Muon: what is it good for?

Wigner RCP, Budapest, 2018.02.16.

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Motivation: muon \((g - 2)\)

\(\mu\) meson \(\rightarrow\) muon

Mirror symmetries and parity violation

High-energy muons in matter

Muons from accelerators

Anomalous magnetic moment

Muon \((g-2)\) experiments at CERN and at BNL

Result of BNL: deviation from Standard Model?

Continue at Fermilab
What makes all this interesting?

The muon anomalous magnetic moment is one of the most precisely measured quantities in particle physics... Since the first results were published, a persisting "discrepancy" between theory and experiment of about 3 standard deviations is observed. It is the largest "established" deviation from the Standard Model seen in a "clean" electroweak observable and thus could be a hint for New Physics to be around the corner.


At present the theoretical and the experimental values are known with a similar relative precision of 0.5 ppm. There is, however, a 3.4 standard deviation difference between the two, strongly suggesting the need for continued experimental and theoretical study.


\[ 3.4\sigma \rightarrow 3.6\sigma \rightarrow 3.5\sigma \]
Why this talk?

In this paper, the comparison between the theory and the experiment is examined by considering the influence of the spacetime curvature to the measurement on the muon $g$-2 experiment using the storage ring on the basis of the general relativity up to the post-Newtonian order of $O(1/c^2)$.


It is my melancholy duty to report that these articles are fundamentally flawed in that they fail to correctly implement the Einstein equivalence principle of general relativity. ...the claimed effect is not compatible with explaining the observed experimental anomaly in the anomalous magnetic moment of the muon.

Nuclear forces: Yukawa interaction?

Interaction range $\sim$ proton diameter
Short range $\rightarrow$ heavy mediator

Yukawa potential:
$$V(r) = -g^2 e^{-Mr/r}$$

H. Yukawa, 1935: meson
$M \sim 100 - 300$ MeV

C. D. Anderson, 1936: Cosmic $\mu$ meson!
Not good, it is a lepton...
C. Powell, 1947: $\pi$ meson, $M = 140$ MeV

Nobel prizes: Anderson, 1936 ($e^+$); Yukawa, 1949; Powell, 1950

Pion exchange is still good approximation in nuclei
Does not work at high energies $\Rightarrow$ QCD
Who needs the muon?

Muon = big brother of the electron at $M_\mu \sim 200 \times M_e$

Sheldon Glashow in 2007, about the questions to be solved by the LHC:

*And why is the muon, a silly particle that was first seen in the 1930s, why is the muon 200 times heavier than the electron, and indeed, why is it there? Who needs the muon?*

([http://www.pbs.org/wgbh/nova/sciencenow/3410/02-bigd-text.html](http://www.pbs.org/wgbh/nova/sciencenow/3410/02-bigd-text.html))
High energy physics needs the muon!

- Time dilatation by special relativity was proven by the survival of cosmic muons to be detected on the surface of Earth.
- Muons helped to observe parity violation, neutrino oscillation, Higgs boson the first time.
- Muons are the most penetrative charged particles, the best messengers of new physics everywhere.
- All HEP experiments encircle their apparati with enormous muon detection systems. In CMS (Compact Muon Solenoid) it weighs 10 k tons.
- Muons facilitate to produce neutrino beams at proton accelerators via letting collimated relativistic pions decay in flight: $\pi \rightarrow \mu + \nu_\mu$ with a subsequent absorption of the muons in earth.
Proton $\Rightarrow$ pion, kaon \[ K \rightarrow \pi, \pi^\pm \rightarrow \mu^\pm \nu_\mu \] \[ K, \pi \text{ stop } \mu^\pm \text{ detection} \]

All particles are relativistic.

Muon slows down and decays, $\mu \rightarrow e \nu_\mu \nu_e$ its neutrinos scatter out

Muon neutrino from pion decay flies forward.

$\nu_\mu$ beam spreads out on 732 km, at LNGS FWHM = 2.8 km
CMS: Compact Muon Solenoid

14000 ton digital camera:
200 M pixel, 40 M pictures/sec, 1000 GB/sec data
Processes cc. 1000 pictures/sec ⇒ intelligent filter!!
\[ H \rightarrow 4\ell \; (\ell = \mu^\pm, e^\pm) \]

Invariant mass spectra at \( \sqrt{s} = 13 \text{ TeV} \)

\[ 124.88 \pm 0.37 \text{(stat)} \pm 0.05 \text{ (syst)} \]

\[ 125.26 \pm 0.20 \text{(stat)} \pm 0.08 \text{ (syst)} \]

Both statistically limited.
CPT Invariance

Charge conjugation: \( C |p(r, t)\rangle = |\bar{p}(r, t)\rangle \)
Space reflection: \( P |p(r, t)\rangle = |p(-r, t)\rangle \)
Time reversal: \( T |p(r, t)\rangle = |p(r, -t)\rangle K \)

Basic assumption of field theory:
\( CPT |p(r, t)\rangle = |\bar{p}(-r, -t)\rangle \sim |p(r, t)\rangle \)
meaning free antiparticle \( \sim \) particle going backwards in space and time.

Giving up \( CPT \) one has to give up:

- **locality** of interactions \( \Rightarrow \) **causality**, or
- **unitarity** \( \Rightarrow \) conservation of matter, information, ... or
- Lorentz invariance
Space reflection: parity

Mirror reflection of all coordinates:
right handed $\Rightarrow$ left handed rendszer

$$f(x) = \frac{1}{2} [f(x) + f(-x)] + \frac{1}{2} [f(x) - f(-x)]$$

$$f(x) = \sum_{k=0}^{\infty} \frac{d^{2k} f(x)}{dx^{2k}} \frac{x^{2k}}{(2k)!} + \sum_{k=0}^{\infty} \frac{d^{2k+1} f(x)}{dx^{2k+1}} \frac{x^{2k+1}}{(2k+1)!}$$

even function.
odd function
Question of Parity Conservation in Weak Interactions*

T. D. Lee, Columbia University, New York, New York
AND
C. N. Yang,† Brookhaven National Laboratory, Upton, New York
(Received June 22, 1956)

The question of parity conservation in $\beta$ decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

How many papers for a Nobel prize?
Parity violation in pion decay

Pion decay:
\[ \pi^+ \rightarrow \mu^+ \nu_\mu \]

Only B realized ⇒ maximal parity violation

Muon production:
\[ \pi^+ \rightarrow \mu^+ \nu_\mu \]

Muon decay:
\[ \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \]

\( \mu \) spin resonance

Parity violation of weak interaction: V–A theory

Vector – axial vector: \( \bar{\psi}(1 - \gamma^5)\gamma^\mu\psi \)

Left-handed particles ⇔ right-handed antiparticles
Lederman’s experiment


\[
\pi^+ \rightarrow \mu^+ \nu_\mu; \quad \mu^+ \rightarrow e^+ \nu_e \overline{\nu}_\mu
\]

Fig. 2. Variation of gated 3-4 counting rate with magnetizing current. The solid curve is computed from an assumed electron angular distribution \(1 - \frac{1}{3} \cos \theta\), with counter and gate-width resolution folded in.
Discovery of parity violation

Experimental Test of Parity Conservation in Beta Decay

C. S. Wu, Columbia University, New York, New York

and

E. Ambler, R. W. Hayward, D. D. Hoffes, and R. P. Hudson,
National Bureau of Standards, Washington, D. C.
(Received January 15, 1957)

The inspiring discussions held with Professor T. D. Lee and Professor C. N. Yang by one of us (C. S. Wu) are gratefully acknowledged.

Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon

Richard L. Garwin,† Leon M. Lederman,
and Marcel Weinrich

Physics Department, Nevis Cyclotron Laboratories,
Columbia University, Irvington-on-Hudson,
New York, New York
(Received January 15, 1957)

The authors wish to acknowledge the essential role of Professor Tsung-Dao Lee in clarifying for us the papers of Lee and Yang. We are also indebted to Professor C. S. Wu for reports of her preliminary results in the Columbia discussions immediately preceding this experiment.

Phys. Rev. 105 (1957) 1413-1414

Nice example of scientific ethics:

Wu’s group worked several months on this experiment.

Lederman’s spent 1 day for data taking, 1 week for analysis, then waited for the Wu result to become public

Lederman received a Nobel prize: $\nu_\mu$, 1963; Wu did not.
Magnetic moment of the electron

\[ \vec{\mu}_e = -g \frac{e}{2m_e} \vec{S} \]

\[ e_R = e + (e_R^{(1)} - e) + \Delta e_R^{(1)} - \Delta e_R^{(1)'} + \ldots \]

\[ g_e = 2 + \frac{\alpha}{\pi} \pm \ldots \]

\( e_R \) contains first 2 terms, 3+4 reduces 2
Magnetic moment of the muon

\[ \vec{\mu}_\mu = -g \frac{e}{2m_\mu} \vec{S} \]

Point-like Dirac fermion: \( g = 2 \)

Anomalous part: \( \alpha = \frac{g-2}{2} \)
Anomalous m.m.: interesting?

\[
a_\mu (\text{SM}) = a_\mu (\text{QED}) + a_\mu (\text{weak}) + a_\mu (\text{hadr}) + a_\mu (??)
\]

Deviation: new physics or incorrect calculation

Particle Data Group, 2017:

<table>
<thead>
<tr>
<th></th>
<th>(a_e \times 10^9)</th>
<th>(a_\mu \times 10^9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED prediction</td>
<td>1159652.4 (4)</td>
<td>1165847.1895 (8)</td>
</tr>
<tr>
<td>hadron corrections</td>
<td>—</td>
<td>1165918.23 (1) (34) (26)</td>
</tr>
<tr>
<td>experiment (BNL)</td>
<td>1159652.186 (4)</td>
<td>(\mu^+): 1165920.4 (6) (5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\mu^-): 1165921.5 (8) (3)</td>
</tr>
</tbody>
</table>

\[
(\bar{a}_\mu^{\text{exp}} - a_\mu^{\text{SM}}) = (268 \pm 63 \pm 43) \times 10^{-11} (3.5\sigma !)
\]
Muon (g-2) deviation: SUSY?

Supersymmetry: many new particles?

Partners for SM fermions + 5 Higgs-bosons

\[ a_\mu^{\text{SUSY}} \approx \pm 130 \cdot 10^{-11} \cdot \left( \frac{100 \text{ GeV}}{m_{\text{SUSY}}} \right)^2 \tan \beta \]

\( m_{\text{SUSY}} \): SUSY mass scale; \( \tan \beta = v_1/v_2 \approx 3 \ldots 40 \)

SUSY particles of \( m = 100 \ldots 500 \text{ GeV} \) could explain the deviation

No evidence for them at LHC

Dark photon of \( m = 10 \ldots 100 \text{ MeV} \) in mixing with photon??

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ATLAS SUSY summary plot

For all results: https://twiki.cern.ch/twiki/bin/view/AtlasPublic/SupersymmetryPublicResults
$(g - 2)_\mu$ measurements

Spin magn. mom.: $\vec{\mu} = g\frac{e\hbar}{2mc}\vec{S}$

Dirac equation: $g = 2$

Larmor precession (non-relativistic, Pauli):

$$\omega_s = \frac{g}{\hbar}\left(\frac{e\hbar}{2mc}\right)B = \frac{eB}{mc} \cdot \frac{g}{2} = \frac{eB}{mc}(1 + \frac{g-2}{2}) = \frac{eB}{mc}(1 + a)$$

Precession of $S = \frac{1}{2}$ particle around its momentum in a $\vec{B}$ field:

spin freq – cyclotron freq $= \omega_a = \omega_S - \omega_c = \omega_S - \frac{eB}{mc} = a\frac{eB}{mc}$

Muon in storage ring: $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$  \hspace{1cm} $\bar{\nu}(e^+) \sim \vec{S}(\mu^+)$

G. Charpak et al., CERN, 1965:

Time spectrum: $N(t) = N_0 e^{-\frac{t}{\tau}}[1 - A \cos(\omega_a t + \phi)]$

$\tau = 2.2\mu s$, $p_\mu = 90$ MeV/$c$, $B=1.6$ T $\Rightarrow a_\mu = 1162(5) \times 10^{-6}$
Muon (g-2) principle

\[ \Delta \omega = \omega_a = \left( \frac{g-2}{2} \right) \frac{eB}{mc} \]
Using relativistic muons

Longer lifetime $\Rightarrow$ more precise measurement

Larmor + Thomas-precession: $\omega_s = g \frac{eB}{mc} + (1 - \gamma) \frac{eB}{mc\gamma}$

Cyclotron frequency: $\omega_c = \frac{eB}{mc\gamma}; \quad \gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

$\omega_a = \omega_s - \omega_c = \frac{eB}{mc} \left[ \frac{g}{2} + \frac{1-\gamma}{\gamma} - \frac{1}{\gamma} \right] = a \frac{eB}{mc}$

J. Bailey et al., CERN, 1972:

$p_\mu = 1.9 \text{ GeV/c}; \quad \gamma = 12; \quad \gamma \tau_\mu = 26 \mu s$

$B = 1.7 \text{ T}, \quad \text{inhomogeneous} \Rightarrow \text{focusing, but broadening}$

Result: $a_\mu = 116616(31) \times 10^{-8}$
Focusing in electric field

\[ \vec{\omega}_a = \frac{e}{mc} [a\vec{B} - (a - \frac{1}{\gamma^2 - 1}) \frac{\vec{v} \times \vec{E}}{|\vec{v}|}] \]

Magic \( \gamma \): \( \gamma - \frac{1}{\gamma^2 - 1} = 0 \Rightarrow \vec{E} \) term eliminated

J. Bailey et al., CERN, 1979:

\[ p_\mu = 3.094 \text{ GeV/c}; \gamma_\mu = 29.37; \gamma\tau_\mu = 64.4 \mu\text{s} \]
\[ B = 1.5 \text{ T}, \text{ homogeneous, el-static focusing} \]

\[ a_\mu = 1165924(85) \times 10^{-9} \]

Brookhaven (g-2) experiment: 1999 - 2006
LIFE OF A MUON: THE $g$-2 EXPERIMENT

Protons from AGS.

Hit Target.

Pions, weighing 1/6 proton, are created.

Pions decay to muons.

Muons are fed into a uniform, doughnut-shaped magnetic field and travel in a circle.

After each circle, muon's spin axis changes by 12°, yet it keeps on traveling in the same direction.

One of 24 detectors see an electron, giving the muon spin direction; $g$-2 is this angle, divided by the magnetic field the muon is traveling through in the ring.

After circling the ring many times, muons spontaneously decay to electron, (plus neutrinos,) in the direction of the muon spin.
Brookhaven (g-2) experiment
BNL (\(g-2\)) result (PDG, 2017)

- SM predictions
  - JN 2009: \(-301 \pm 65\)
  - HLMNT 2011: \(-263 \pm 49\)
  - DHMZ 2011: \(-289 \pm 49\)
  - DHMZ 2017: \(-268 \pm 43\)

- Measurement
  - BNL-E821 (world average): \(0 \pm 63\)

\[ a_\mu - a_\mu^{\text{exp}} \times 10^{-11} \]
Way of \((g - 2)\) ring to Fermilab

Muon g-2 ring
BNL to FNAL, 2013
Muon \((g-2)\) started at Fermilab

Aim: \(21 \times \) statistics, \(1/4 \times \) systematics, up to \(7\sigma\) (if!!)
Summary (not conclusion!)

- There is a $3.5\sigma$ difference between theory and experiment for the muon ($g-2$)
- Its source is not known.
- There were many attempts to explain it, mere speculations or refuted.
- Fermilab hopes to measure it at higher precision.
- We always hope for deviations from the Standard Model: new physics!

Thanks for your attention!