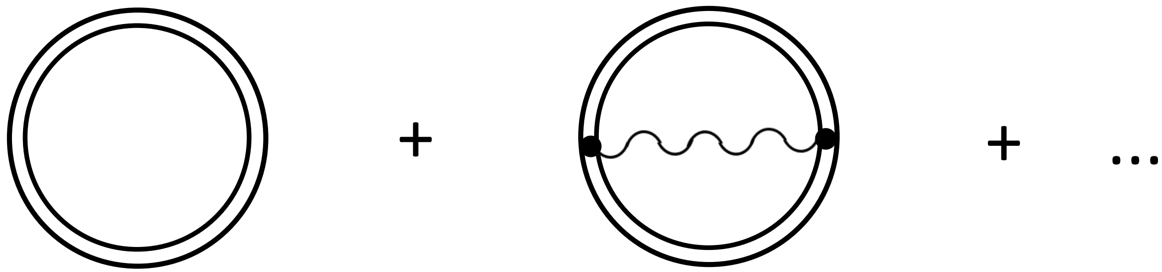


Adapted fig. 111.1 in: A. Ringwald, L.J Rosenberg and G. Rybka, "Axions and other similar particle," Chapter 111 in: M. Tanabashi *et al.* [Particle Data Group], "Review of Particle Physics," Phys. Rev. D **98** (2018) no.3, 030001.

Toward stronger EM fields…

Radiative corrections

1st order alpha corrections include internal photon line diagram



This correcting diagram contains electron propagator in external field modified by self-energy, producing in weak-field limit anomalous electron spin g-factor of $g=2+\alpha/\pi$, but in strong external fields, finite field-dependent self-energy corrections arise in both mass and g.

The problem is how to include AMM into relativistic QM: KGP vs DP? We choose KGP as used in EH, Weisskopf and Schwinger, due to inability to sum over Landau levels and plug into EH action with DP method (without Landau level-dependent AMM).

Radiative corrections via AMM

Prior to Casimir energy computation of shift in vacuum energy in presence of two parallel conducting plates, Weisskopf computed EH action via shift in vacuum energy in presence of a constant homogeneous magnetic field.

'Squared' Dirac equation (multiplying Dirac operator with opposite mass sign): the Klein-Gordon-Pauli (KGP) equation:

$$\left((i\partial_\mu - eA_\mu)^2 - m^2 - \frac{g}{2}\frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\Psi = 0\ ,$$

$$\frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu} = i\gamma^5\vec{\sigma}\cdot\mathcal{E} - \vec{\sigma}\cdot\vec{\mathcal{B}}\ , \quad \vec{\sigma} = \gamma^5\gamma^0\vec{\gamma}.$$

giving Landau energy levels, with sum periodic in *g*

$$\boxed{E_n^{\pm}(\mathcal{B}) = \pm\sqrt{p_x^2 + m^2 + 2e\mathcal{B}\left(n + \frac{1}{2} - \frac{g}{2}s\right)}\ ,}$$

$$\vec{\mathcal{B}} = \mathcal{B}\hat{x}, \quad s = \pm 1/2.$$

- Lance Labun and Johann Rafelski. arXiv 1205.1835
- Andrew Steinmetz, Martin Formanek, Johann Rafelski. Eur.Phys.J A55 (2019) no. 3, 40

Radiative corrections via AMM

EH action for periodically reset *g*:

$$\mathcal{L}_{\rm EH} = \frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-im^2 s} \times \left(\frac{eas\cosh[\frac{g_k-4k}{2}eas]}{\sinh[eas]} \frac{ebs\cos[\frac{g_k-4k}{2}ebs]}{\sin[ebs]} - 1 \right)$$

$$g = g_k - 4k \ , \qquad -2+4k < g_k < 2+4k \ ,$$

$$-1 \to -1 + (a^2 - b^2)(g^2/8 - 1/6).$$

Self-adjoint only in domain

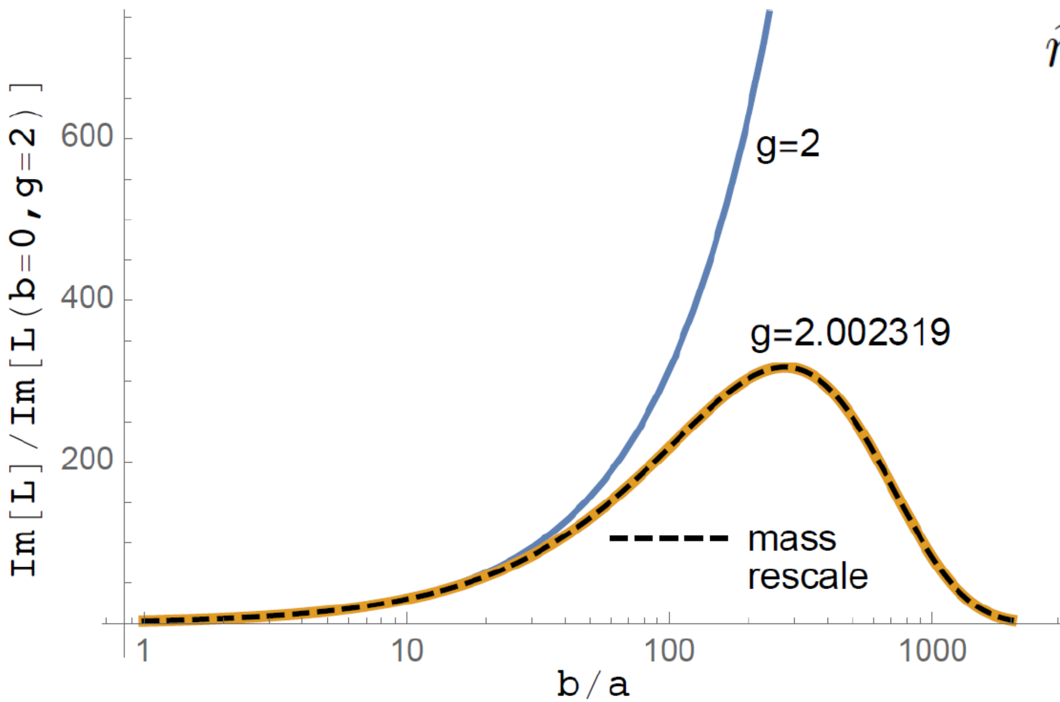
$$|e\mathcal{B}| < \frac{m^2}{|g/2-1|} \sim 862 m^2 \ , \qquad g = -2{+}0.002319$$

Radiative corrections via AMM

Imaginary part of action suppressed in strong magnetic fields due to magnetic moment:

$$\Im[\mathcal{L}_{\rm EH}] \sim \frac{(ea)(eb)}{8\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cos[\frac{g}{2}n\pi] e^{-n\pi\widetilde{m}^2/ea} \;,$$

$$\widetilde{m}^2 = m^2 + \left|\frac{|g_k - 4k|}{2} - 1\right| eb$$

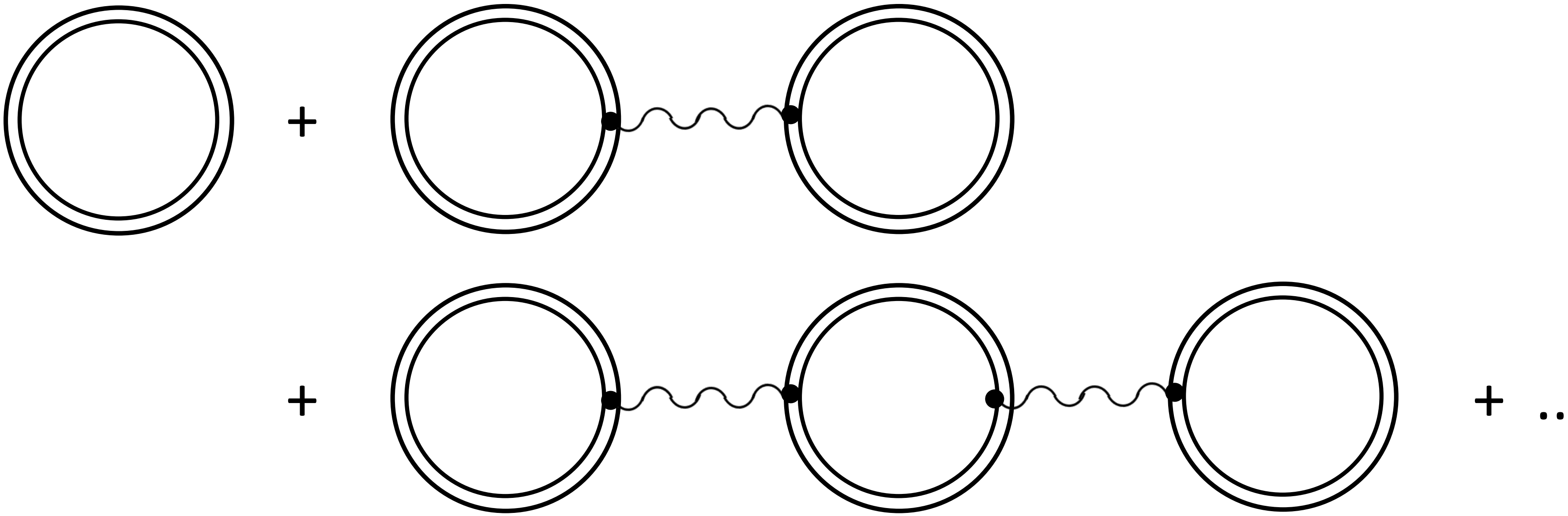


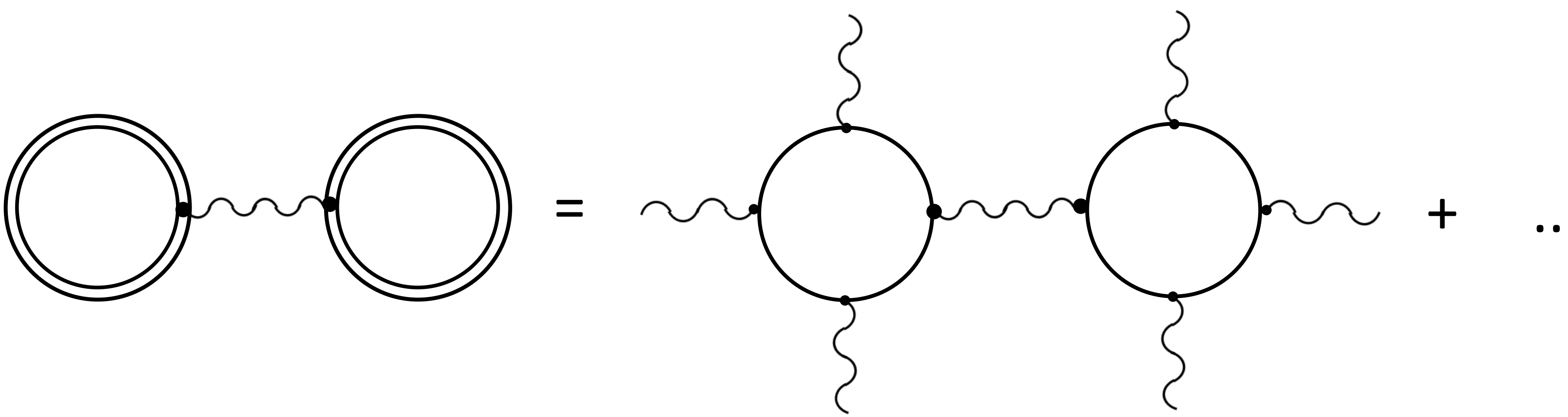
Generalization possible for:

$$g \rightarrow g_f(a,b),\; m \rightarrow m_f(a,b)$$

Stefan Evans, Johann Rafelski Phys.Rev. D98 (2018) no.1, 016006

Additional diagrams in extreme fields



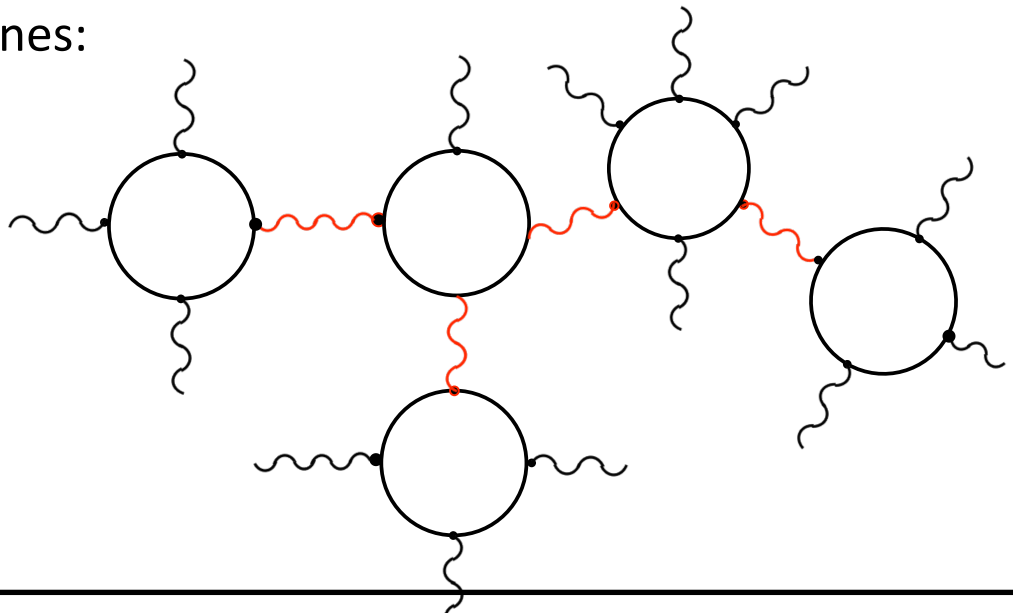


Connecting photon lines

In addition to internal photon lines appearing in diagrams, we have photons connecting diagrams: modification suggested by Weisskopf in 1936:

"one can by no means separate the external field from the field that is created by the vacuum electrons themselves, such that the field that enters into the calculations implicitly partially includes the action of other vacuum electrons."

We distinguish perturbative in external field from perturbative in connecting photon lines:



- Holger Gies and Felix Karbstein, JHEP 1703 (2017) 108
- Felix Karbstein, arXiv:1903.06998

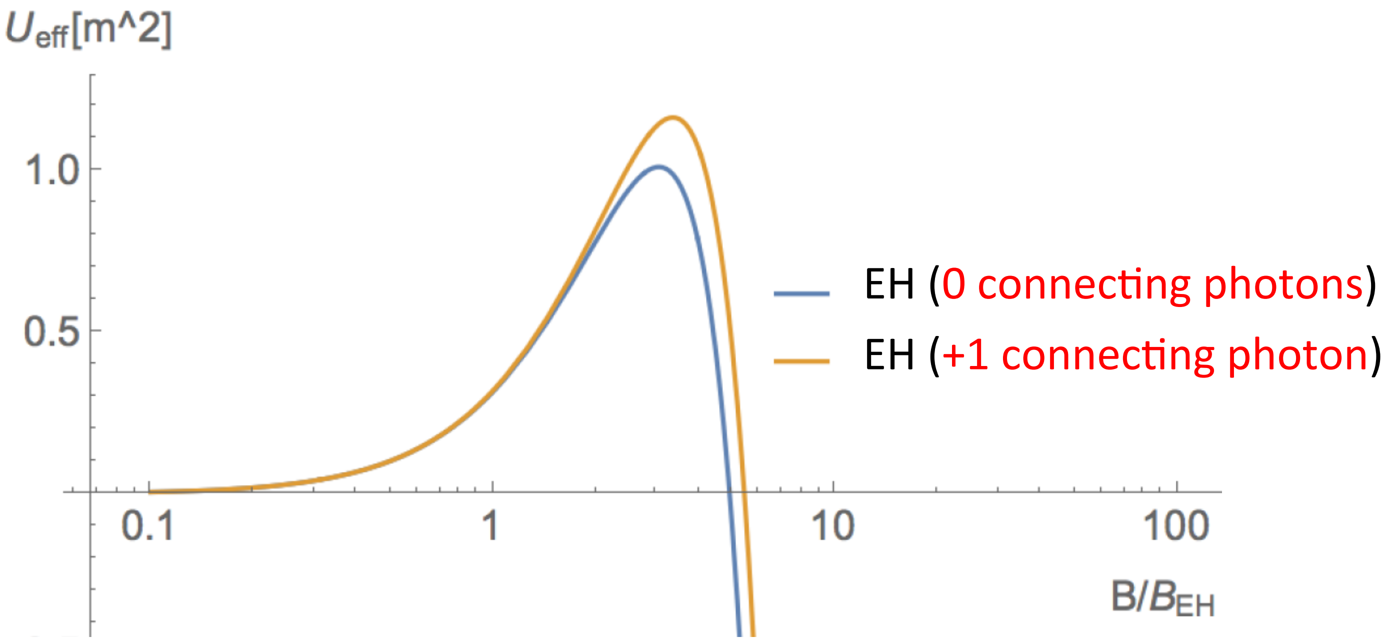
Connecting photon lines

When is interaction with connecting photon lines nonperturbative?

$$f(\mathcal{B}) = \frac{\alpha B^2}{6\pi} \ln\left[\frac{4\sqrt{\alpha\pi}}{m^2}\mathcal{B}\right] , \qquad \frac{\alpha}{6\pi} \ln\left[\frac{4\sqrt{\alpha\pi}}{m^2}\mathcal{B}\right] < 1 ,$$

When expansion parameter on right-hand side is larger than 1:

$$\alpha \to \tilde{\alpha} = 1/2$$

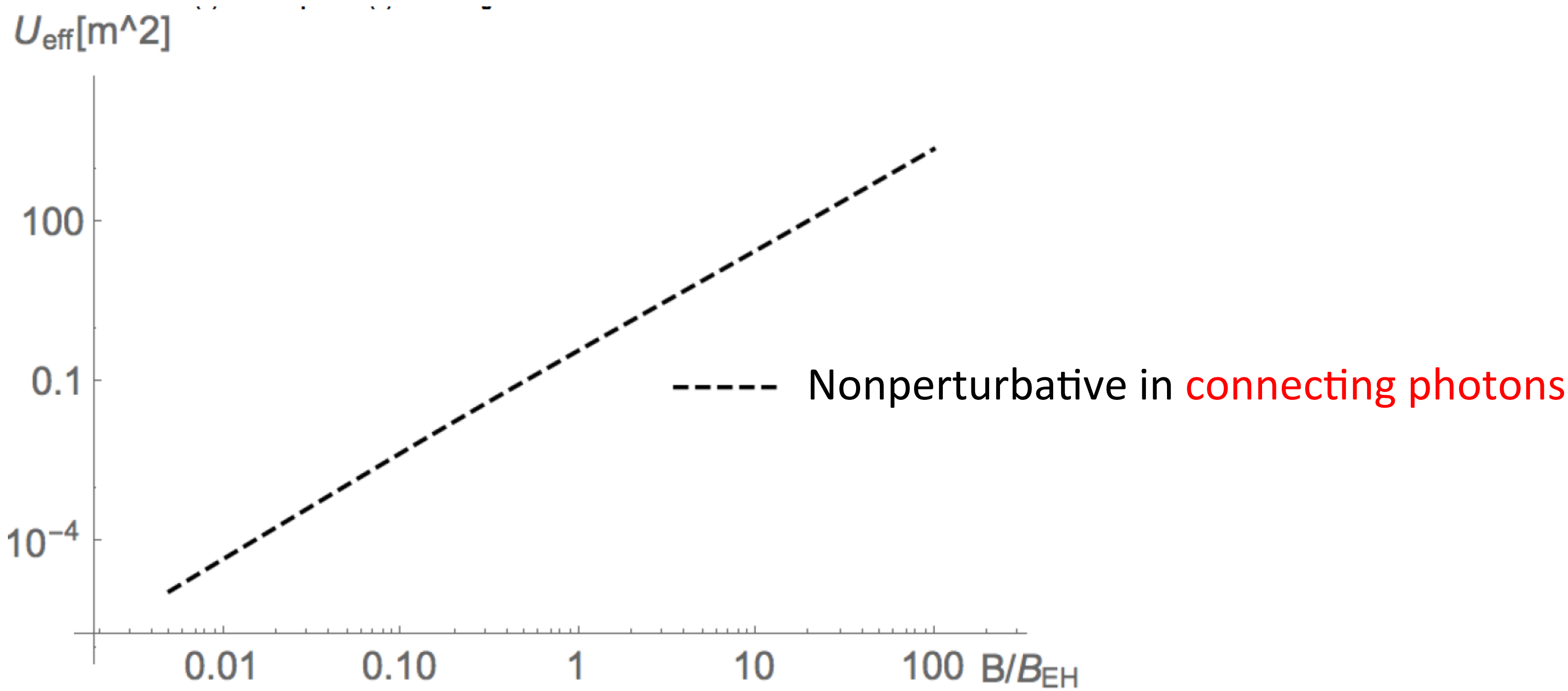


Connecting photon lines

Stabilized vacuum for

$$\boxed{\frac{\alpha}{6\pi}\ln\left[\frac{4\sqrt{\alpha\pi}}{m^2}\mathcal{B}\right] > 1}$$

$$\alpha \to \tilde{\alpha} = 1/2$$



Thank you