Quantum state redistribution with local coherence

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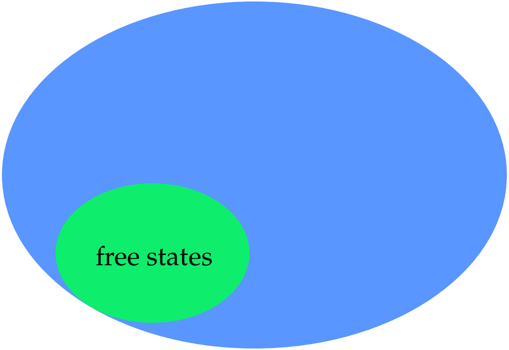
Budapest University of Technology and Economics
September 26, 2018

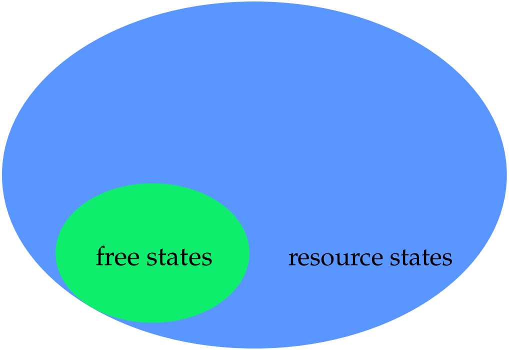
Quantum resource theories[a]



Free states:
quantum states which are easy to create

[a] Horodecki and Oppenheim, Int. J. Mod. Phys. B **27**, 1345019 (2013)

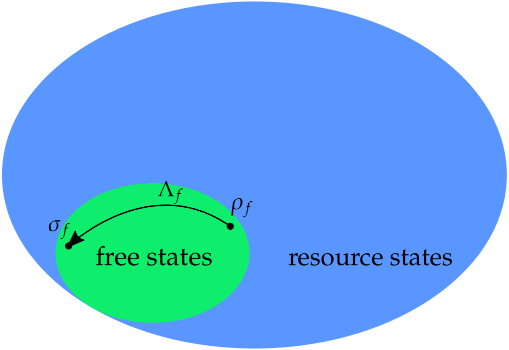
Quantum resource theories[a]



Resource states:
quantum states which are not free

[a] Horodecki and Oppenheim, Int. J. Mod. Phys. B **27**, 1345019 (2013)

Quantum resource theories[a]



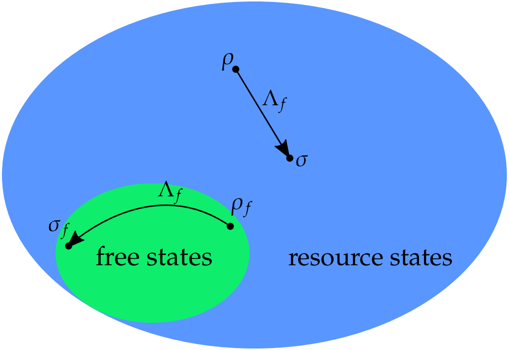
Free operations:

easy implementable state transformations,

transform free states into free states

[a] Horodecki and Oppenheim, Int. J. Mod. Phys. B **27**, 1345019 (2013)

Quantum resource theories[a]

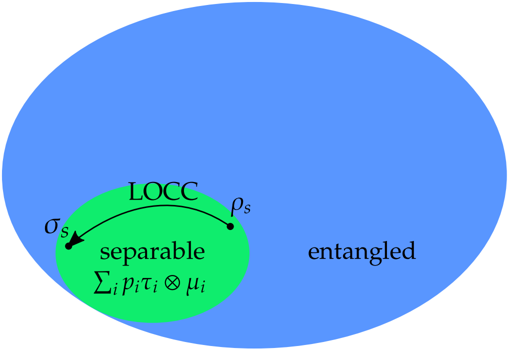


State conversion problem:

determining if a state σ can be obtained from ρ

[a] Horodecki and Oppenheim, Int. J. Mod. Phys. B **27**, 1345019 (2013)

Quantum resource theories[a]



Resource theory of entanglement:

Local operations and classical communication

+ separable states

[a]Horodecki and Oppenheim, Int. J. Mod. Phys. B **27**, 1345019 (2013)

Quantum resource theories

- **Entanglement**[a]**:** LOCC + separable states

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Quantum resource theories

- **Entanglement**[a]**:** LOCC + separable states
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Quantum resource theories

- **Entanglement**[a]**:** LOCC + separable states
- **Quantum coherence**[b]**:** incoherent operations + states
- **Quantum thermodynamics**[c]**:** thermal operations + states

[a]Horodecki *et al.*, Rev. Mod. Phys. **81**, 865 (2009)
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[c]Lostaglio *et al.*, Phys. Rev. X **5**, 021001 (2015)

Quantum resource theories

- **Entanglement[a]:** LOCC + separable states
- **Quantum coherence[b]:** incoherent operations + states
- **Quantum thermodynamics[c]:** thermal operations + states
- **Purity[d]:** unital operations + maximally mixed state

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[b]Streltsov *et al.*, Rev. Mod. Phys. **89**, 041003 (2017)
[c]Lostaglio *et al.*, Phys. Rev. X **5**, 021001 (2015)
[d]Horodecki *et al.*, Phys. Rev. A **67**, 062104 (2003)

Resource theory of quantum coherence[a][b]

Free states: incoherent (diagonal) states

$$\hat{\rho} = \sum_i p_i \, |i\rangle\langle i| = \begin{pmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{pmatrix}$$

[a] Baumgratz, Cramer, Plenio, Phys. Rev. Lett. **113**, 140401 (2014)
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Maximally coherent state:

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

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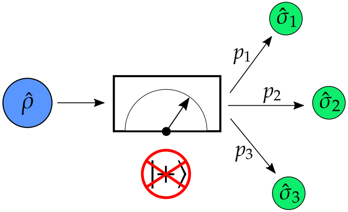
Incoherent operations:



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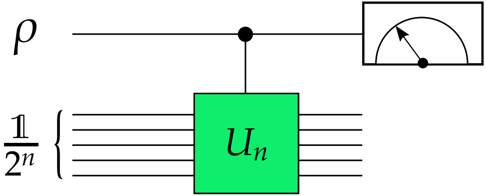
Quantum coherence in quantum technology



Quantum computation: direct relation between coherence and precision of DQC1 algorithm, noisy quantum computation with little entanglement[a][b]

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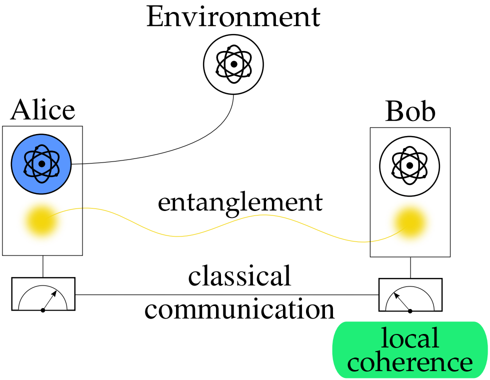
[a] Knill and Laflamme, Phys. Rev. Lett. **81**, 5672 (1998)
[b] Matera *et al.*, Quantum Sci. Technol. **1** 01LT01 (2016)

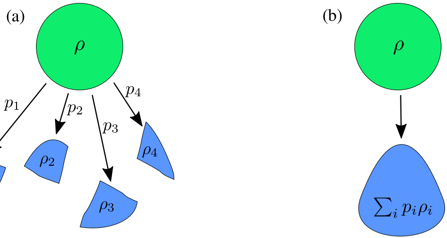
Quantum coherence in quantum technology



Quantum communication: local coherence reduces entanglement consumption in quantum communication[a]

[a] Streltsov, Chitambar, Rana, Bera, Winter, Lewenstein, Phys. Rev. Lett. **116**, 240405 (2016)

Resource theories of quantum coherence



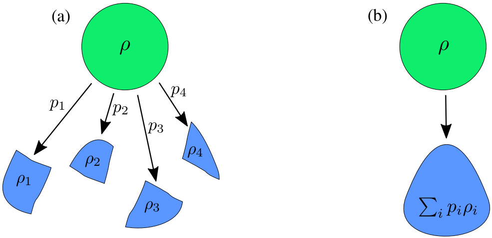
a) **Incoherent operations (IO)**[a]:

$$\Lambda_{\mathrm{IO}}[\rho] = \sum_i K_i \rho K_i^\dagger \qquad (1)$$

with incoherent Kraus operators $K_i \,|m\rangle \sim |n\rangle$

[a] Baumgratz *et al.*, Phys. Rev. Lett. **113**, 140401 (2014)

Resource theories of quantum coherence



b) **Maximally incoherent operations (MIO)**[b]:

$$\Lambda_{\mathrm{MIO}}[\hat{\rho}] = \hat{\sigma} \tag{2}$$

with incoherent states $\hat{\rho}$ and $\hat{\sigma}$

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[a] Baumgratz *et al.*, Phys. Rev. Lett. **113**, 140401 (2014)
[b] Aberg, arXiv:quant-ph/0612146

Coherence and entanglement in quantum communication

Shared entanglement and **local coherence** are resources for quantum communication[a][b]

[a] Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993)
[b] Streltsov *et al.*, Phys. Rev. Lett. **116**, 240405 (2016)

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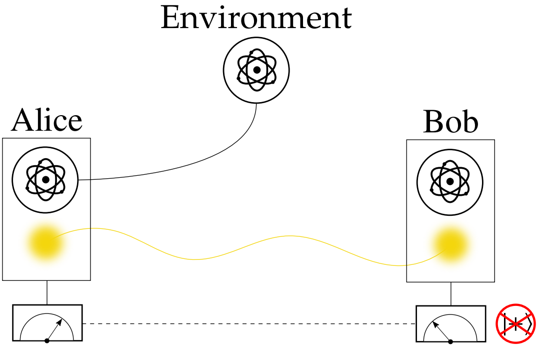

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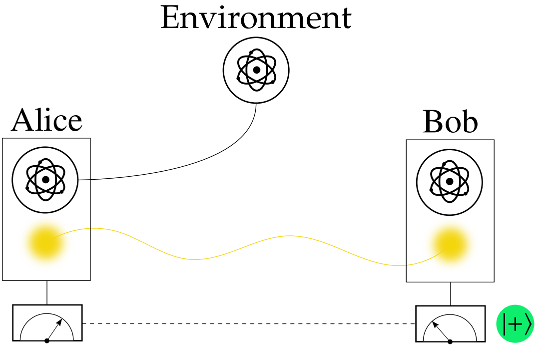

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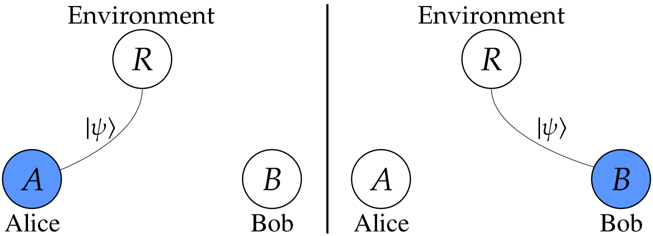
Quantum teleportation[a]



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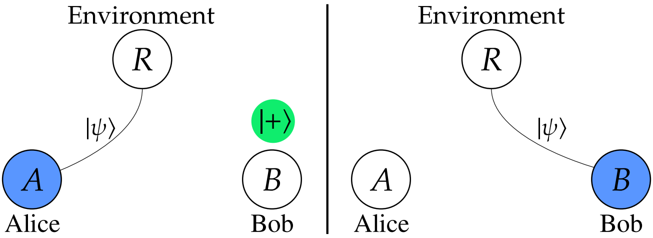

Entanglement consumption rate:

- with local coherence[b]: $E = S(\rho^A)$

[a]Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993)
[b]Schumacher, Phys. Rev. A **51**, 2738 (1995)

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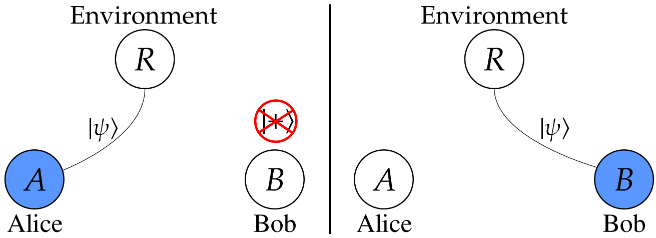

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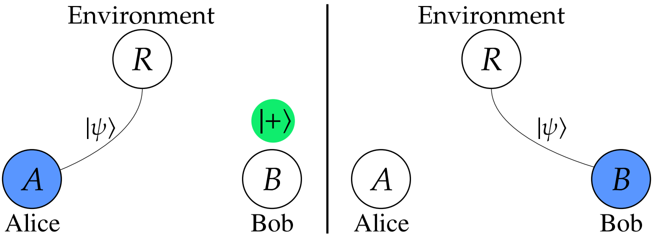

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- Tradeoff[c]: $E + C \geq S(\rho^A_{\mathrm{diag}})$

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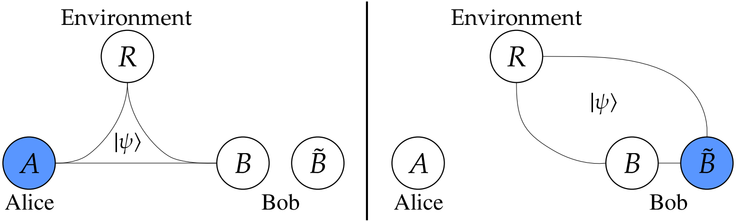
Quantum state merging[a]



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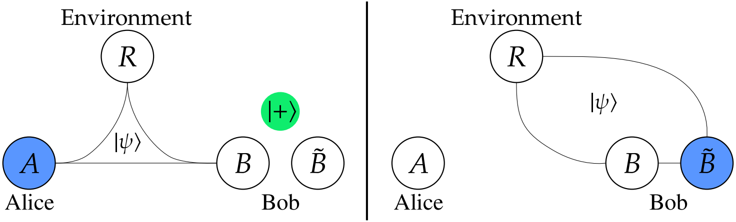
Quantum state merging[a]



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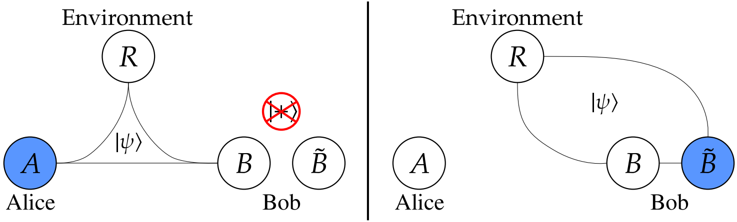

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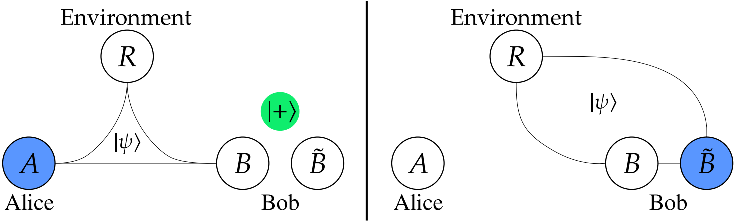

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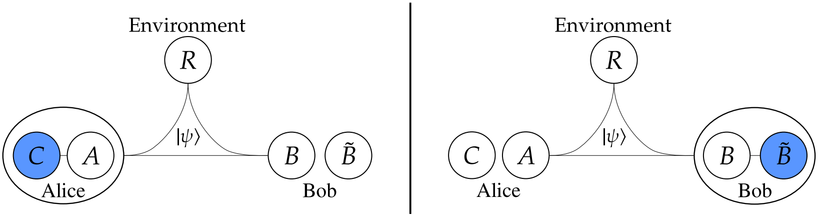
Quantum state redistribution[a]



[a] Devetak and Yard, Phys. Rev. Lett. **100** , 230501 (2008)

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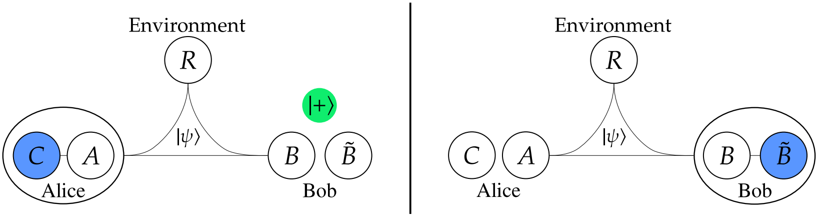

Quantum communication rate:

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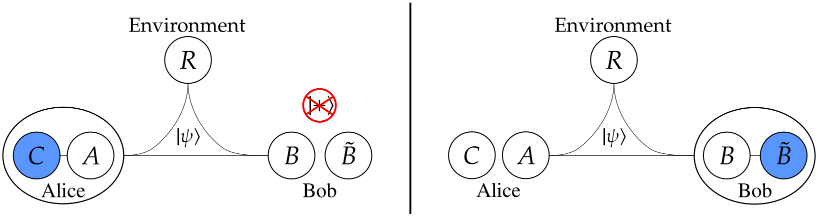

Quantum communication rate:

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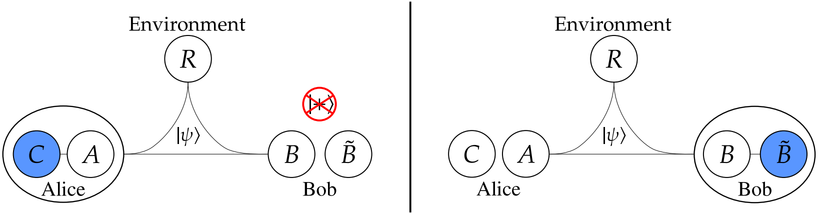
[a] Devetak and Yard, Phys. Rev. Lett. **100** , 230501 (2008)
[b] Anshu, Jain, Streltsov, arXiv:1804.04915

Quantum state redistribution[a]



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- Conditional mutual information:
  $I(C:R|B) = S(R|B) - S(R|BC)$

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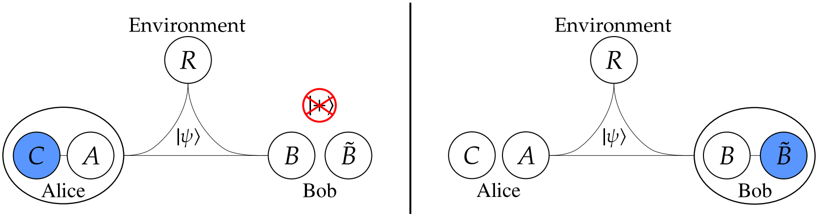

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- Relative entropy of coherence[cd]: $R_c(\rho) = S(\rho_{\mathrm{diag}}) - S(\rho)$

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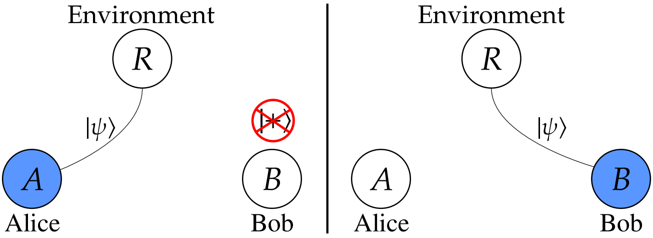
[a] Devetak and Yard, Phys. Rev. Lett. **100** , 230501 (2008)
[b] Anshu, Jain, Streltsov, arXiv:1804.04915
[c] Baumgratz, Cramer, Plenio, Phys. Rev. Lett. **113**, 140401 (2014)
[d] Winter and Yang, Phys. Rev. Lett. **116**, 120404 (2016)

Quantum state redistribution

without side information and coherence



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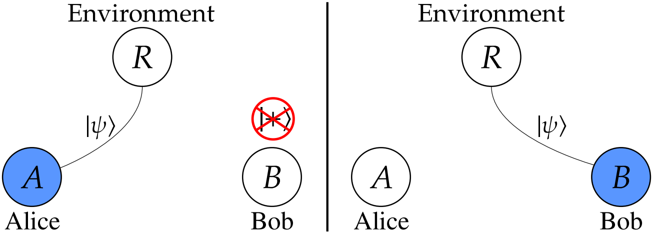

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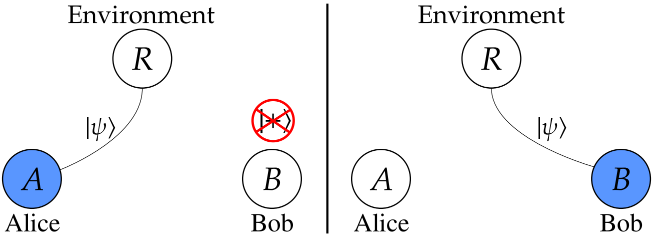

- Quantum communication rate[a]: $Q = \frac{1}{2}\left\{S(\rho^A) + S(\rho^A_{\mathrm{diag}})\right\}$
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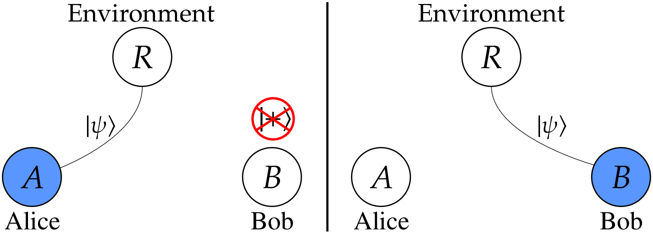


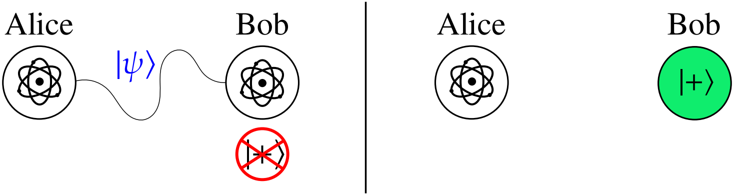
- Quantum communication rate[a]: $Q = \frac{1}{2}\left\{S(\rho^A) + S(\rho^A_{\rm diag})\right\}$
- Optimal singlet rate[b]: $E = S(\rho^A)$
- Singlet rate in absence of coherence[c]: $E = S(\rho^A_{\rm diag})$

[a] Anshu, Jain, Streltsov, arXiv:1804.04915

[b] Schumacher, Phys. Rev. A **51**, 2738 (1995)

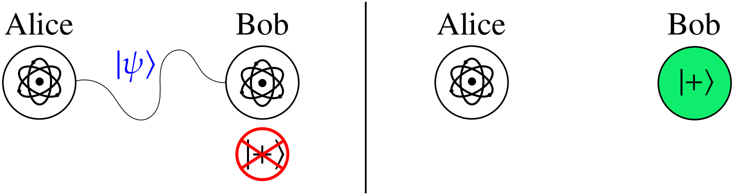
[c] Streltsov *et al.*, Phys. Rev. Lett. **116**, 240405 (2016)

Assisted distillation of coherence



Entanglement is useful for local coherence extraction

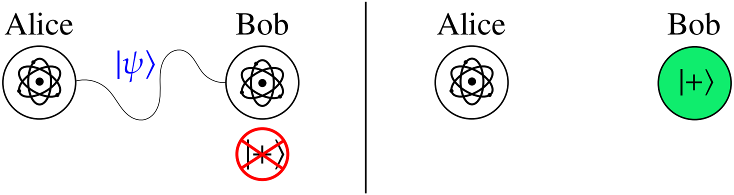
Assisted distillation of coherence



Entanglement is useful for local coherence extraction

- Alice and Bob share a pure state $|\psi\rangle$

Assisted distillation of coherence



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- With assistance of Alice[a,b]: $C_a = S(\rho^B_{\mathrm{diag}})$

[a] Chitambar *et al.*, Phys. Rev. Lett. **116**, 070402 (2016)
[b] Streltsov *et al.*, Phys. Rev. X **7**, 011024 (2017)

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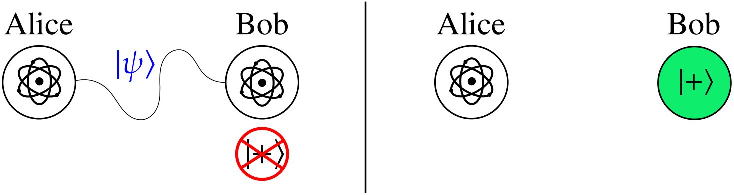

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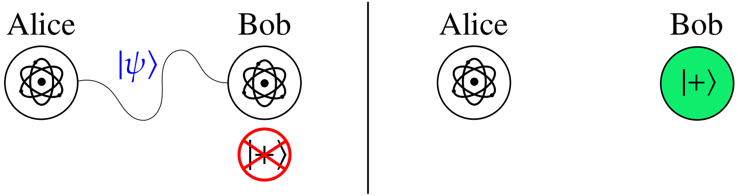

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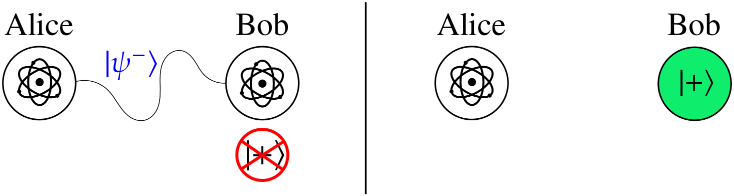
- Non-asymptotic assisted coherence distillation[d]

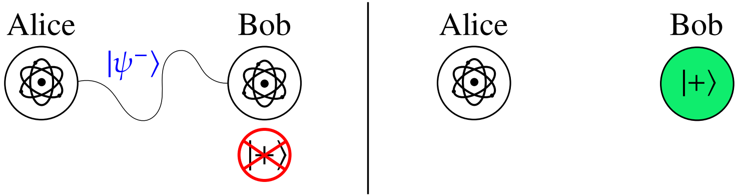
[a] Chitambar *et al.*, Phys. Rev. Lett. **116**, 070402 (2016)
[b] Streltsov *et al.*, Phys. Rev. X **7**, 011024 (2017)
[c] Winter and Yang, Phys. Rev. Lett. **116**, 120404 (2016)
[d] Regula, Lami, Streltsov, arXiv:1807.04705

Assisted distillation of coherence



Conversion of entanglement into coherence

Assisted distillation of coherence

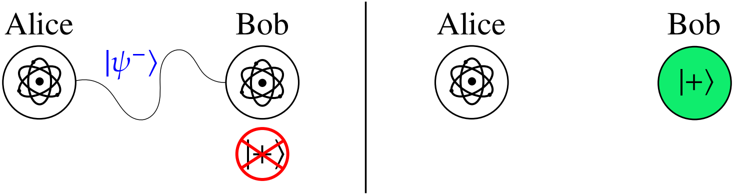


Conversion of entanglement into coherence

- Free classical communication:
  E shared singlets → *E* maximally coherent states[a]

[a]Chitambar *et al.*, Phys. Rev. Lett. **116**, 070402 (2016)

Assisted distillation of coherence



Conversion of entanglement into coherence

- Free classical communication:
  E shared singlets $\rightarrow$ E maximally coherent states[a]

- No free classical communication:
  E shared singlets and Q qubits of quantum communication
  $\rightarrow C = Q + \min\{E, Q\}$ maximally coherent states[b]

[a] Chitambar *et al.*, Phys. Rev. Lett. **116**, 070402 (2016)
[b] Anshu, Jain, Streltsov, arXiv:1804.04915

Summary

- **Local quantum coherence useful** for quantum state merging and quantum state redistribution

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[a] Anshu, Jain, Streltsov, arXiv:1804.04915
[b] Streltsov *et al.*, Phys. Rev. Lett. **116**, 240405 (2016)

Summary

- **Local quantum coherence useful** for quantum state merging and quantum state redistribution
- Both processes are possible also **without local coherence at a higher cost**

[a] Anshu, Jain, Streltsov, arXiv:1804.04915
[b] Streltsov *et al.*, Phys. Rev. Lett. **116**, 240405 (2016)

Summary

- **Local quantum coherence useful** for quantum state merging and quantum state redistribution

- Both processes are possible also **without local coherence at a higher cost**

- **Quantum state redistribution:** quantum communication cost without local coherence[a]

$$Q = \frac{1}{2}\left\{I(C:R|B) + R_c(\rho^{BC}) - R_c(\rho^{B})\right\} \tag{3}$$

[a] Anshu, Jain, Streltsov, arXiv:1804.04915

[b] Streltsov *et al.*, Phys. Rev. Lett. **116**, 240405 (2016)

Summary

- **Local quantum coherence useful** for quantum state merging and quantum state redistribution

- Both processes are possible also **without local coherence at a higher cost**

- **Quantum state redistribution:** quantum communication cost without local coherence[a]

$$Q = \frac{1}{2}\left\{I(C:R|B) + R_c(\rho^{BC}) - R_c(\rho^{B})\right\} \qquad (3)$$

- **Quantum state merging:** entanglement cost without local coherence[b]

$$E = S(A|B)_{\rho_{\mathrm{diag}}} \qquad (4)$$

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[a] Anshu, Jain, Streltsov, arXiv:1804.04915

[b] Streltsov *et al.*, Phys. Rev. Lett. **116**, 240405 (2016)