# Symmetries in Nuclear Physics - Wigner and Neutron Star Interiors

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1. **QCD Phase Diagram: Quark-Hadron Continuity ?**
2. **Neutron Star Interiors: Strong Phase Transition?**
3. **Mass-Radius Constraints: GW170817 & NICER**
4. **Hadron Dissociation: "Breit-Wigner Squared"**

**wigner – 115, Budapest, 15. November 2017**

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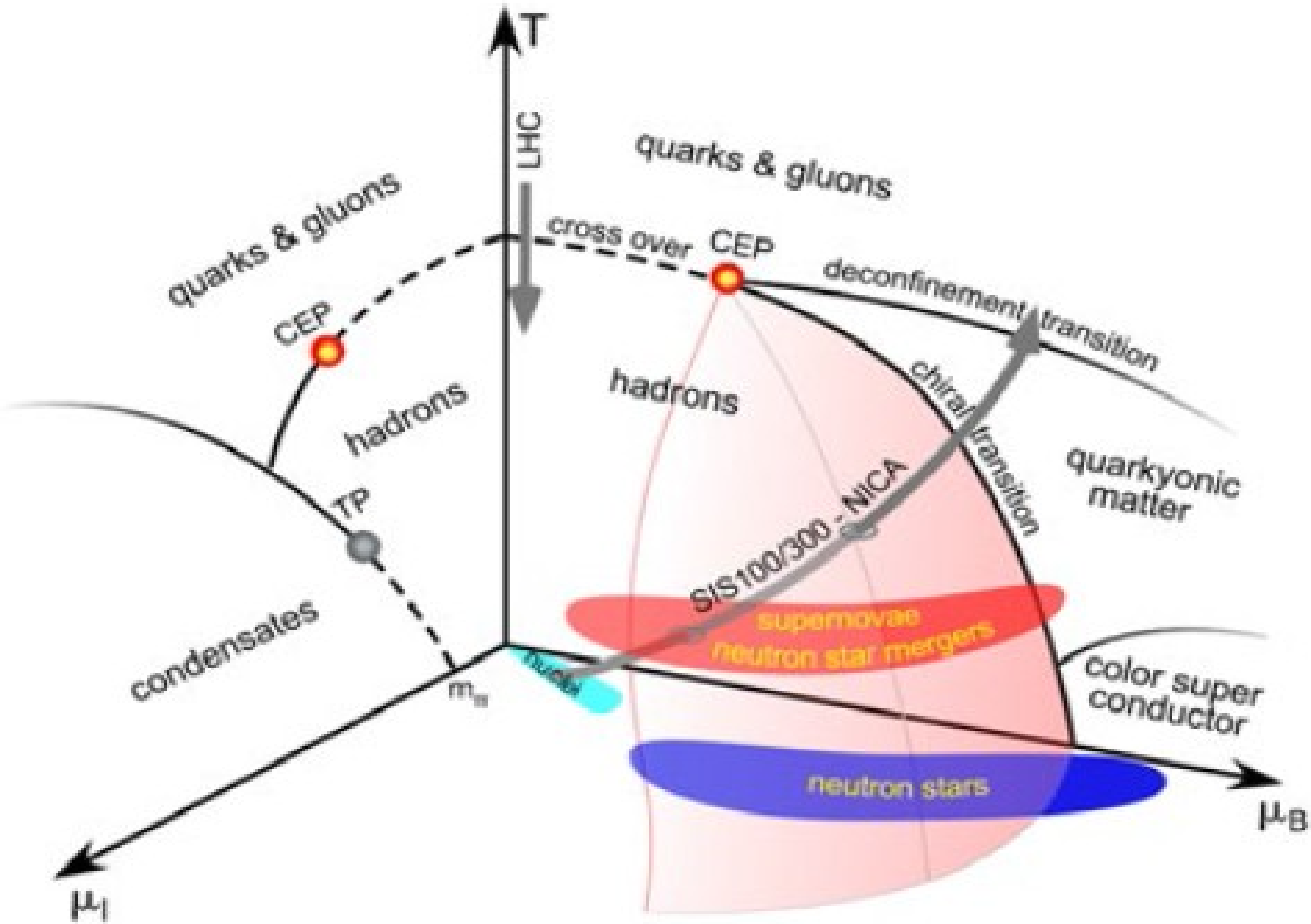
RSF Russian Science Foundation

# CEP in the QCD phase diagram: HIC vs. Astrophysics

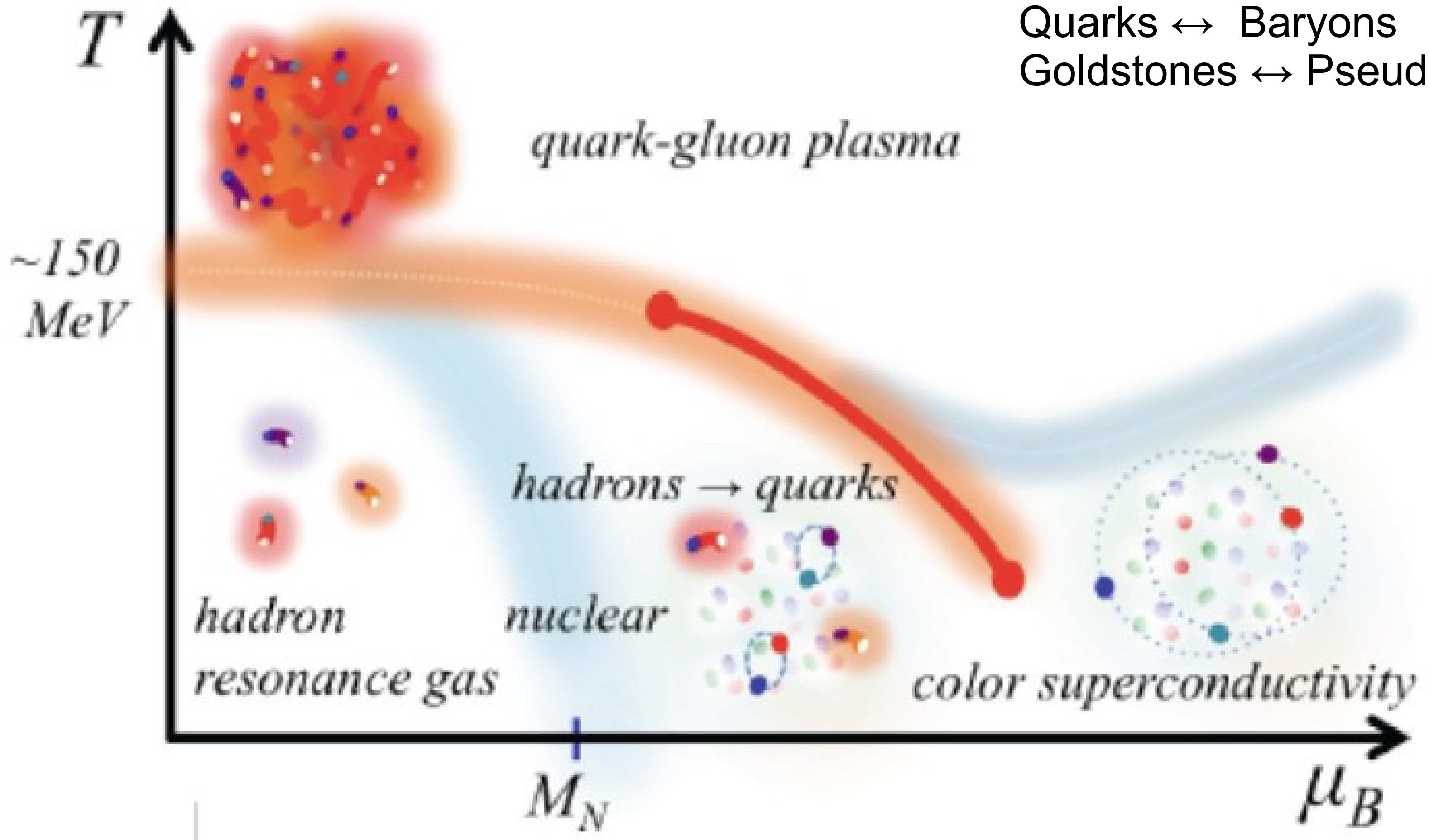



NuPECC Long Range Plan 2017; http://www.nupecc.org

A. Andronic, D. Blaschke, et al., "Hadron production ...", Nucl. Phys. A 837 (2010) 65 - 86

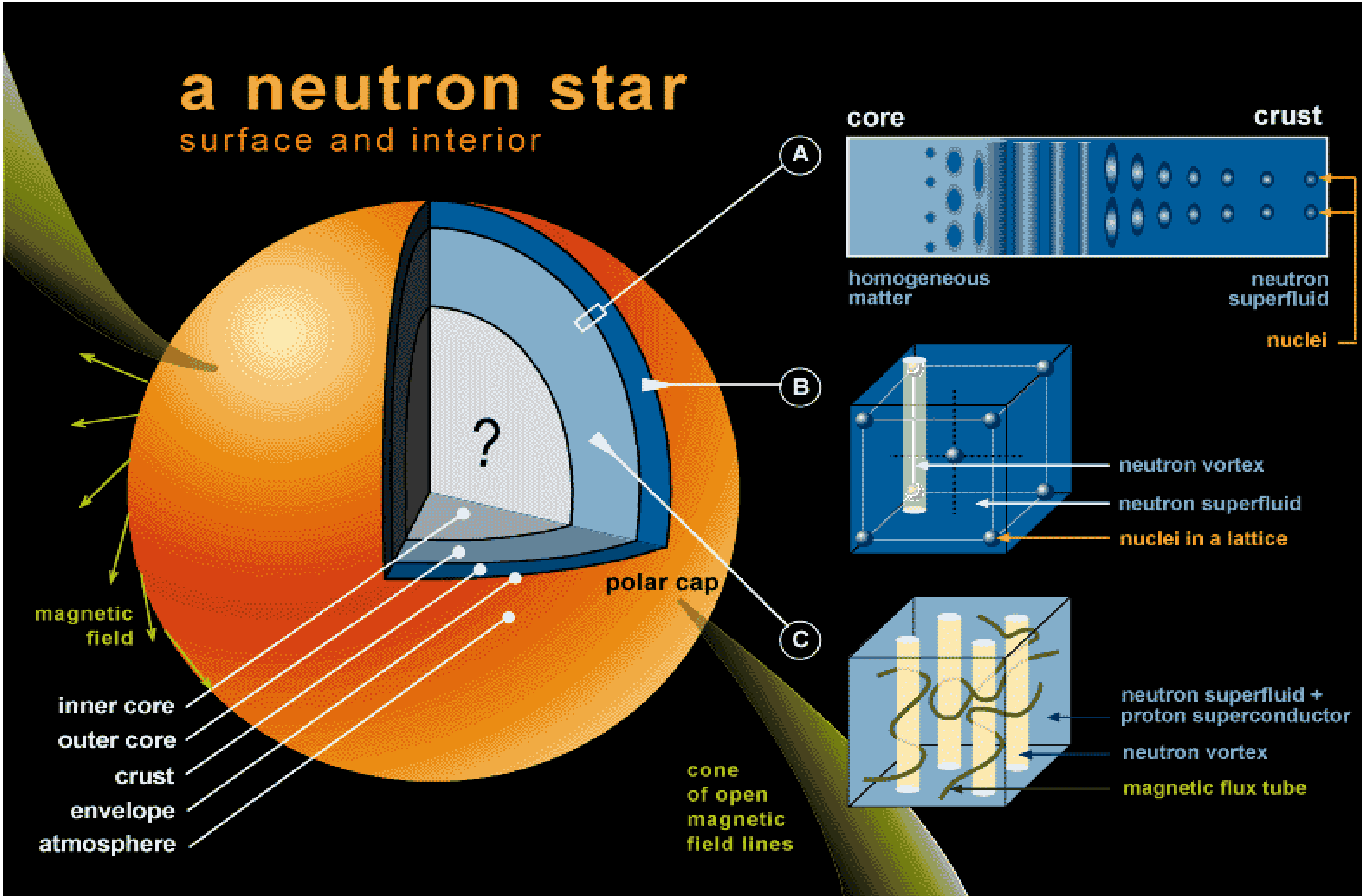
# $2^{nd}$ CEP in QCD phase diagram: Quark-Hadron Continuity?

Gluons ↔ Vector mesons
Quarks ↔ Baryons
Goldstones ↔ Pseudoscalar mesons




T. Schaefer & F. Wilczek, Phys. Rev. Lett. 82 (1999) 3956
C. Wetterich, Phys. Lett. B 462 (1999) 164
T. Hatsuda, M. Tachibana, T. Yamamoto & G. Baym, Phys. Rev. Lett. 97 (2006) 122001

# Neutron Star Interiors: Strong Phase Transition?



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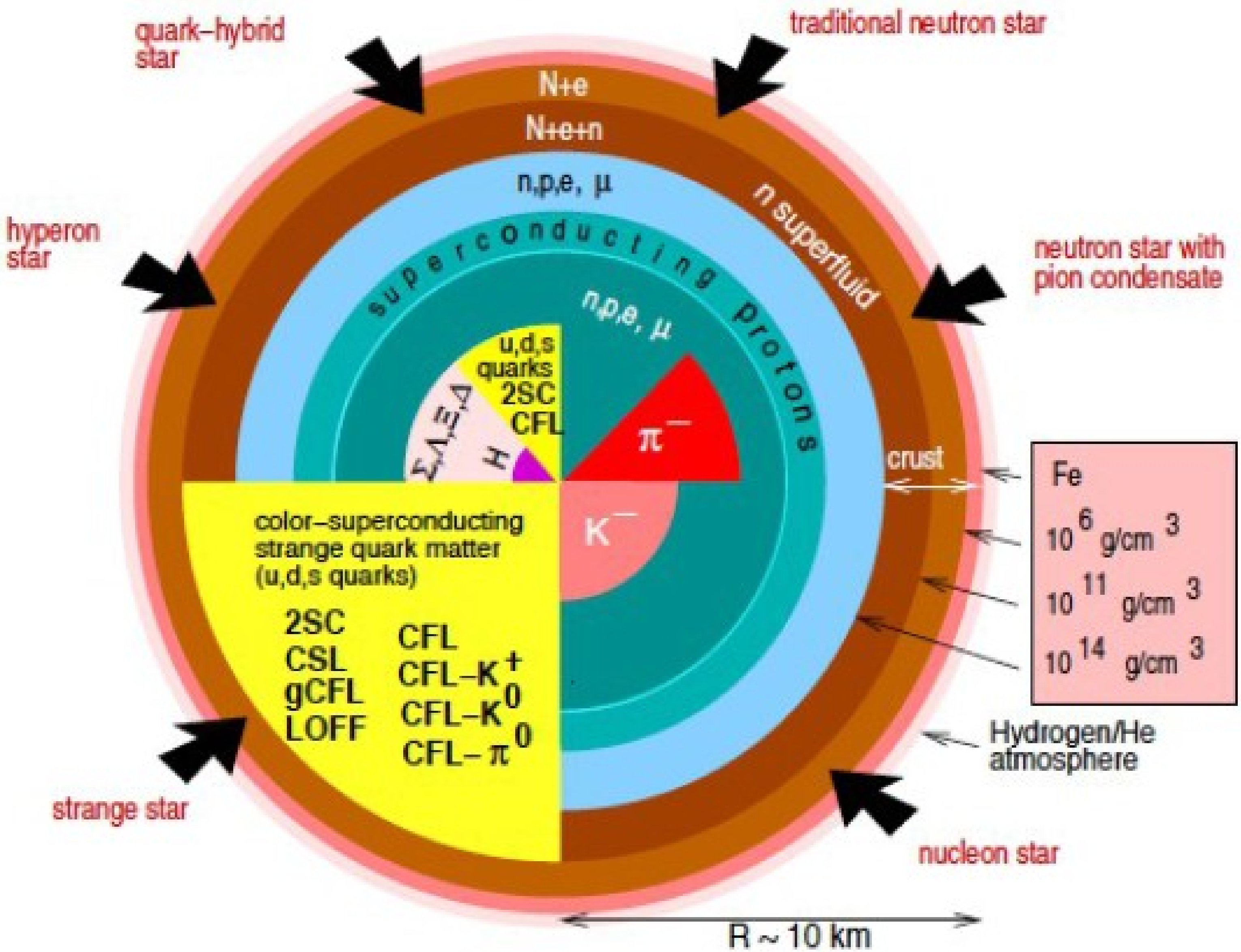
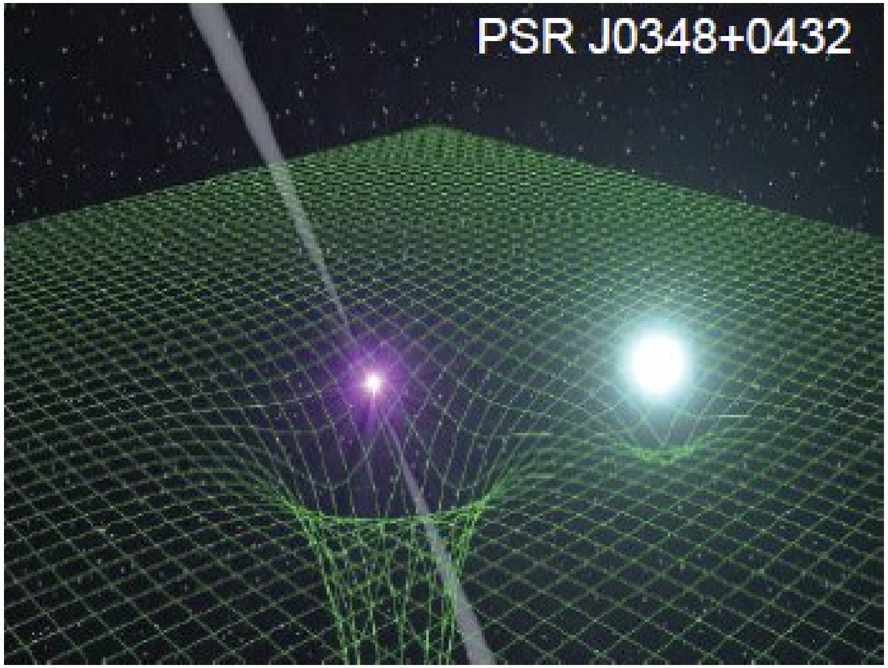


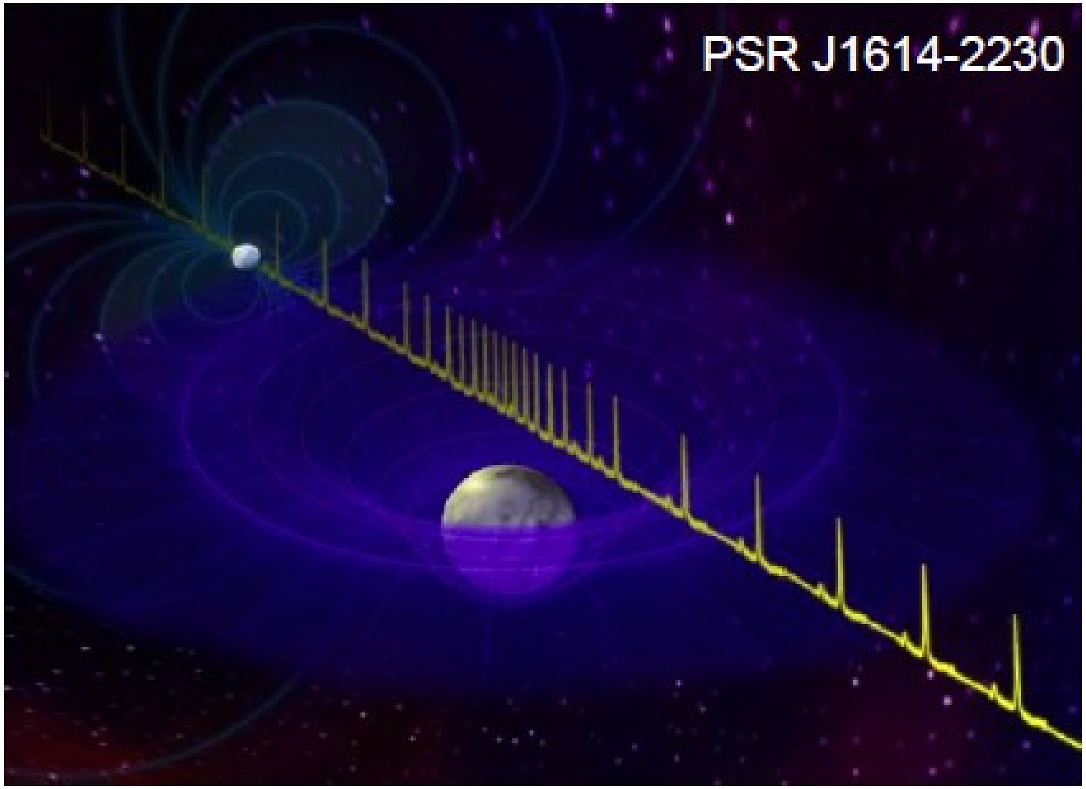

F. Weber:
"Neutron Stars -
Cosmic Labs ..."
IoP Bristol, 1999

# Neutron Star Interiors: Strong Phase Transition?

M=2.01 +/- 0.04 Msun




M=1.928 +/- 0.017 Msun

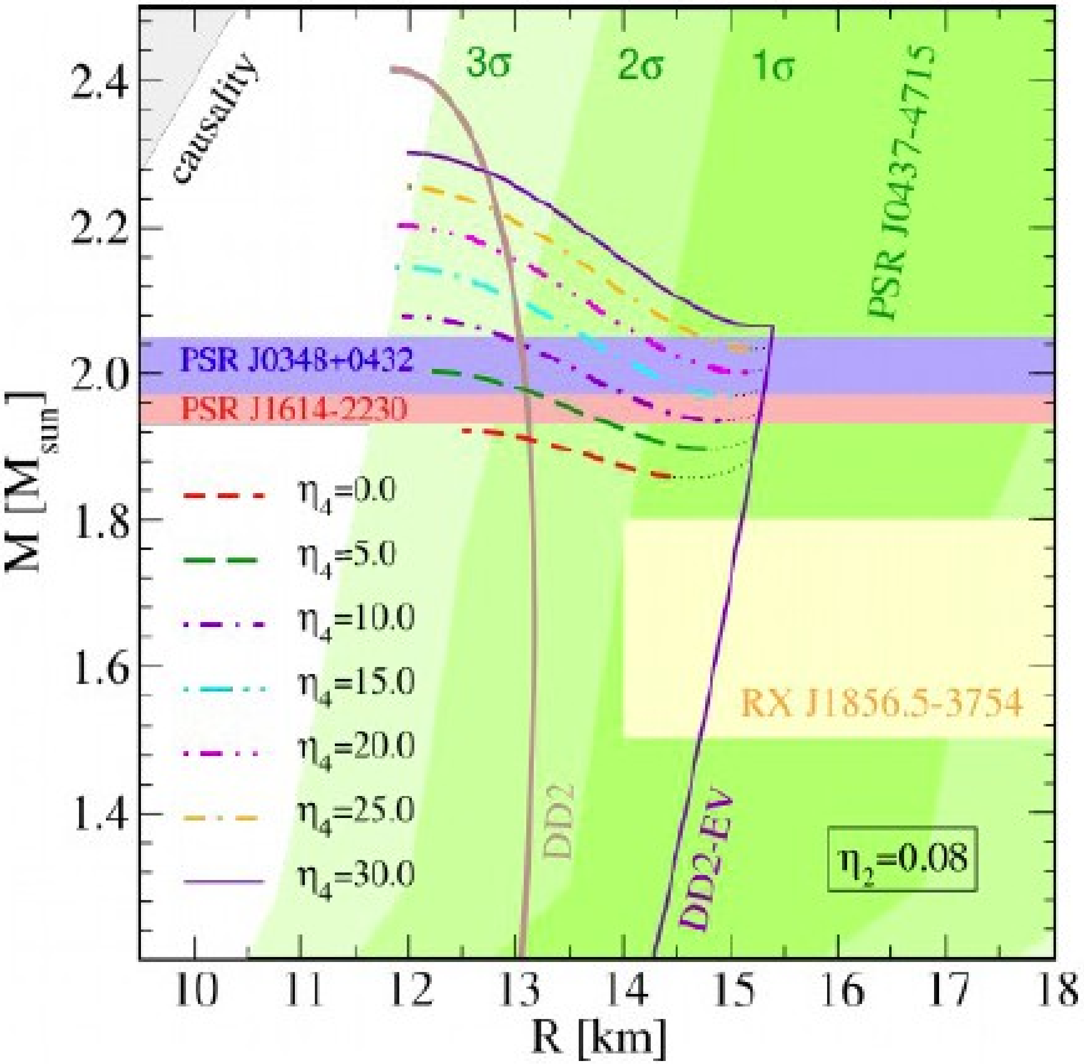



Antoniadis et al., Science 340 (2013) 448
Demorest et al., Nature 467 (2010) 1081
Fonseca et al., arxiv:1603.00545

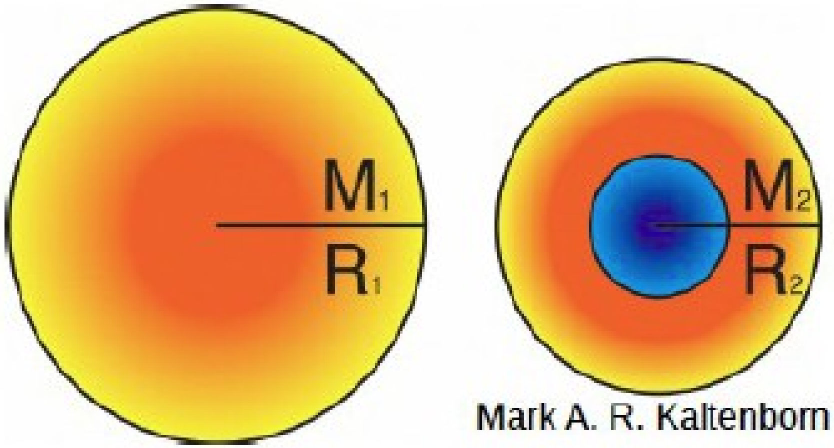
**What if they were high-mass twin stars?**

→ radius measurement required ! → NICER (2017)

# Neutron Star Interiors: Strong Phase Transition?




- Star configurations with same masses, but different radii




- **New class of EOS, that features high mass twins**
- NASA NICER mission: radii measurements ~ 0.5 km
- Existence of twins implies 1st order phase-transition and hence a critical point

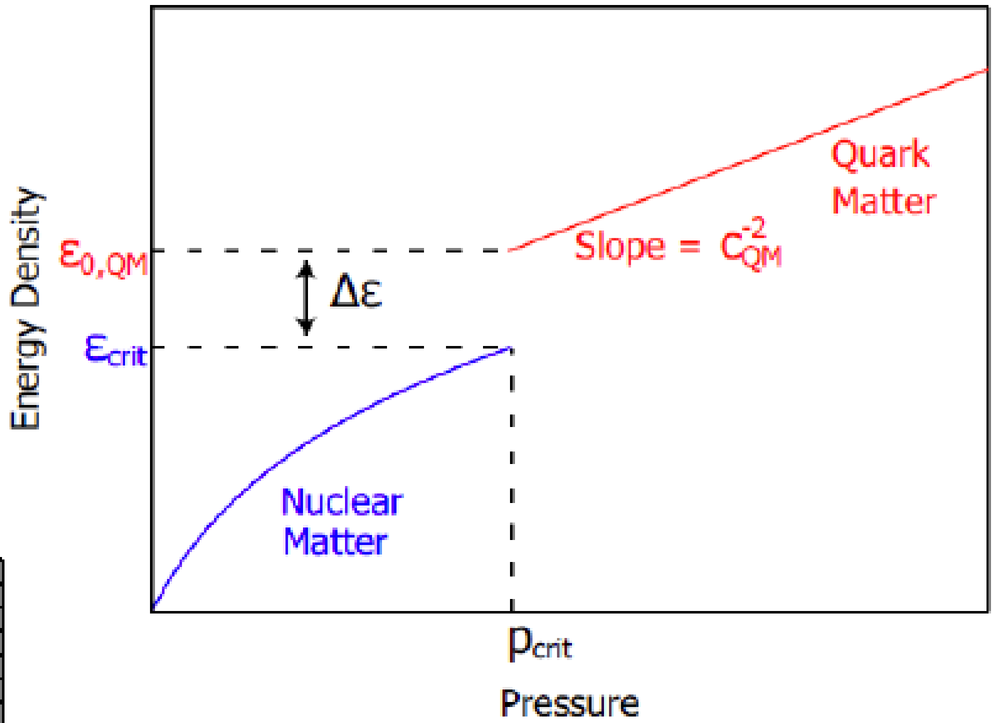


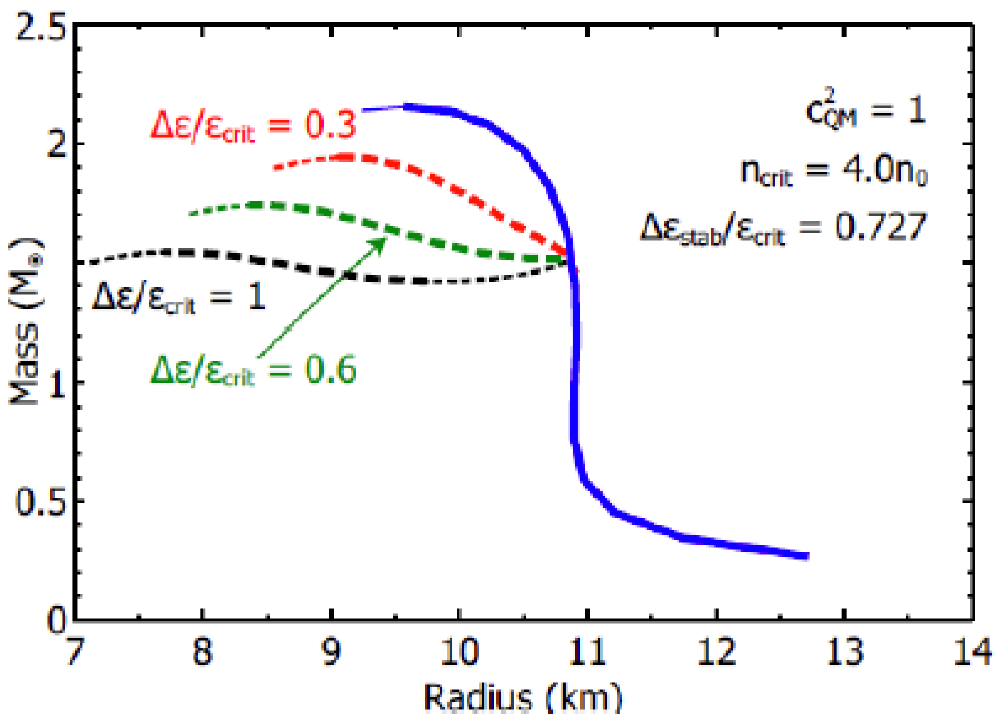
Benic, Blaschke, Alvarez-Castillo, Fischer, Typel, A&A 577, A40 (2015)

# Neutron Star Interiors: Strong Phase Transition?

**Alford, Han, Prakash, arxiv:1302.4732**

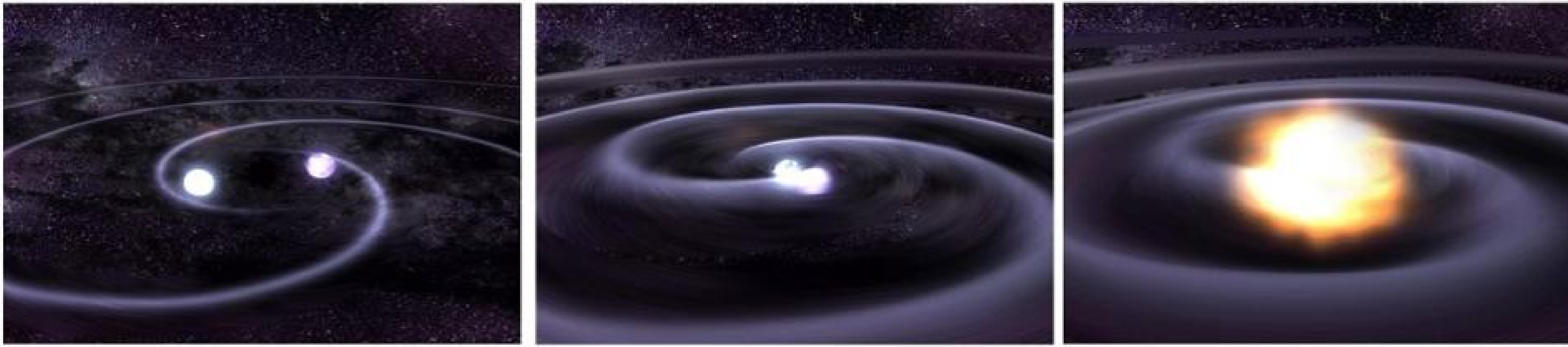
First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → **“third family of CS”**.

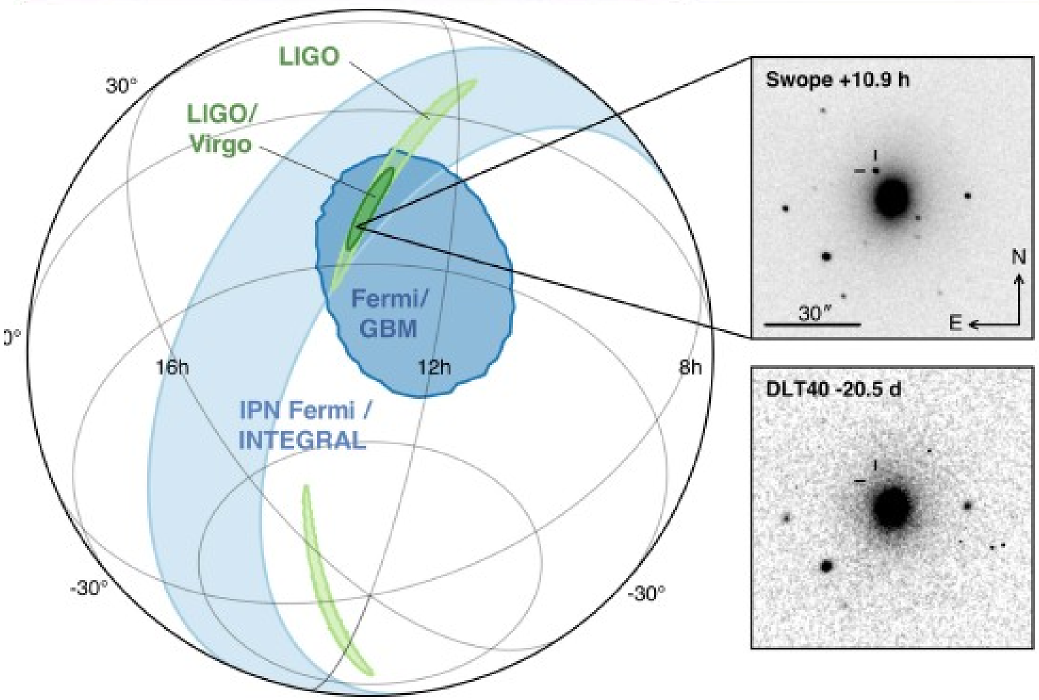


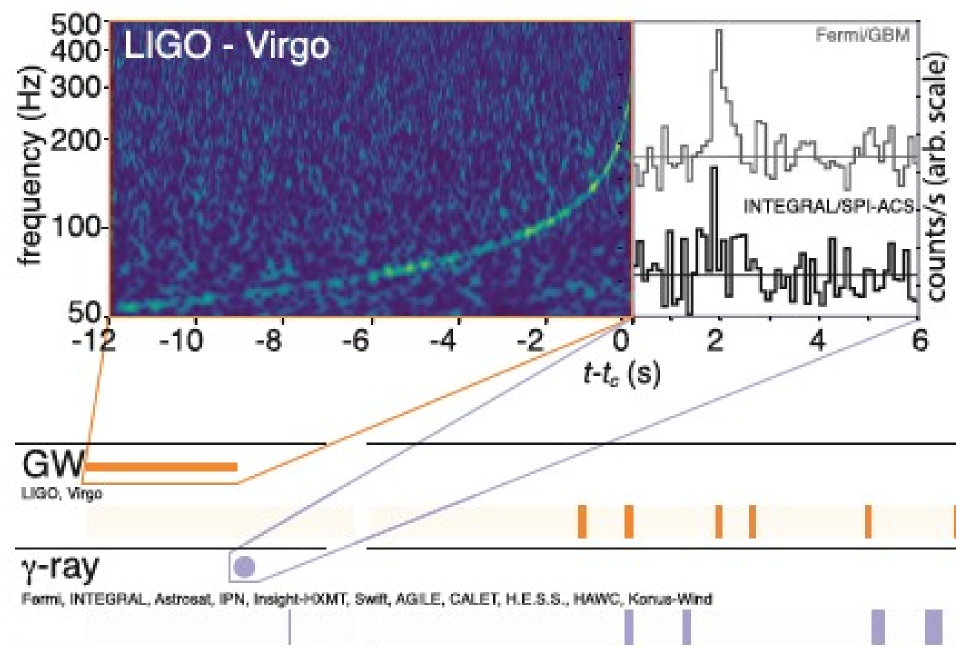




Measuring two **disconnected populations** of compact stars in the M-R diagram would be the **detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP) in the QCD phase diagram**!

# Mass-Radius Constraints: GW170817 & NICER






GW170817, announced on 16.10.2017

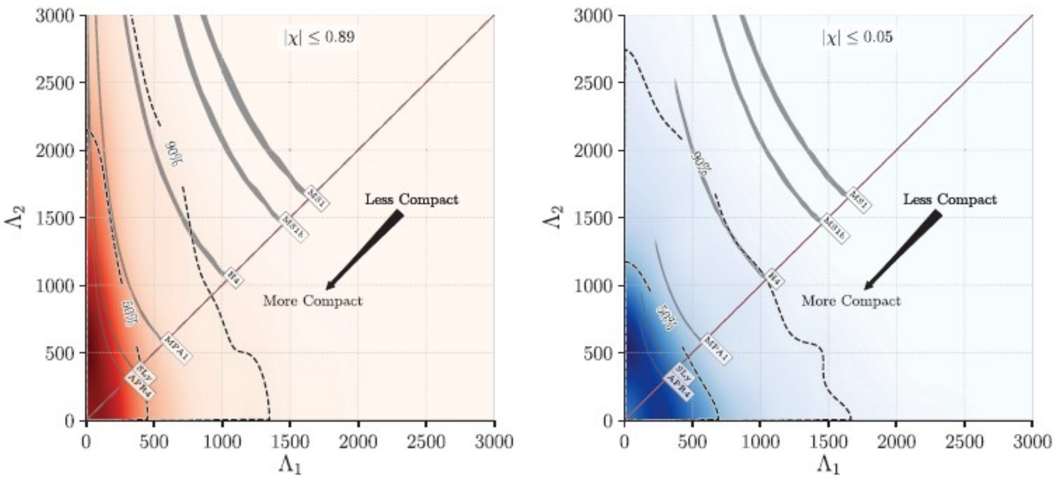
B.P. Abbott et al. [LIGO/Virgo Collab.], PRL 119, 161101 (2017); ApJLett 848, L12 (2017)

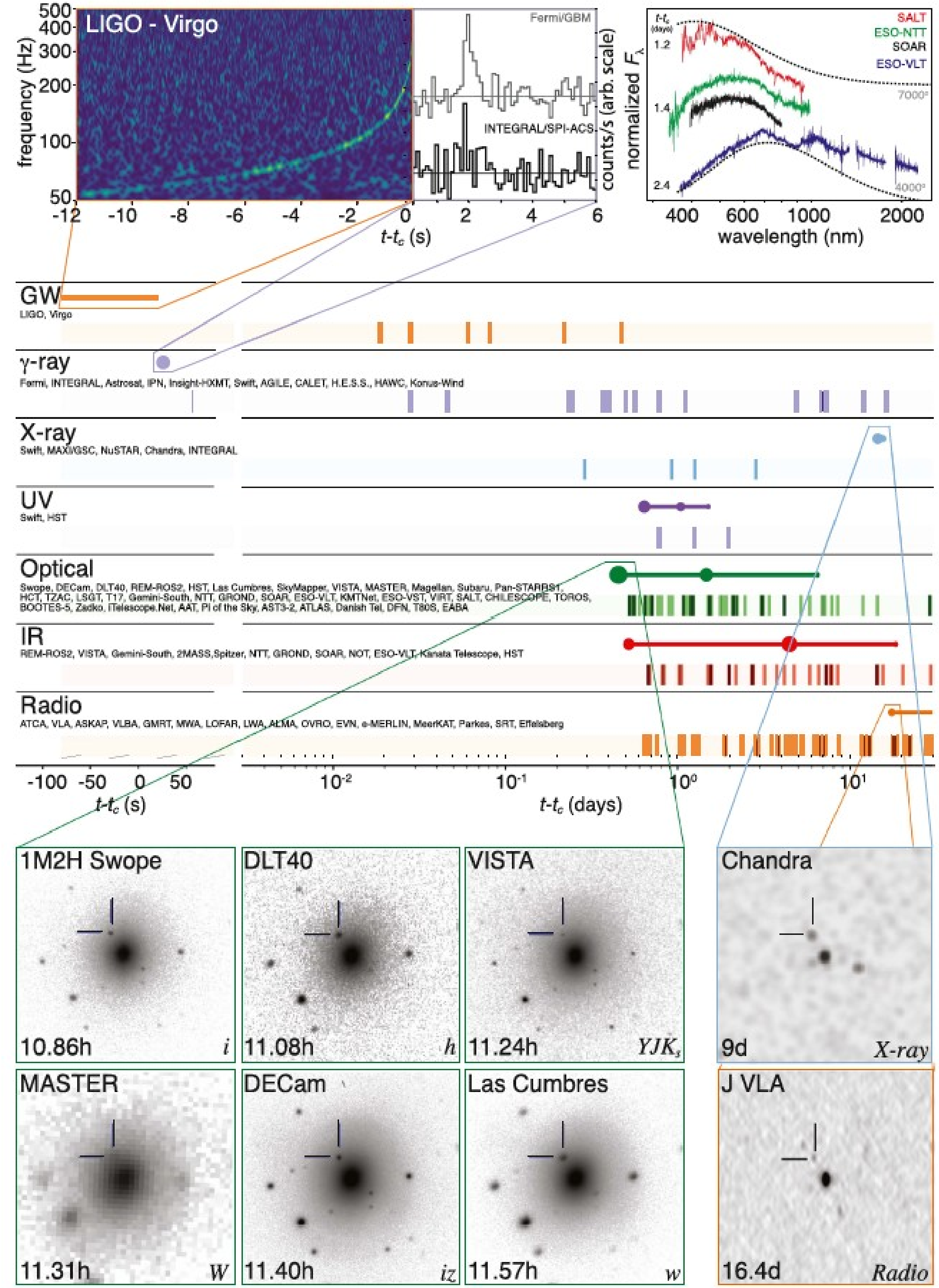
# GW170817: NS-NS Merger

Multi-Messenger Astrophysics !!

M < 2.17 M_sun (arxiv:1710.05938)

| | Low-spin priors ($|\chi| \leq 0.05$) |
|---|---|
| Primary mass $m_1$ | $1.36–1.60\ M_\odot$ |
| Secondary mass $m_2$ | $1.17–1.36\ M_\odot$ |
| Chirp mass $\mathcal{M}$ | $1.188^{+0.004}_{-0.002} M_\odot$ |
| Mass ratio $m_2/m_1$ | $0.7–1.0$ |
| Total mass $m_{tot}$ | $2.74^{+0.04}_{-0.01} M_\odot$ |
| Radiated energy $E_{rad}$ | $> 0.025 M_\odot c^2$ |
| Luminosity distance $D_L$ | $40^{+8}_{-14}$ Mpc |

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# GW170817: NS-NS Merger – Equation of State Constraints

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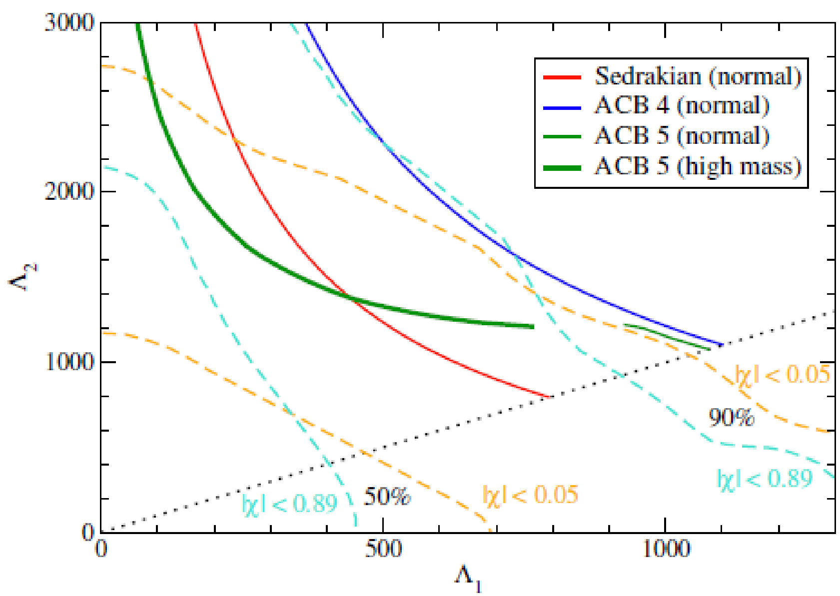
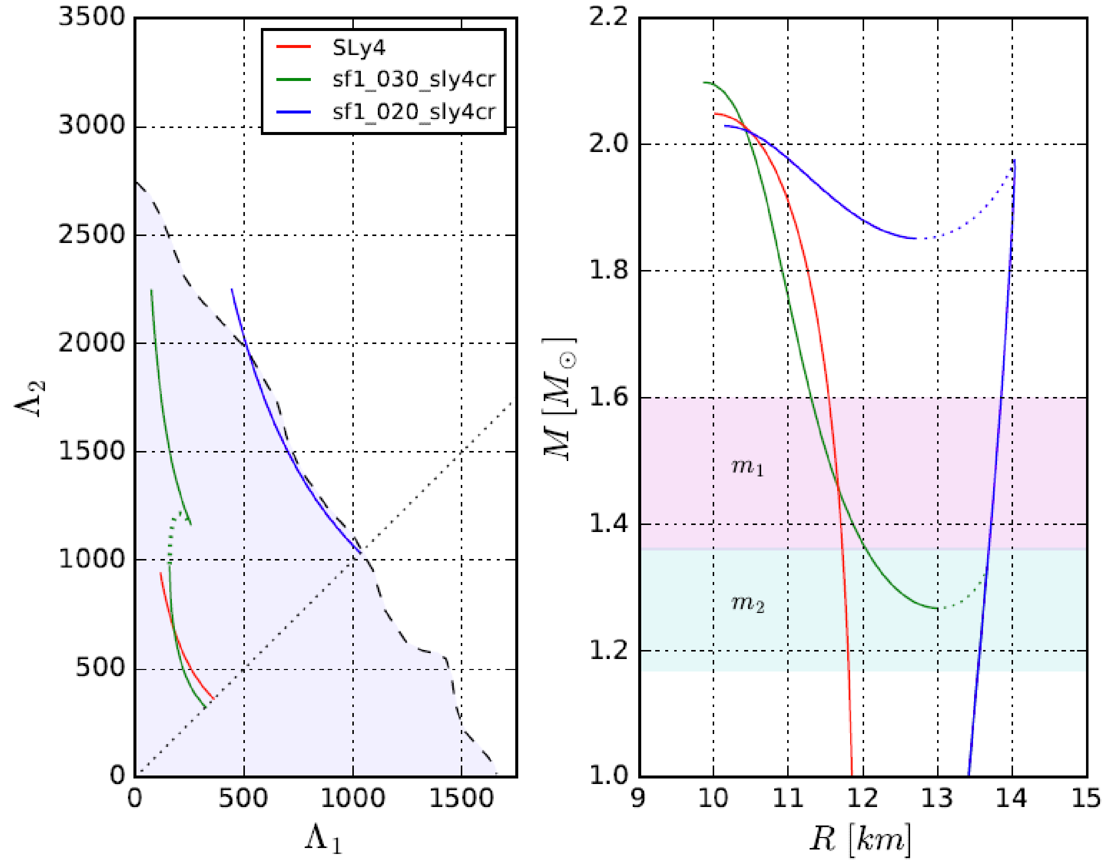



M. Bejger, D.B., et al., in preparation (2017)

D. Alvarez-Castillo, D.B., K. Yagi et al. (2017)

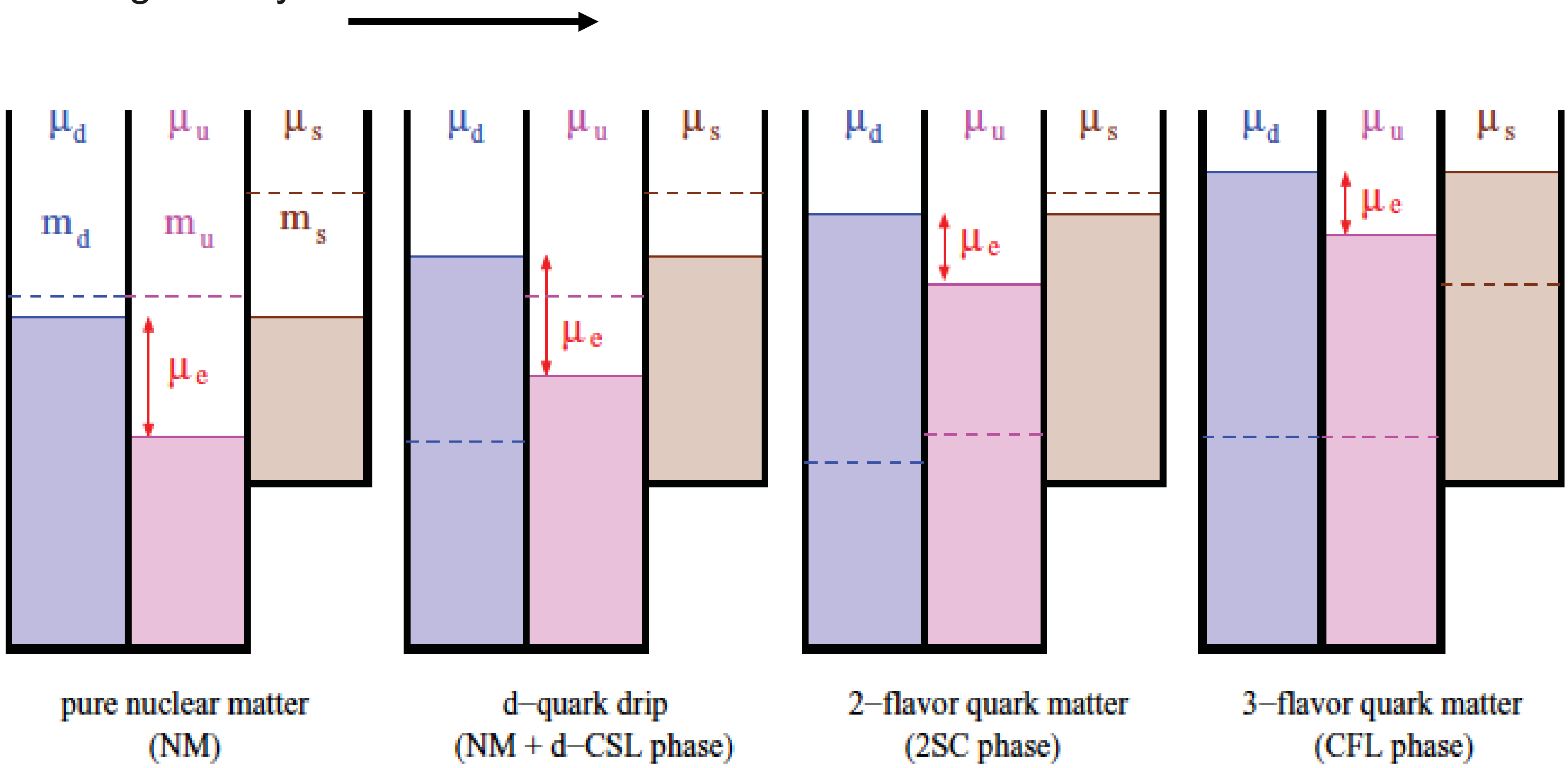
**Question:** can the heavier NS be a member of
The “third family” of hybrid stars with quark core?

# Neutron Star Interiors: Sequential Phase Transitions?

How likely is it that s-quarks (and no s-bar) exist and survive in neutron stars in a QGP or in hyperons. How large is then the ratio s/(u+d) in neutron stars and in the Universe?

There could also be single flavor quark matter, mixed with nuclear matter (d-quark dripline)

Increasing density




D.B., F. Sandin, T. Klaehn, J. Berdermann, PRC 80 (2009) 065807

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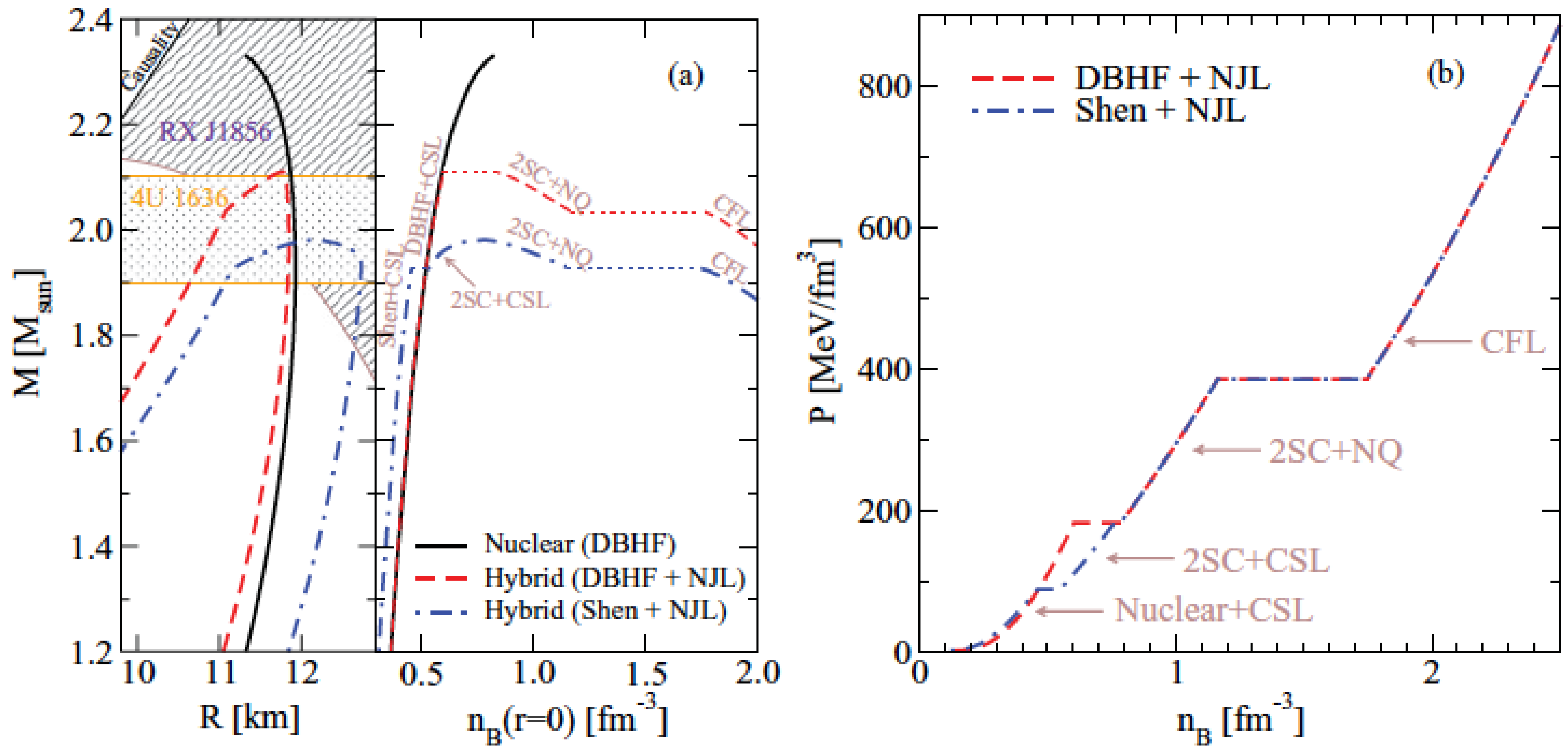
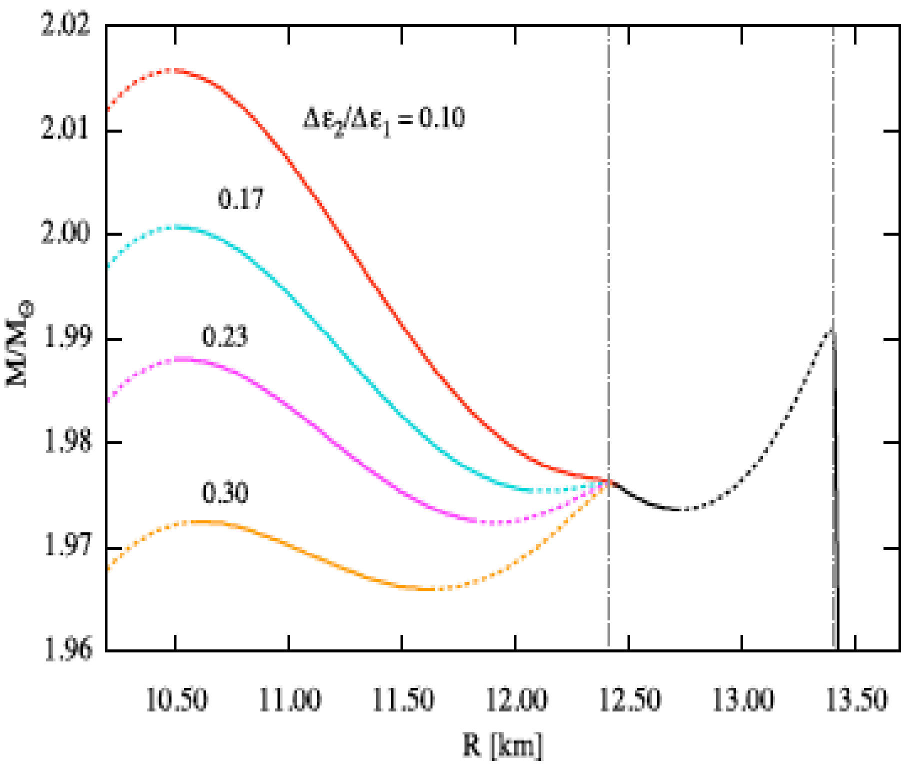
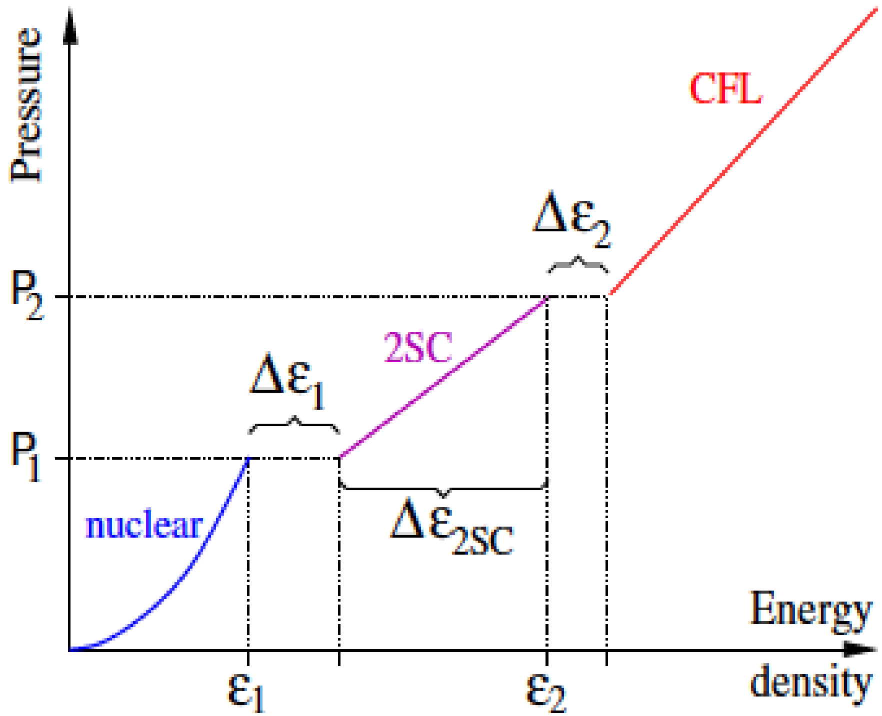



D.B., F. Sandin, T. Klaehn, J. Berdermann, PRC 80 (2009) 065807

# Neutron Star Interiors: Sequential Phase Transitions?

Measuring Mass vs. Radius ⟷ Equation of state



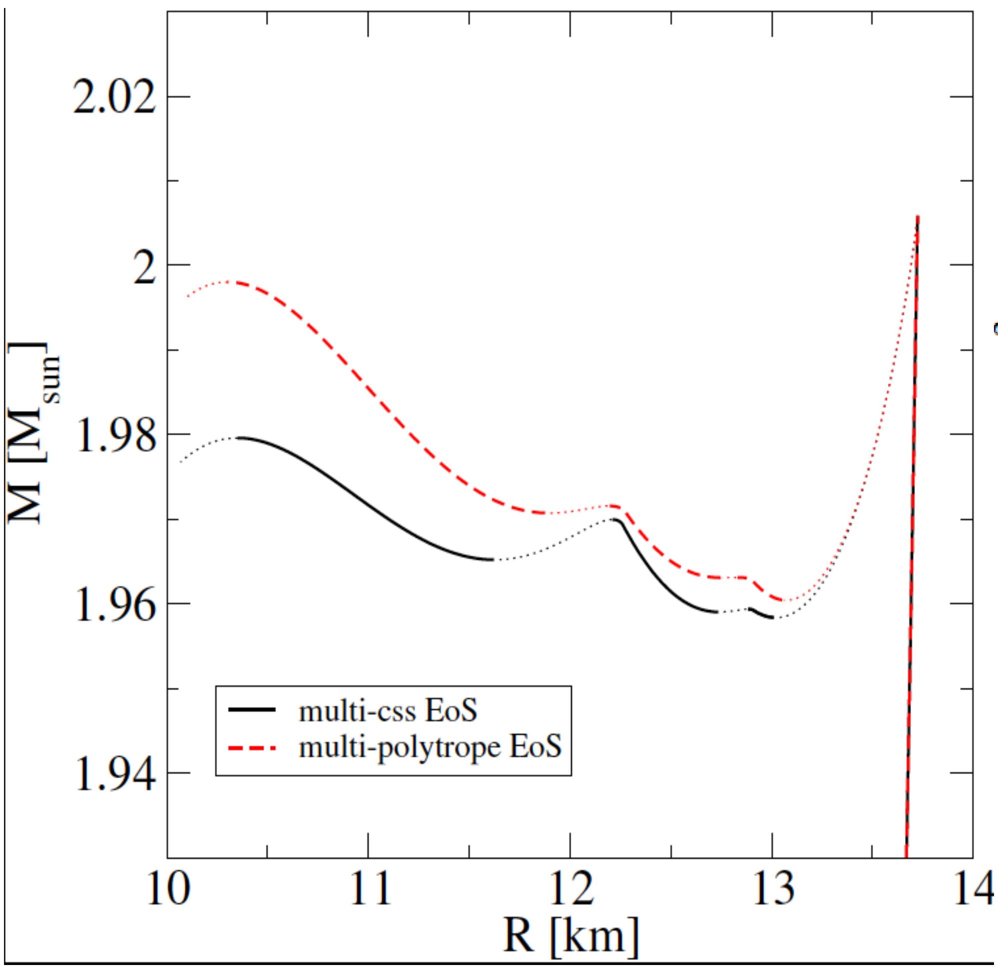



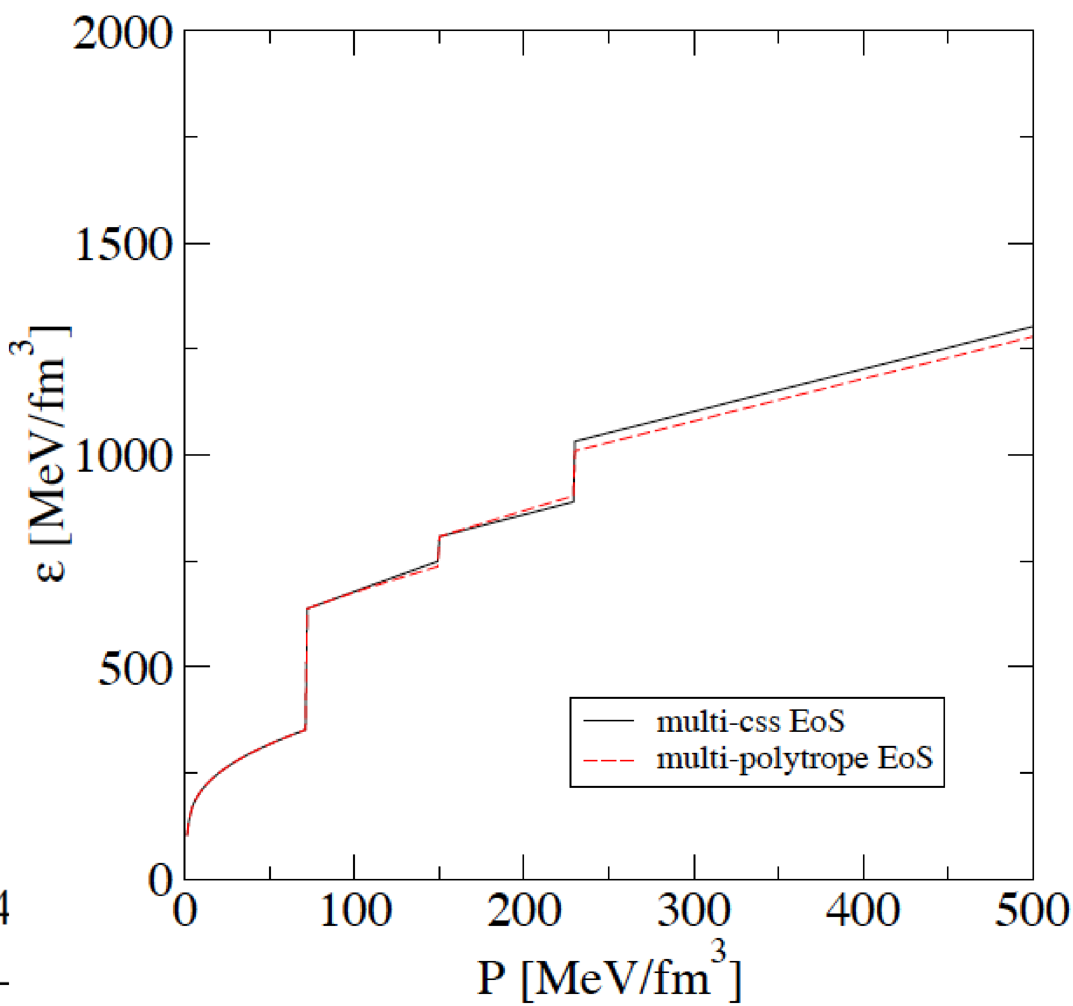

**High-mass twins:**
D. Blaschke et al., PoS CPOD 2013
S. Benic et al., A&A 577 (2015) A50

**High-mass triples and fourth family:**
M. Alford and A. Sedrakian, arxiv:1706.01592
PRL 119 (2017)

# Neutron Star Interiors: Sequential Phase Transitions?

Measuring Mass vs. Radius ⟷ Equation of state

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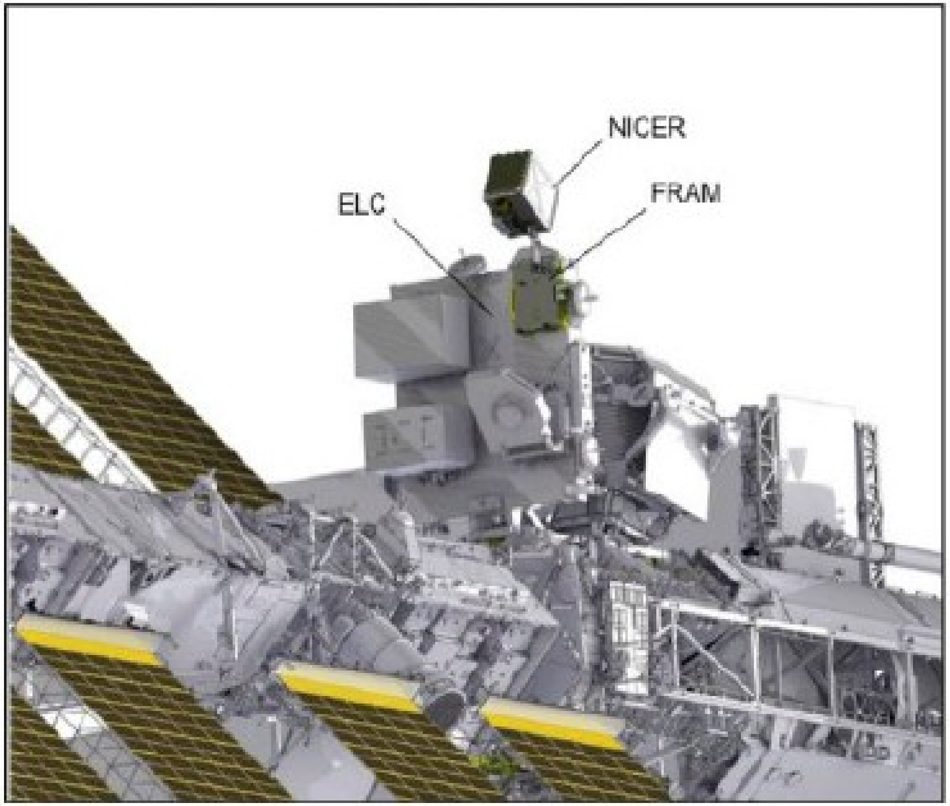
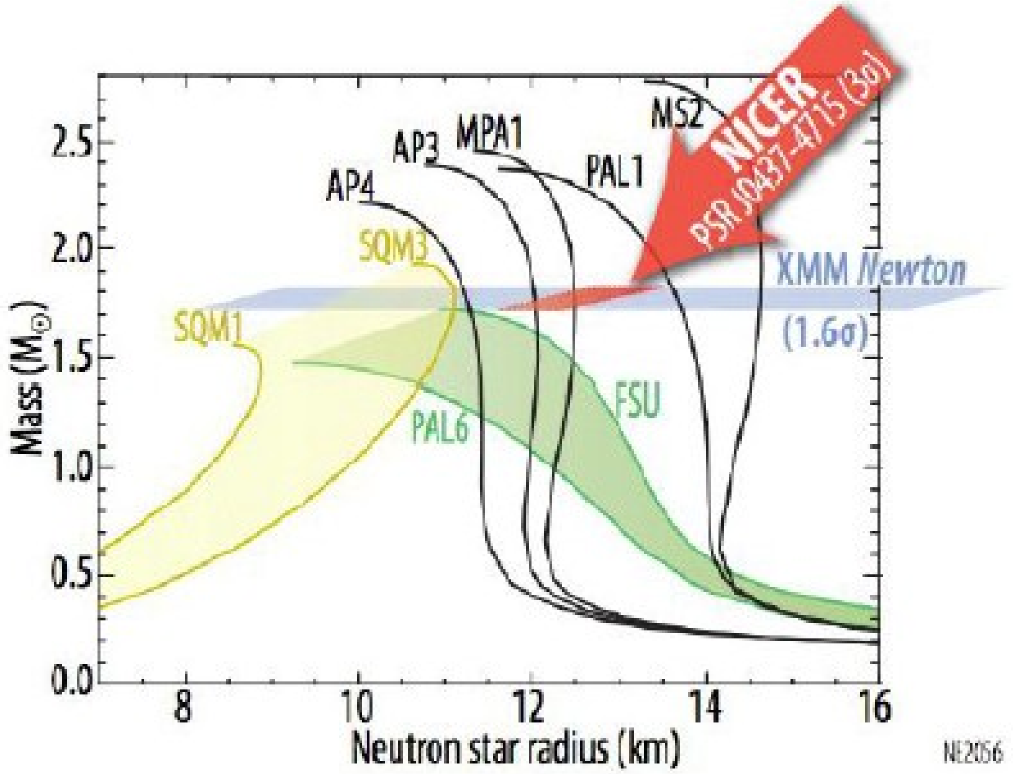
**High-mass triples and fifth family:**
A. Ayriyan, D.B., H. Grigorian, in preparation (2017)

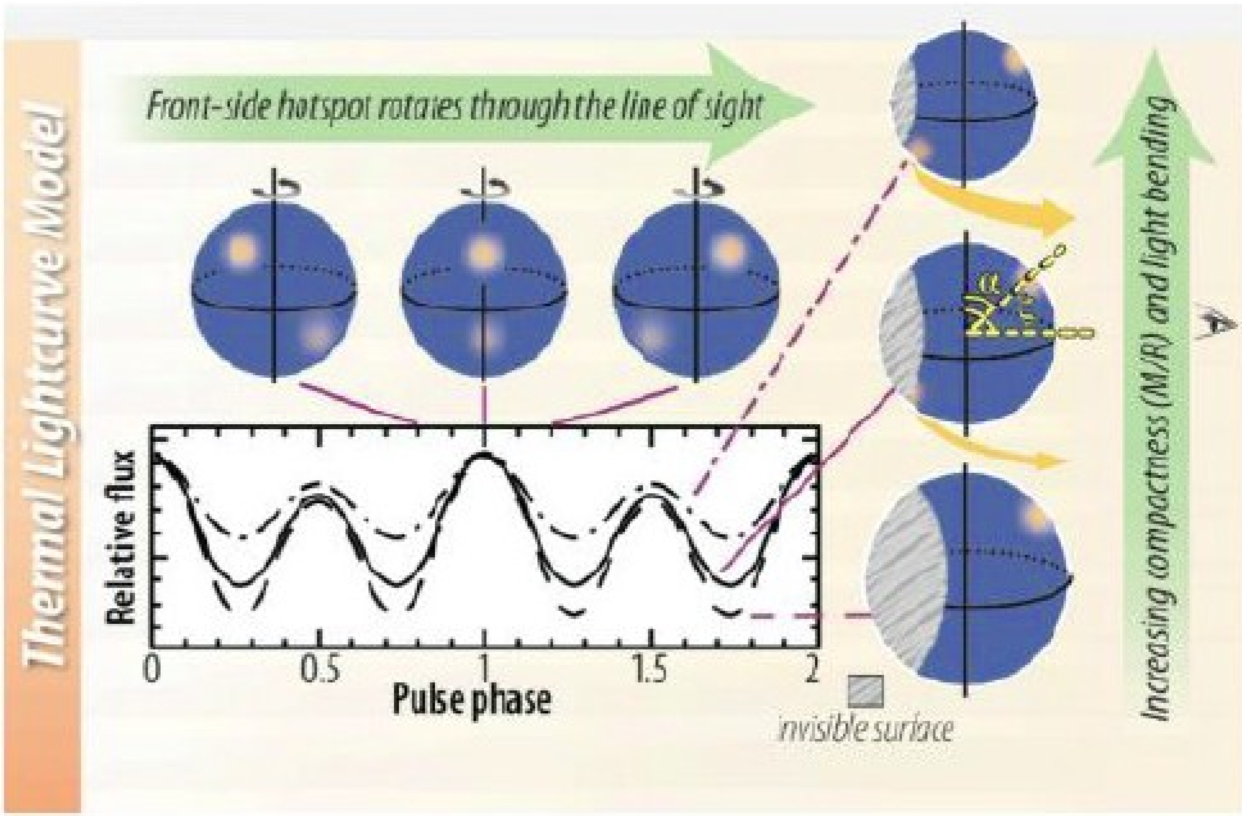



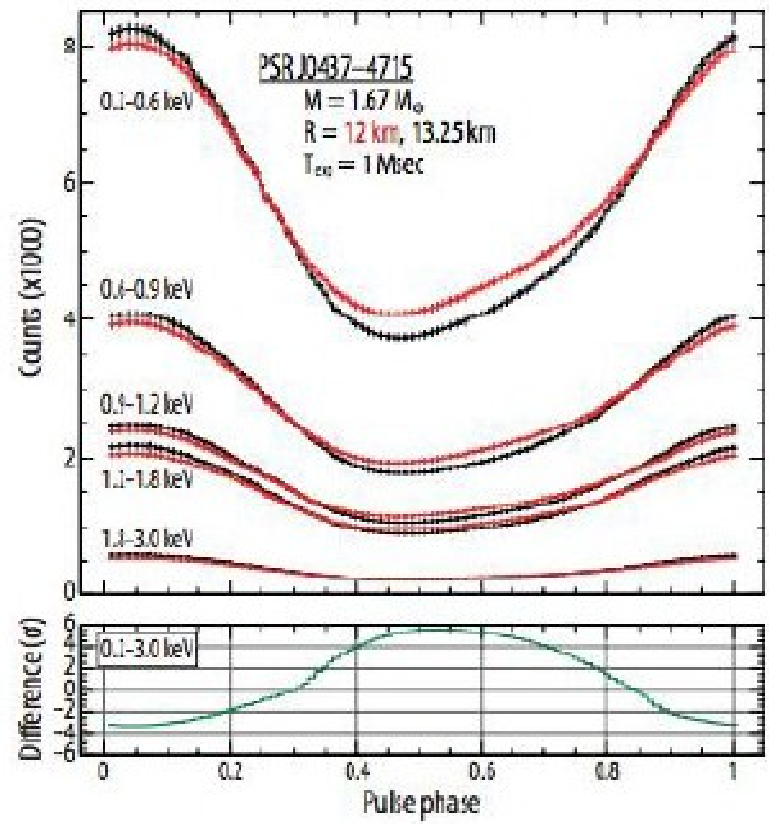




# NICER 2017

Gendreau, K. C., Arzoumanian, Z., & Okajima, T. 2012, Proc. SPIE, 8443, 844313

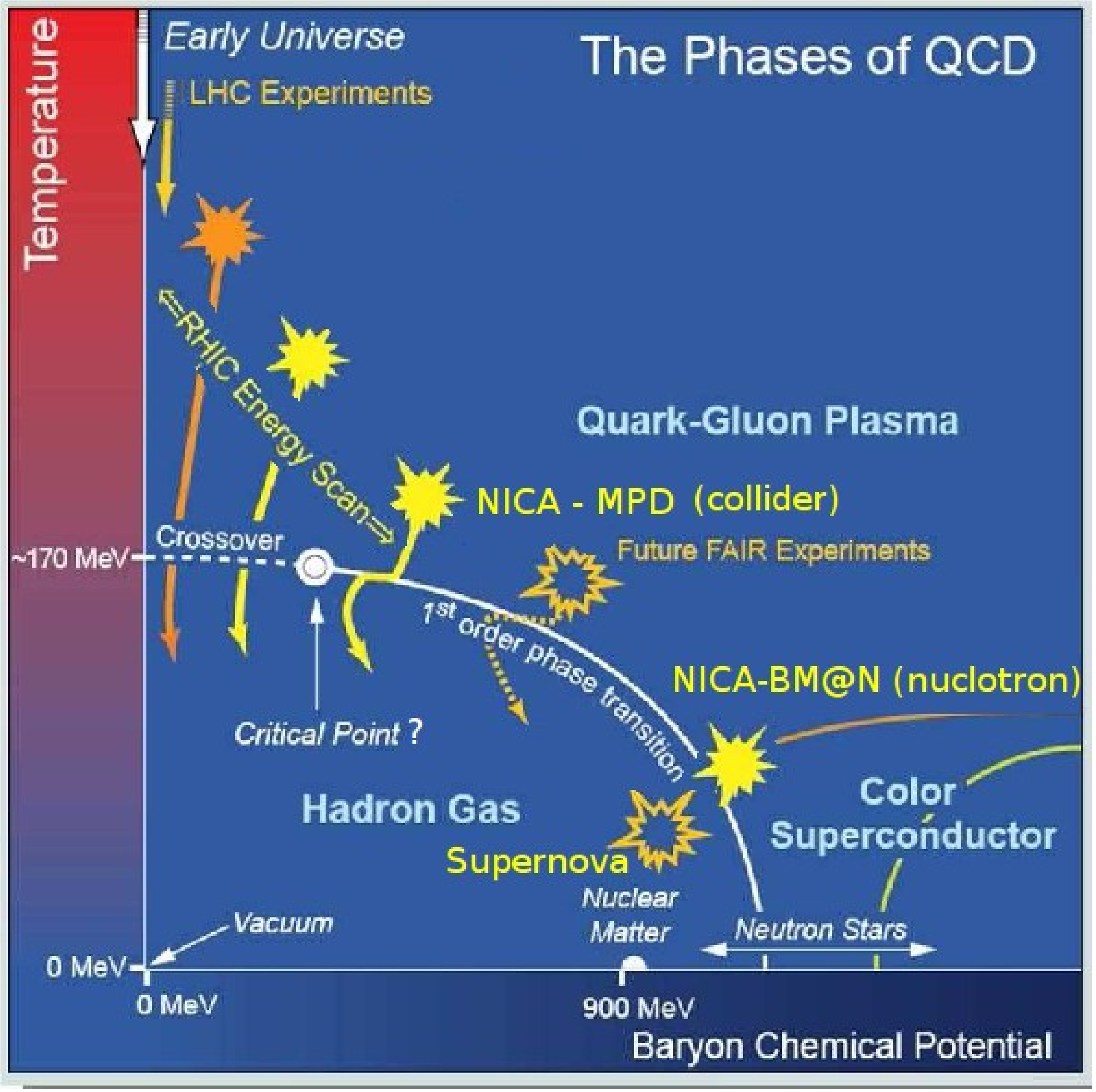

NICER
Neutron star Interior Composition ExploreR


Thermal Lightcurve Model
Front-side hotspot rotates through the line of sight
Increasing compactness (M/R) and light bending
Relative flux
Pulse phase
0
0.5
1
1.5
2
invisible surface


PSR J0437−4715
M = 1.67 M☉
R = 12 km, 13.25 km
T = 1 Msec
0.1–0.6 keV
0.6–0.9 keV
0.9–1.2 keV
1.1–1.8 keV
1.8–3.0 keV
Counts (x1000)
Difference (σ)
0.1–3.0 keV
Pulse phase

# Hot Spots

# Goal: Hadron Dissociation in the QCD Phase Diagram



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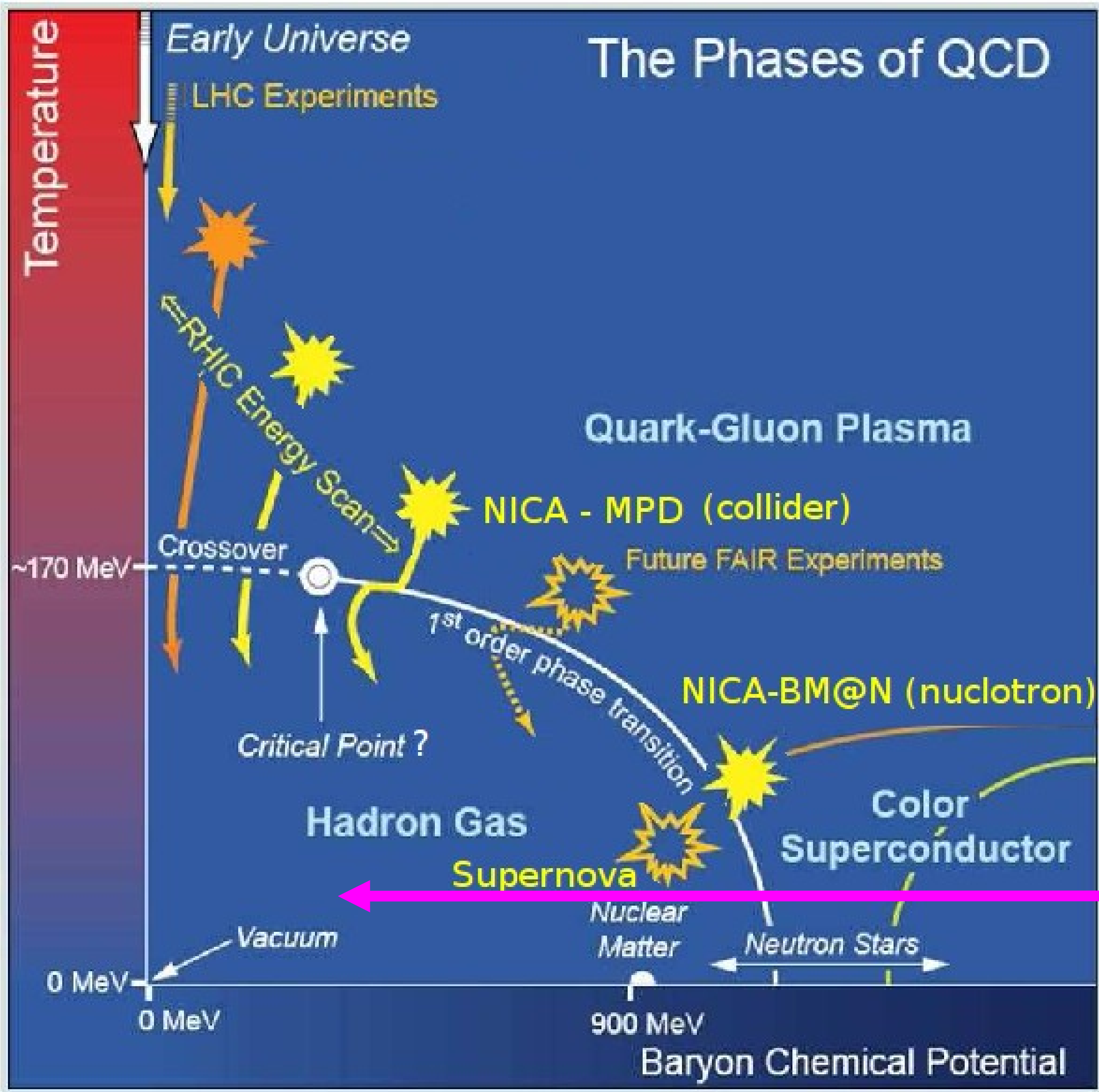


**Statistical Model of Hadron Resonance Gas**

Well established for Description of chemical freezeout

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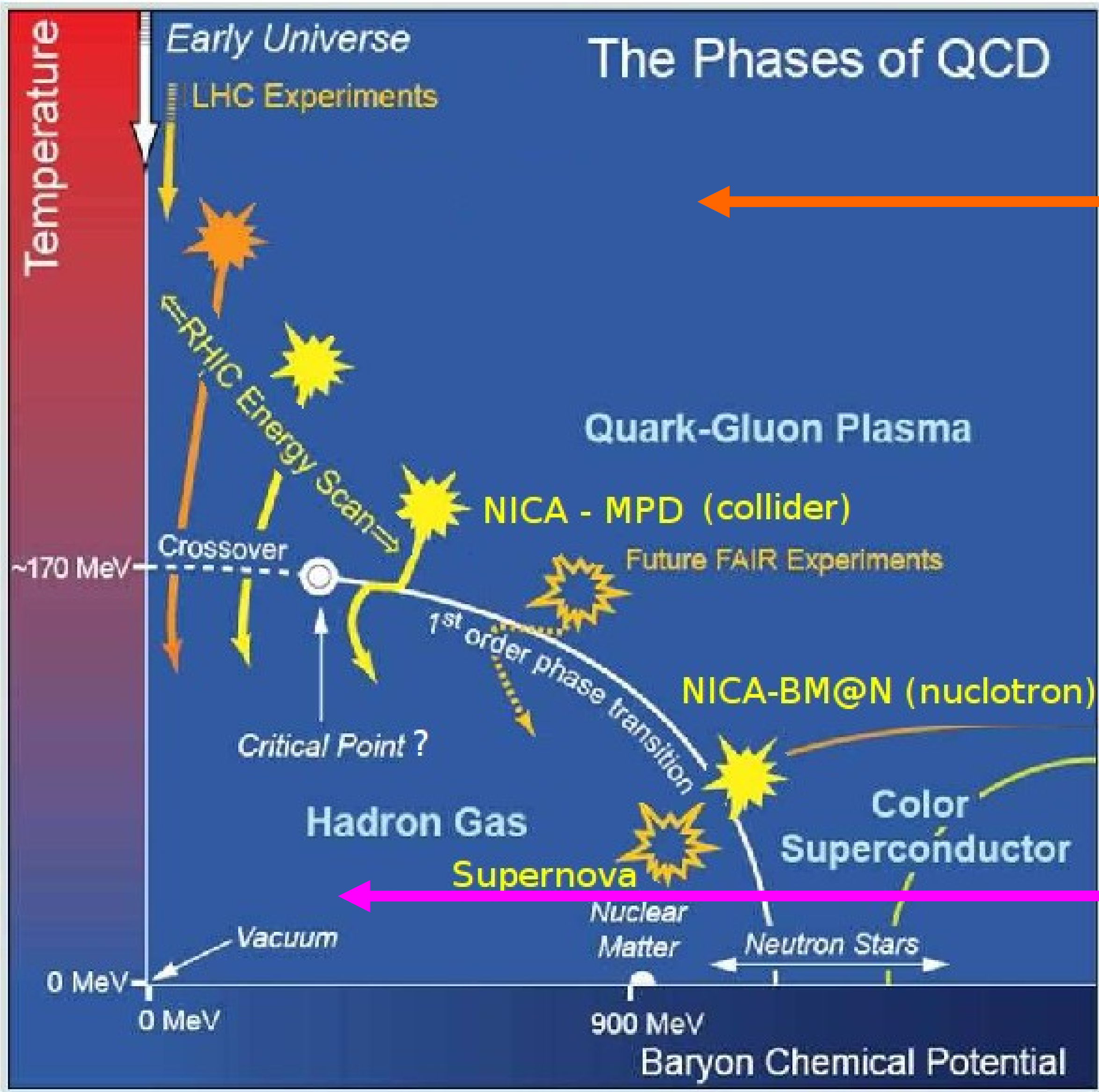


**Perturbative QCD**

Approximately selfconsistent HTL resummation
($T > 2.5\, T_c$ , $\mu > 1500$ MeV)

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Well established for Description of chemical freezeout

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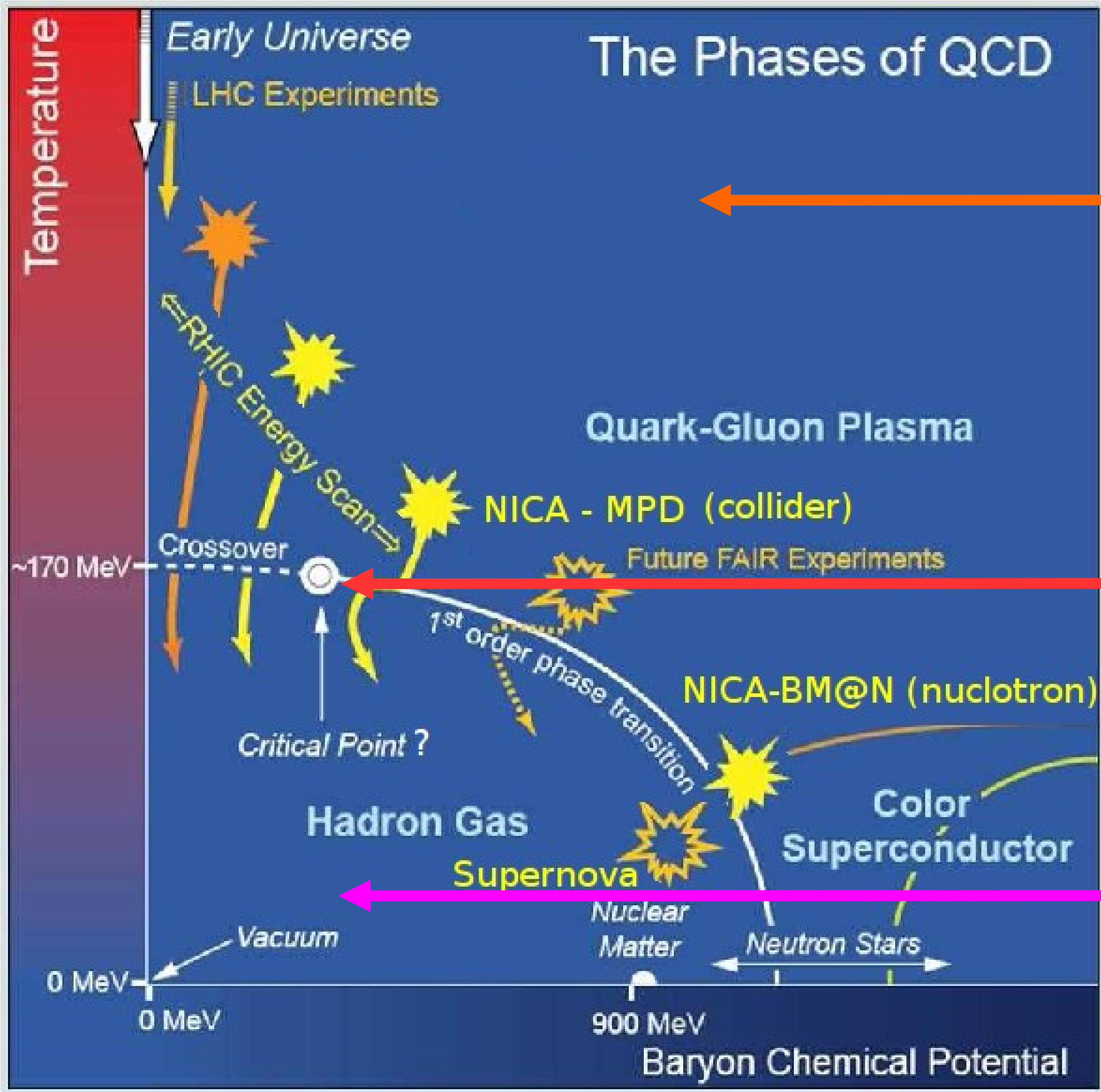



**Perturbative QCD**

Approximately selfconsistent HTL resummation ($T > 2.5\ T_c$ , $\mu > 1500$ MeV)

**QCD Phase transition(s)**

Mott dissociation of hadrons, Deconfinement, $\chi$SR

**Statistical Model of Hadron Resonance Gas**

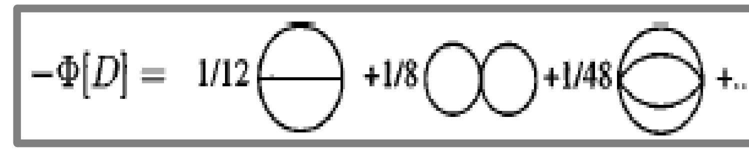
Well established for Description of chemical freezeout

# Φ-derivable approach, 2-loop approximation

J.-P. Blaizot, E. Iancu, A. Rebhan, Phys. Rev. D 63 (2001) 065003

**Skeleton expansion for thermodynamic potential and entropy**

$$\beta\Omega[D] = -\log Z = \frac{1}{2}\mathrm{Tr}\log D^{-1} - \frac{1}{2}\mathrm{Tr}\,\Pi D + \Phi[D]$$




Inv. Temp: 1/T — trace in conf. Space — self-energy related to D

**Dyson equation:** $D^{-1} = D_0^{-1} + \Pi$ Free propagator Do is known

Essential property of Ω[D] is Stationarity under variation of D: δ Ω[D] / δ D = 0

This implies δ Φ[D] / δ D = 1/2 Π

Physical propagator and selfenergy are defined self-consistently !

**Self-consistent approximations are defined by the choice of Φ**

→ **Φ – derivable theories**

G. Baym, Phys. Rev. 127 (1962) 1391; Vanderheyden & Baym; J. Stat. Phys. 93, 843 (1998)

# Approximately selfconsistent thermodynamics

Matsubara summation:

$$\Omega/V = \int \frac{d^4k}{(2\pi)^4} n(\omega)[\mathrm{Im}\log(-\omega^2+k^2+\Pi) - \mathrm{Im}\,\Pi D] \;+ T\Phi[D]/V$$

Analytic properties:

$$D(\omega,k) = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{\rho(k_0,k)}{k_0-\omega}, \qquad \mathrm{Im}\,D(\omega,k) \equiv \mathrm{Im}\,D(\omega+i\epsilon,k) = \frac{\rho(\omega,k)}{2}.$$

Thermodynamics from entropy density: $\mathcal{S} = -\partial(\Omega/V)/\partial T$

$$\mathcal{S} = -\int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Im}\log D^{-1}(\omega,k) \;+ \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Im}\,\Pi(\omega,k) \mathrm{Re}\,D(\omega,k) + \mathcal{S}'$$

$$\mathcal{S}' \equiv -\left.\frac{\partial(T\Phi/V)}{\partial T}\right|_D + \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Re}\,\Pi\,\mathrm{Im}\,D \longrightarrow 0$$

for two-loop skeleton diagrams

Loosely speaking: S' accounts for residual interactions of "independent quasiparticles"

$d/d\omega\,[\,\mathrm{Im}\log D^{-1} + \mathrm{Im}\Pi\,\mathrm{Re}D\,] = 2\,\mathrm{Im}\,[\,D\,\mathrm{Im}\Pi\,(d/d\omega\,D^*)\,\mathrm{Im}\Pi] = 2\sin^2\delta\, d\delta/d\omega$ , for $D = |D|e^{i\delta}$

D. B., G. Baym & G. Roepke, in preparation (2017)

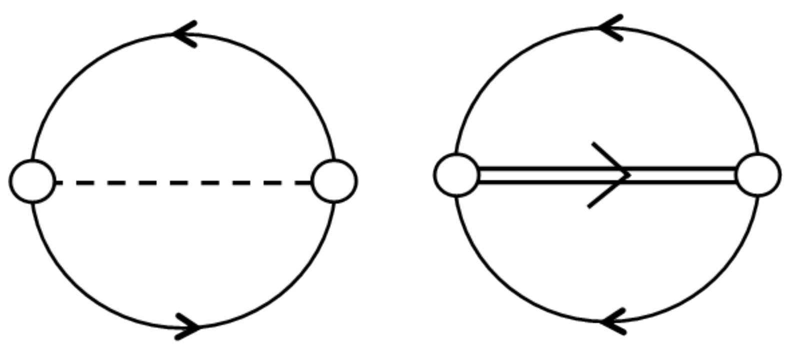
# Φ-derivable Q-M-D PNJL model, 2-loop approximation

$$\Omega = \frac{1}{2}\frac{T}{V}\sum_{i=Q,M,D} c_i \,\mathrm{Tr}\left\{\ln\left[S_i^{-1}\right] + \left[S_i \Pi_i\right]\right\} + \Phi\left[S_Q, S_M, S_D\right] \ ,$$

$$S_i^{-1}(\mathrm{i}z_n, \mathbf{q}) = S_{i,0}^{-1}(\mathrm{i}z_n, \mathbf{q}) - \Pi_i(\mathrm{i}z_n, \mathbf{q}) \ , \qquad \frac{\delta\Omega}{\delta S_i} = 0 \ , \quad \text{if} \quad \Pi_i = \frac{\delta\Omega}{\delta S_i} \ .$$

$$\Omega \ = \ \frac{1}{2}T \sum_{i=Q,M,D} \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f_i(\omega)\,\mathrm{Tr}\left\{\mathrm{Im}\ln\left[S_i^{-1}\right] + \left[\mathrm{Re}\, S_i\, \mathrm{Im}\,\Pi_i\right]\right\} + \tilde{\Omega}$$

$$\tilde{\Omega} \ = \ \Phi\left[S_Q, S_M, S_D\right] - \frac{1}{2}T \sum_{i=Q,M,D} \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f_i(\omega)\,\mathrm{Tr}\left\{\left[\mathrm{Im}\, S_i\, \mathrm{Re}\,\Pi_i\right]\right\} \ ,$$

$\Phi =$ 

$$\mathcal{S} = -\frac{\partial\Omega}{\partial T} = \sum_i \mathcal{S}_i + \tilde{\mathcal{S}}$$

$$\mathcal{N} = -\frac{\partial\Omega}{\partial\mu} = \sum_i \mathcal{N}_i + \tilde{\mathcal{N}} \ .$$

# Φ-derivable Q-M-D PNJL model, 2-loop approximation

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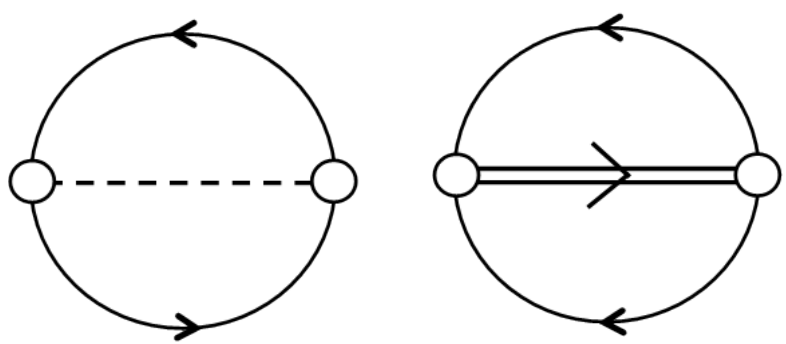
$\Phi =$ 

$$\mathcal{S} = -\frac{\partial\Omega}{\partial T} = \sum_i \mathcal{S}_i + \tilde{\mathcal{S}}$$

$$\mathcal{N} = -\frac{\partial\Omega}{\partial\mu} = \sum_i \mathcal{N}_i + \tilde{\mathcal{N}} .$$

# Φ-derivable Q-M-D PNJL model, 2-loop approximation

$$\left(\mathrm{Im}\ln S^{-1}\right)' = -\,\mathrm{Im}\,(S\Pi') = \underbrace{S'_R\Pi_I - S_I\Pi'_R}_{2\,\mathrm{Im}(S\Pi_I S^{*\prime}\Pi_I)} - \underbrace{(\Pi_I S'_R + S_R\Pi'_I)}_{(\Pi_I S_R)'} \ ,$$

Use optical theorems ...

$$S\Pi_I \ = \ \sin\delta\,\mathrm{e}^{\mathrm{i}\delta} \ , \quad S^{*\prime}\Pi_I = -\mathrm{i}\delta'\sin\delta\,\mathrm{e}^{-\mathrm{i}\delta} \ , \quad 2\,\mathrm{Im}(S\Pi_I S^{*\prime}\Pi_I) = -2\delta'\sin^2\delta \ .$$

## Generalized Beth-Uhlenbeck EoS

$$\Omega \ = \ -\sum_{i=Q,M,D} d_i \int \frac{\mathrm{d}^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, T\ln[1-\mathrm{e}^{-(\omega-\mu_i)/T}]\,\sin^2\delta_i(\omega,\mathbf{q})\frac{\partial\delta_i(\omega,\mathbf{q})}{\partial\omega}$$

Effect of the sin^2 term ... example: Breit-Wigner ...

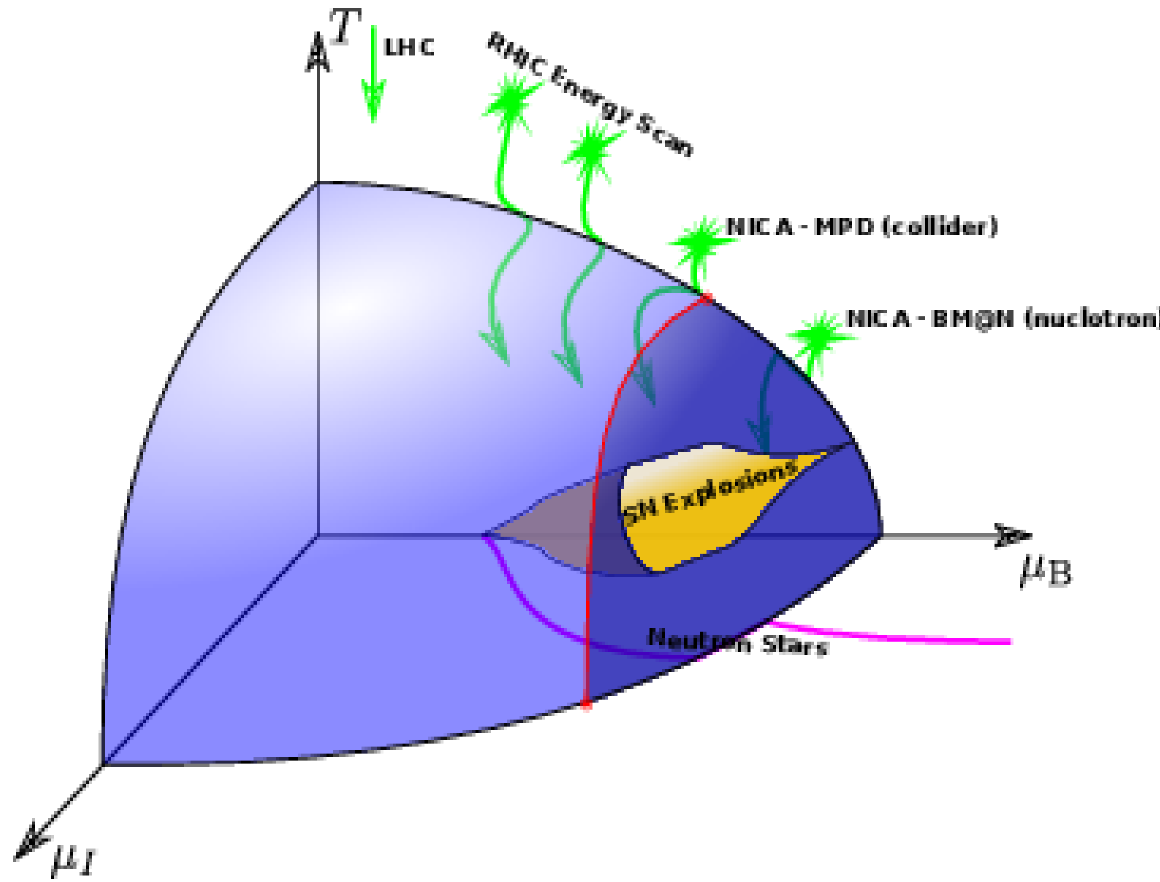
$$\delta_i(\omega) = -\,\arctan\left[\frac{\omega_i\Gamma_i}{\omega^2-\omega_i^2}\right] \ , \quad \frac{\partial\delta_i(\omega)}{\partial\omega} = \frac{2\omega\omega_i\Gamma_i}{(\omega^2-\omega_i^2)^2+\omega_i^2\Gamma_i^2} \ ,$$

$$\sin^2\delta_i(\omega)\frac{\partial\delta_i(\omega)}{\partial\omega} \ = \ \frac{2\omega(\omega_i\Gamma_i)^3}{[(\omega^2-\omega_i^2)^2+\omega_i^2\Gamma_i^2]^2} \ .$$

“Squared Breit-Wigner” ...
Vanderheyden & Baym (1998)
Morozov & Roepke (2009)

# Conclusion:

High-mass twins (HMTs) with quark matter cores can be obtained within different hybrid star EoS models, e.g.,
- constant speed of sound
- higher order NJL
- piecewise polytrope
- density functional

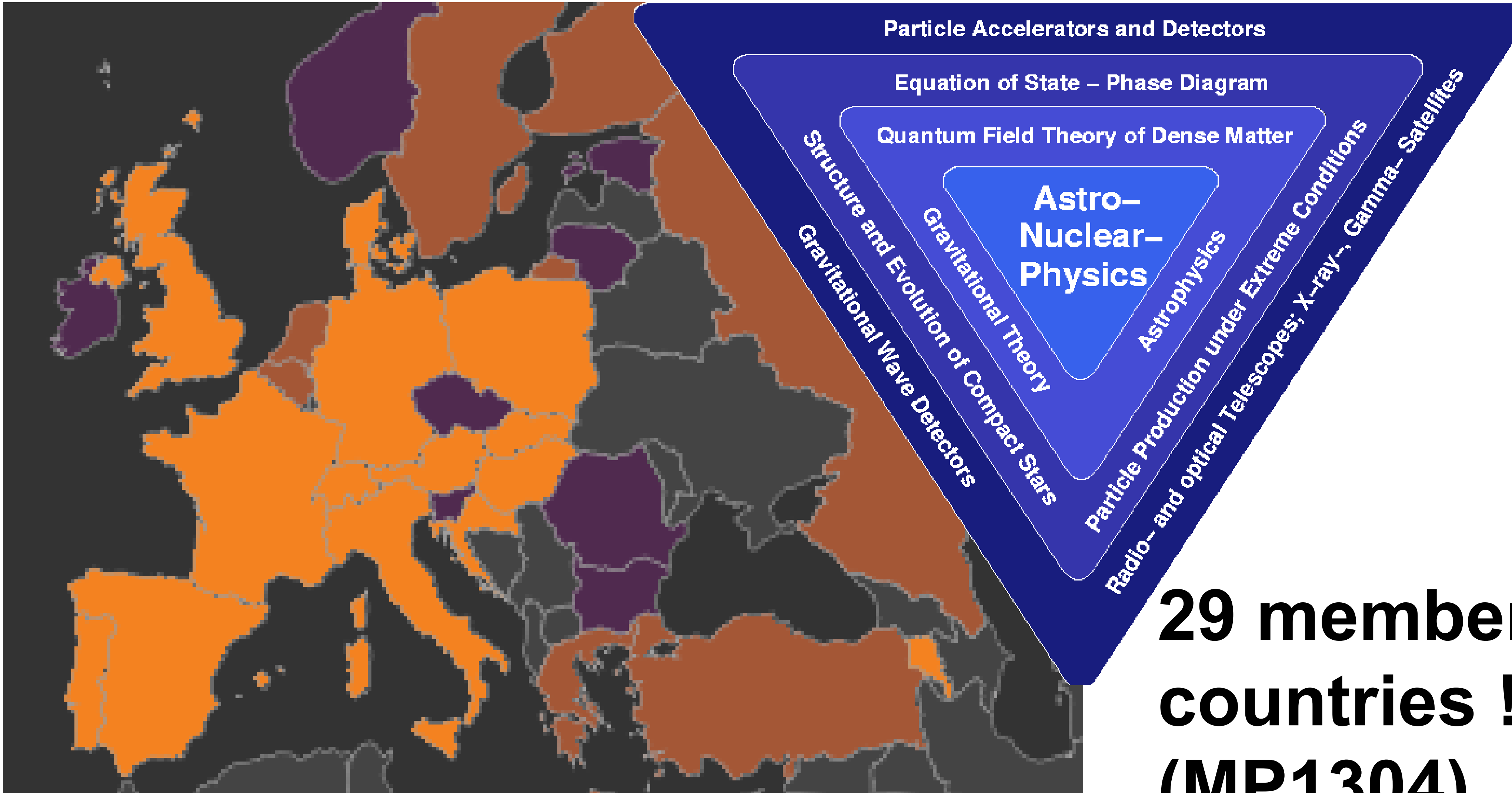



HMTs require stiff hadronic and quark matter EoS with a strong phase transition (PT)

Existence of HMTs can be verified, e.g., by precise compact star mass and radius observations (and a bit of good luck) → Indicator for strong PT !!

Extremely interesting scenarios possible for dynamical evolution of isolated (spin-down and accretion) and binary (NS-NS merger) compact stars

**Critical endpoint search in the QCD phase diagram with Heavy-Ion Collisions goes well together with Compact Star Astrophysics**



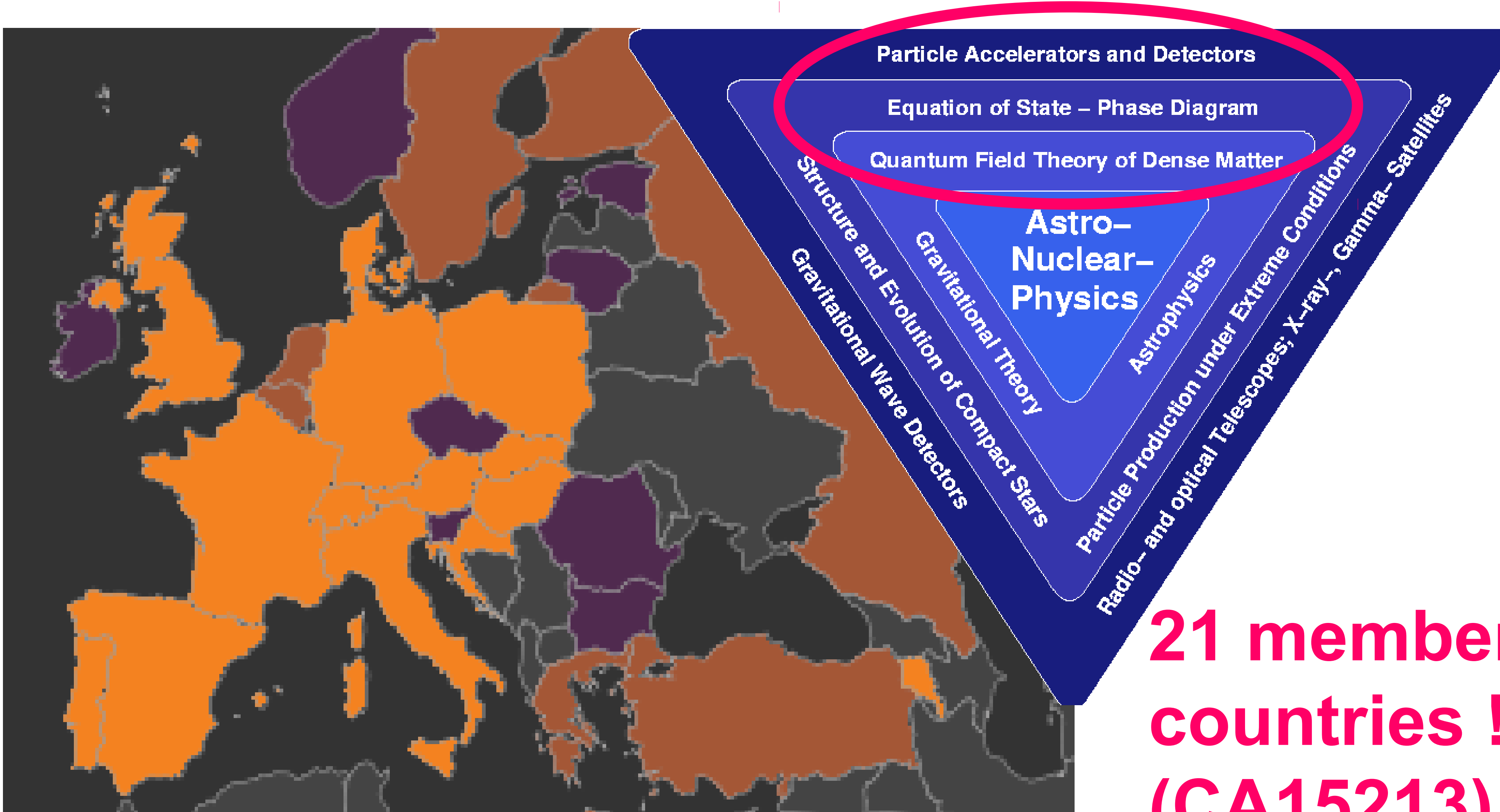

# 29 member countries !! (MP1304)

**New** compstar **!**

cost

EUROPEAN COOPERATION
IN SCIENCE AND TECHNOLOGY

**Kick-off: Brussels, November 25, 2013**




**21 member countries ! (CA15213)**

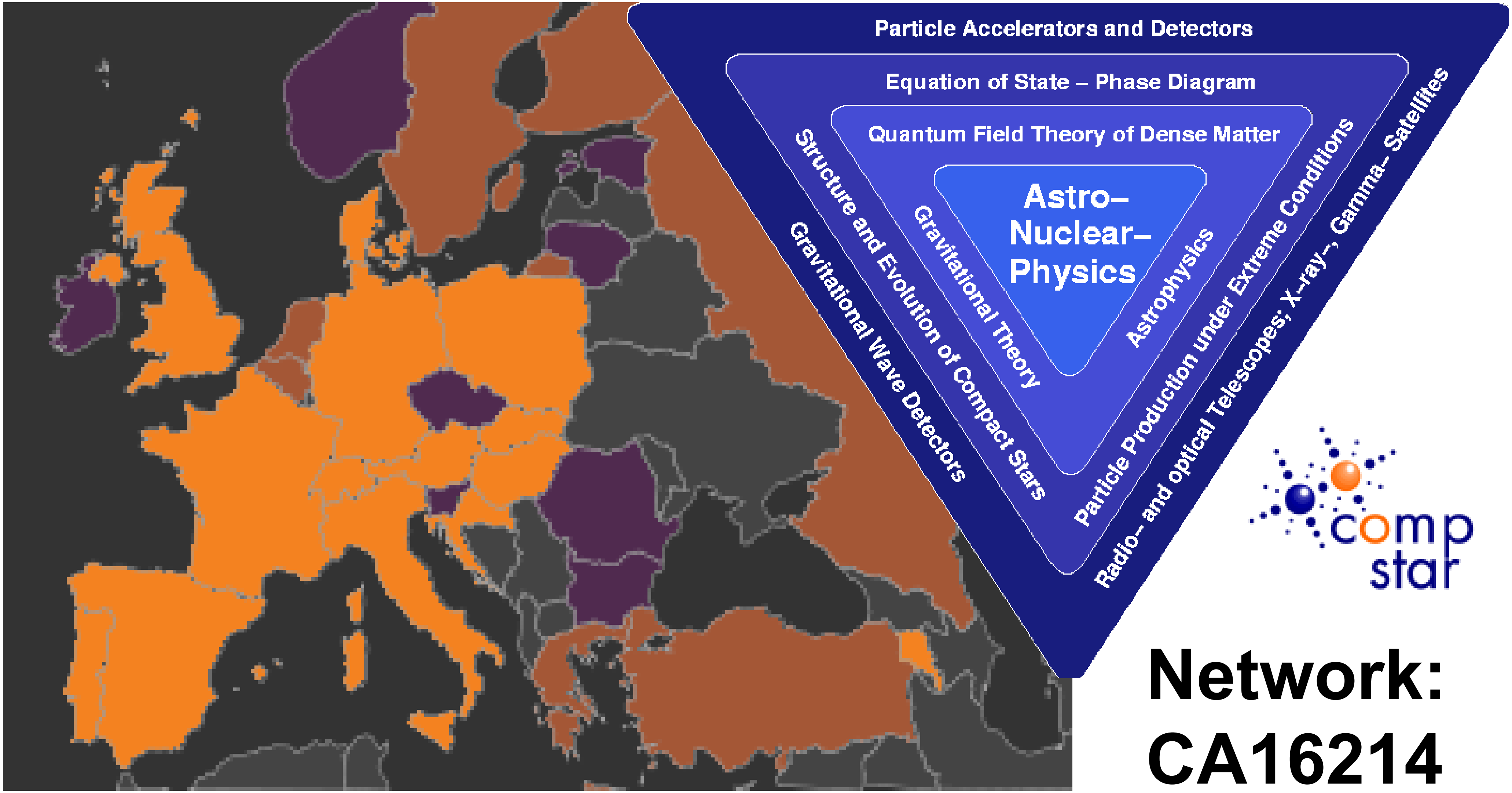
"Theory of HOt Matter in Relativistic Heavy-Ion Collisions"

# New: THOR!

cost

EUROPEAN COOPERATION
IN SCIENCE AND TECHNOLOGY

**Kick-off: Brussels, October 17, 2016**




comp star

# Network: CA16214

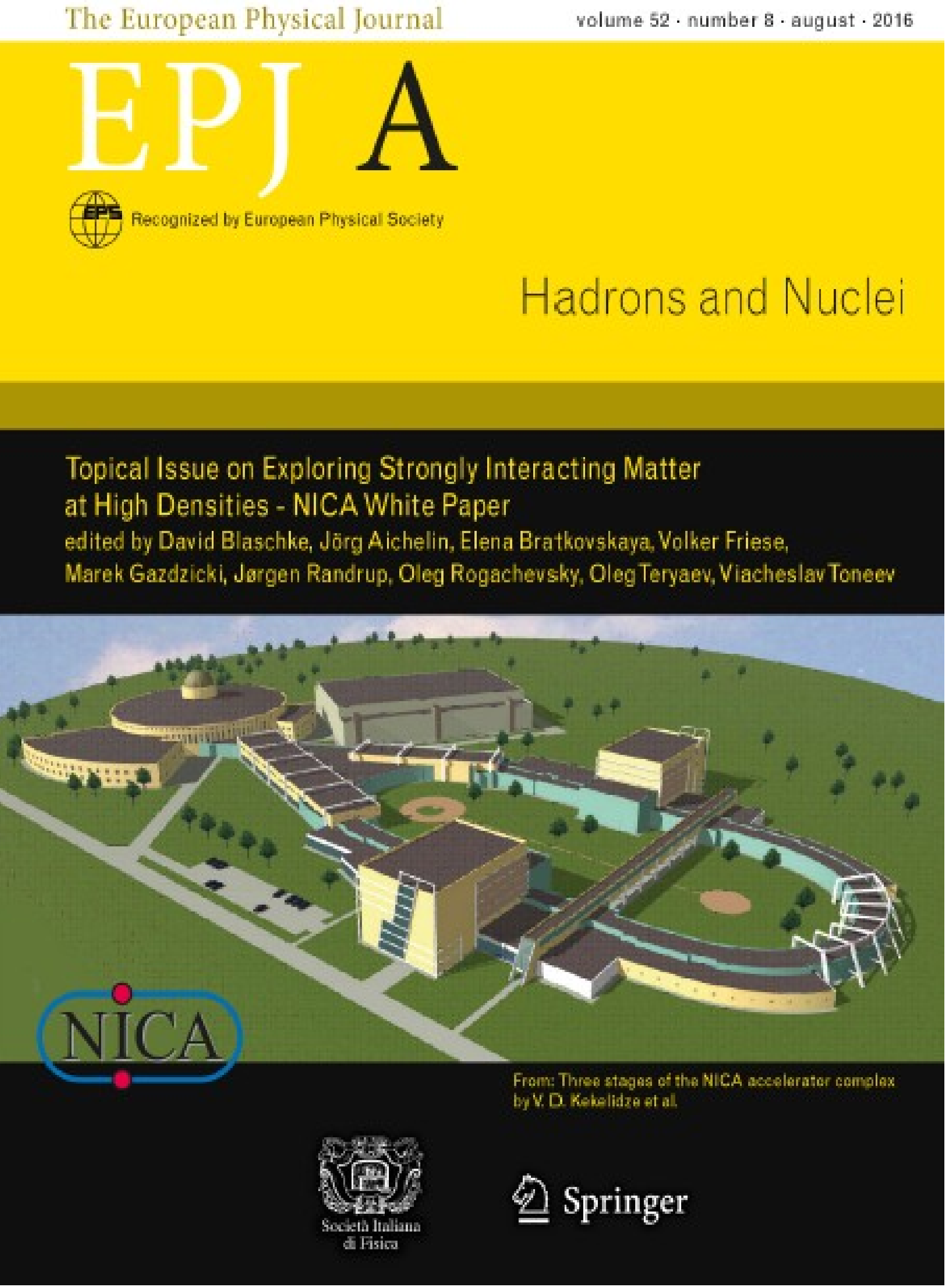
# Newest: PHAROS

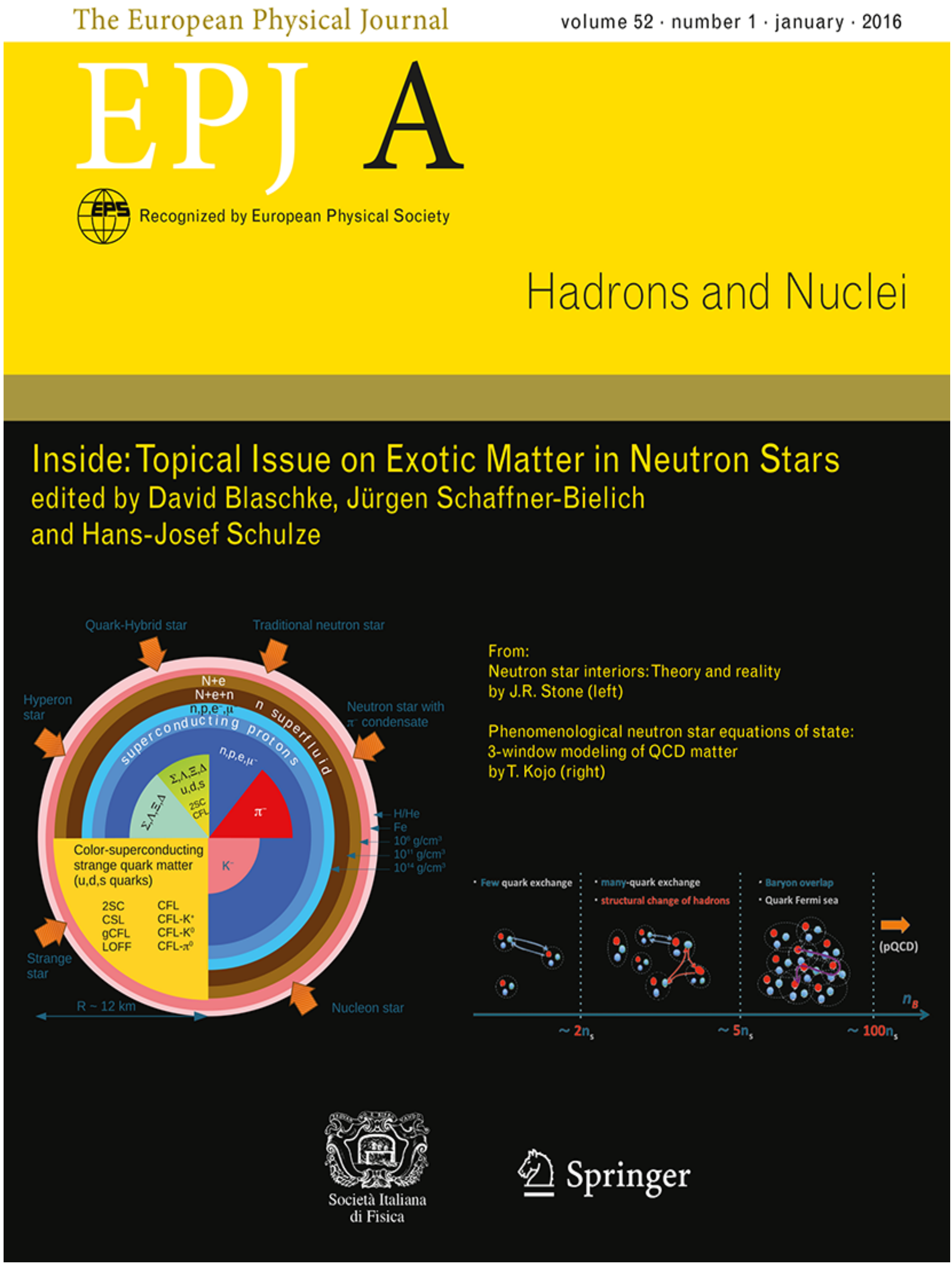
cost
EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

http://www.cost.eu/COST_Actions/ca/CA16214

**Kick-off: Brussels, 22.11. 2017**

International Conference “Critical Point and Onset of Deconfinement”
University of Wroclaw, May 29 – June 4, 2016




EPJA Topical Issues can be found at → http://epja.epj.org/component/list/?task=topic

# Backup slides

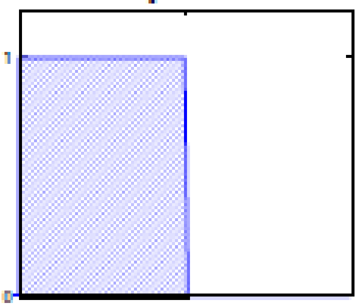
# QCD symmetries - breaking and restoration in compact stars?

- Cooper instability
- Quark condensates
- Symmetries and pairing patterns
- Two-flavor color superconductors → 2SC phase
- Three-flavor color superconductors → CFL phase
- NJL model and Nambu-Gorkov formalism
- Mean field gap equations and solutions
- Thermodynamic potential
- Phase diagram
- EoS and TOV equations – Hybrid stars with CC
- NS phenomenology – GW170817 & NICER

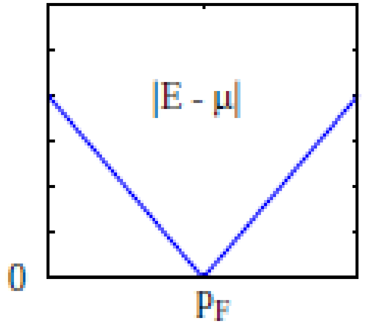
# Cooper instabilities

- ideal Fermi gas:
  - ➜ pair creation @ Fermi surface with no free energy
- (arbitrarily small) attraction:
  - ➜ instability: condensation of Cooper pairs

occupation #



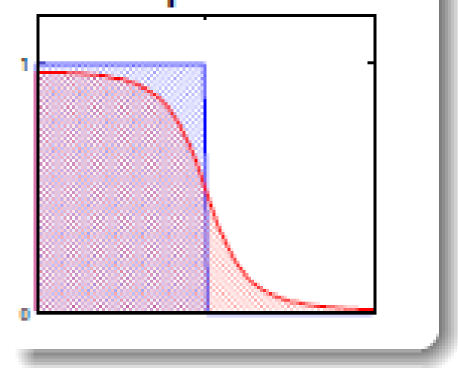
free energy



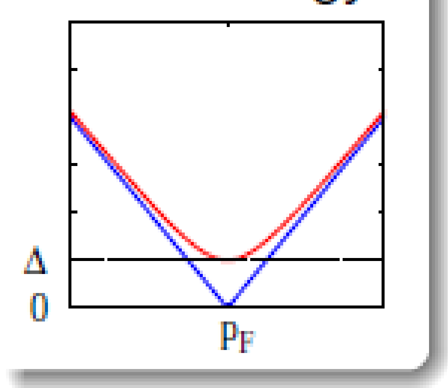
# Cooper instabilities

- ideal Fermi gas:
  - ➜ pair creation @ Fermi surface with no free energy
- (arbitrarily small) attraction:
  - ➜ instability: condensation of Cooper pairs
  - ➜ reorganisation of the Fermi surface
  - ➜ gaps

occupation #




free energy




- QCD: attractive $qq$ interaction ➜ diquark condensates

# Field operators

- quark field operator: $q(x) = \begin{pmatrix} q_1(x) \\ \vdots \\ q_{4N_fN_c}(x) \end{pmatrix}$
  - 4 Dirac $\times\, N_f$ flavor $\times\, N_c$ color components
  - annihilates a quark or creates an antiquark
- transposed operator: $q^T = (q_1, \ldots, q_{4N_fN_c})$
- adjoint operator: $q^\dagger, \qquad \bar{q} = q^\dagger \gamma^0$
  - annihilates an antiquark or creates a quark

# Quark-antiquark condensates

- quark-antiquark condensates: $\langle \bar{q}\, \hat{\mathcal{O}}\, q \rangle$
  - $\hat{\mathcal{O}}$ = operator in color, flavor, and Dirac space (including derivatives)

- examples:
  - "chiral condensate": $\langle \bar{q} q \rangle$
  - quark number density: $\langle \bar{q} \gamma^0 q \rangle = \langle q^\dagger q \rangle$
  - electric charge density:

    $\langle \bar{q}\, \hat{Q} \gamma^0\, q \rangle = \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s, \qquad \hat{Q} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}_f$
  - color charge densities

# Diquark condensates

- diquark condensates: $\langle q^T \hat{O} q \rangle \equiv \langle g.s.| q^T \hat{O} q |g.s.\rangle$
  - $qq$ annihilates two quarks
    ➜ baryon number (formally) not conserved!
    (ground state does not have fixed baryon number.)

- Bogoliubov rotation:

$$|g.s.\rangle = \prod_{\vec{p},s,c,c'} \left[\cos\theta_s^b(\vec{p}) + \varepsilon_{3cc'}\, e^{i\xi_s^b(\vec{p})}\, \sin\theta_s^b(\vec{p})\, b^\dagger(\vec{p},s,u,c)\, b^\dagger(-\vec{p},s,d,c')\right]$$
$$\left[\cos\theta_s^d(\vec{p}) + \varepsilon_{3cc'}\, e^{i\xi_s^d(\vec{p})}\, \sin\theta_s^d(\vec{p})\, d^\dagger(\vec{p},s,u,c)\, d^\dagger(-\vec{p},s,d,c')\right]|0\rangle$$

  - quasiparticles = superpositions of particles and holes (+ antiparticles) which annihilate $|g.s.\rangle$

# Diquark condensates

- diquark condensates: $\langle q^T \hat{\mathcal{O}} q \rangle$

- Pauli principle: $q_i q_j = -q_j q_i$

$$\Rightarrow q^T \hat{\mathcal{O}} q = q_i \hat{\mathcal{O}}_{ij} q_j = -q_j \hat{\mathcal{O}}_{ij} q_i = -q_j \hat{\mathcal{O}}^T_{ji} q_i = -q^T \hat{\mathcal{O}}^T q$$

➜ $\hat{\mathcal{O}}$ must be totally antisymmetric: $\hat{\mathcal{O}}^T = -\hat{\mathcal{O}}$

# Operators in flavor and color space

- Pauli matrices (for two flavors):

$$\underbrace{\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{\text{symmetric triplet}},\quad \underbrace{\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}_{\text{antisymm. singlet}}$$

- Gell-Mann matrices (for three flavors or colors):

$$\underbrace{\mathbb{1},\ \lambda_1,\ \lambda_3,\ \lambda_4,\ \lambda_6,\ \lambda_8,}_{\text{symmetric sextet}}\qquad \underbrace{\lambda_2,\ \lambda_5,\ \lambda_7}_{\text{antisymmetric antitriplet}}$$

  - antitriplet: The vector $\langle q^T \begin{pmatrix} \lambda_7 \\ -\lambda_5 \\ \lambda_2 \end{pmatrix} q \rangle$ transforms like an antiquark $\bar{q} = \begin{pmatrix} \bar{r} \\ \bar{g} \\ \bar{b} \end{pmatrix}$ under $SU(3)_c$.

# Operators in Dirac space

- hermitean basis of $4 \times 4$ matrices: $\mathbb{1}, \; i\gamma_5, \; \gamma^\mu, \; \gamma^\mu\gamma_5, \; \sigma^{\mu\nu}$
- charge conjugation matrix: $C = i\gamma^2\gamma^0$
  - properties: $C = C^* = -C^T = -C^\dagger = -C^{-1}$
- antisymmetric:
  - $C\gamma_5$ (scalar)
  - $C$ (pseudoscalar)
  - $C\gamma^\mu\gamma_5$ (vector)
- symmetric:
  - $C\gamma^\mu$ (axial vector)
  - $C\sigma^{\mu\nu}$ (tensor)

# Combined operators

| | symmetric | antisymmetric |
|---|---|---|
| Dirac | $C\gamma^\mu$, $C\sigma^{\mu\nu}$<br>A T | $C$, $C\gamma_5$, $C\gamma_5\gamma^\mu$<br>P S V |
| $U(2)$ | $\underbrace{\mathbb{1}, \tau_1, \tau_3}_{3}$ | $\underbrace{\tau_2}_{1}$ |
| $U(3)$ | $\underbrace{\mathbb{1}, \lambda_1, \lambda_3, \lambda_4, \lambda_6, \lambda_8}_{6}$ | $\underbrace{\lambda_2, \lambda_5, \lambda_7}_{\bar{3}}$ |

- combination: Dirac $\otimes$ flavor $\otimes$ color

  $$\text{totally antisymmetric} = \begin{cases} (\text{antisymmetric})^3 \\ (\text{symmetric})^2 \times \text{antisymmetric} \end{cases}$$

➜ many possibilities . . .

# Two-flavor color superconductors

- important example: $\langle q^T C\gamma_5 \tau_A \lambda_{A'} q \rangle$
  - spin 0, antisymmetric in color and flavor
  - 2 flavors: $q = \begin{pmatrix} u \\ d \end{pmatrix}$, $\tau_A = \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
  - 3 colors: $q = \begin{pmatrix} r \\ g \\ b \end{pmatrix}$, $\lambda_{A'} \overset{e.g.}{=} \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$\langle q^T C\gamma_5 \tau_2 \lambda_2 q \rangle \sim \underbrace{(\uparrow\downarrow - \downarrow\uparrow)}_{\text{spin}} \otimes \underbrace{(ud - du)}_{\text{flavor}} \otimes \underbrace{(rg - gr)}_{\text{color}}$$

# Symmetry properties: color

- only red and green quarks are paired:

$$\langle q^T \lambda_2 q \rangle \sim \langle rg - gr \rangle \hat{=} \langle \bar{b} \rangle$$ "antiblue"

→ $SU(3)_c$ "spontaneously" broken to $SU(2)_c$

→ 5 of 8 gluons receive a mass ("Meissner effect")

- more general ansatz?

$$\langle q^T (\alpha\lambda_2 + \beta\lambda_5 + \gamma\lambda_7)\, q \rangle = \alpha\langle \bar{b} \rangle - \beta\langle \bar{g} \rangle + \gamma\langle \bar{r} \rangle \equiv \begin{pmatrix} \gamma \\ -\beta \\ \alpha \end{pmatrix}$$

  - can always be rotated into the "antiblue" direction by a global color transformation $q \to \exp(i\theta_a \cdot \frac{\lambda_a}{2})\, q$

→ equivalent to the "simple" ansatz

# Symmetry properties: global symmetries

- $\delta \equiv \langle q^T C\gamma_5 \tau_2 \lambda_2 q \rangle$

- baryon number:

$$B: \quad q \to e^{i\alpha} q \quad \Rightarrow \quad \delta \to e^{2i\alpha}\delta$$ broken

but: There is a conserved *"modified baryon number"*:

$$\tilde{B} \quad q \to e^{i\alpha(1-\sqrt{3}\lambda_8)} q \quad \Rightarrow \quad \delta \to \delta$$

- chiral symmetry:
  - $SU(2)_V$: $\quad q \to e^{i\theta_a \frac{\tau_a}{2}} q \quad \Rightarrow \quad \delta \to \delta$
  - $SU(2)_A$: $\quad q \to e^{i\theta_a \frac{\tau_a}{2}\gamma_5} q \quad \Rightarrow \quad \delta \to \delta$ conserved

# Three-flavor color superconductors

- scalar color-antitriplet condensates:
  - $s_{AA'} = \langle q^T C\gamma_5 \tau_A \lambda_{A'} q \rangle$
  - notation:
    - $\tau_A =$ antisymmetric flavor generator
    - $\lambda_{A'} =$ antisymmetric color generator
- two flavors, three colors:
  - $\tau_A = \tau_2, \quad A' \in \{2,5,7\} \quad \Rightarrow \quad \vec{s} = (s_{22}, s_{25}, s_{27})$
  - can always be rotated into "antiblue" direction:

$$\vec{s} \to \vec{s}\,' = \vec{s}\,U = (s'_{22}, 0, 0), \quad U \in SU(3)_c$$

- three flavors, three colors:
  - $A, A' \in \{2,5,7\} \quad \Rightarrow \quad s = (s_{AA'}) = \begin{pmatrix} s_{22} & s_{25} & s_{27} \\ s_{52} & s_{55} & s_{57} \\ s_{72} & s_{75} & s_{77} \end{pmatrix}$

# Three-flavor color superconductors

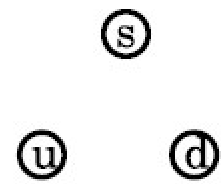
- three flavors, three colors:
  - $A, A' \in \{2,5,7\} \quad \Rightarrow \quad s = (s_{AA'}) = \begin{pmatrix} s_{22} & s_{25} & s_{27} \\ s_{52} & s_{55} & s_{57} \\ s_{72} & s_{75} & s_{77} \end{pmatrix}$
  - $SU(3)_c$ rotation: $\quad \rightarrow \; s' = s\,U = \begin{pmatrix} s_{22} & s_{25} & s_{27} \\ 0 & s_{55} & s_{57} \\ 0 & 0 & s_{77} \end{pmatrix}$
  - In general, that's all we can do ...
- three degenerate flavors: $M_u = M_d = M_s$
  - ➜ $SU(3)_f$-symmetric
  - ➜ diagonalization by combined color and flavor rotations:

$$s \rightarrow s' = V\,s\,U = \begin{pmatrix} s_{22} & 0 & 0 \\ 0 & s_{55} & 0 \\ 0 & 0 & s_{77} \end{pmatrix}, \quad U \in SU(3)_c,\; V \in SU(3)_f$$

# Pairing patterns

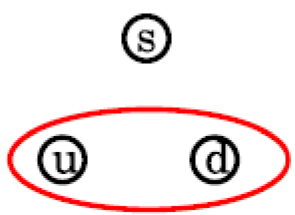
- eight possible phases:

normal quark matter (NQ)
$s_{22} = s_{55} = s_{77} = 0$



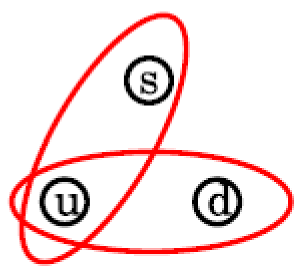

2SC phase
$s_{22} \neq 0, \quad s_{55} = s_{77} = 0$

+ two more phases of this kind

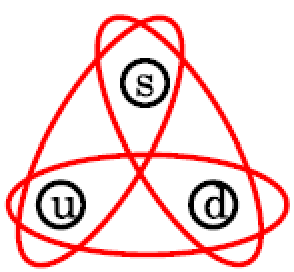


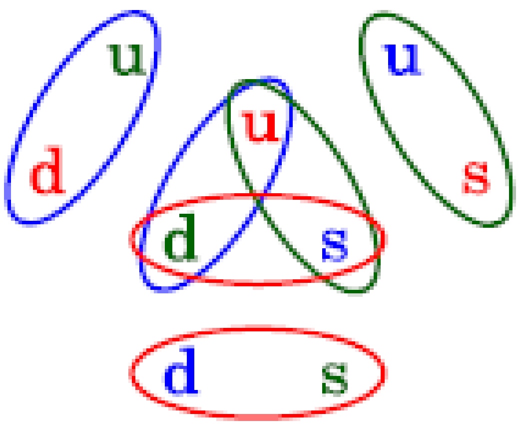

uSC phase
$s_{22},\ s_{55} \neq 0, \quad s_{77} = 0$

+ two more phases of this kind




CFL phase
$s_{22},\ s_{55},\ s_{77} \neq 0$




- CFL pairing pattern (more explicitly):

$$(\uparrow\downarrow - \downarrow\uparrow) \otimes \Big( \Delta_2\,(ud - du) \otimes (r\,g - g\,r) + \Delta_5\,(ds - sd) \otimes (g\,b - b\,g) + \Delta_7\,(su - us) \otimes (b\,r - r\,b) \Big)$$

# Color-flavor locking

- symmetries:
  - color: $SU(3)_c$ completely broken → 8 massive gluons
  - chiral: $SU(3)_A$ " → 8 Goldstone bosons
    $SU(3)_V$ "

but: symm. under "locked" color-flavor rotations $q \to e^{i\theta_a(\tau_a - \lambda_a^T)} q$

  - baryon #: broken → 1 scalar Goldstone boson

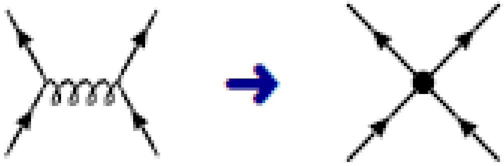
- electromagnetism:
  - invariant under (local) $q \to \exp(i\alpha\tilde{Q})q$
    $\tilde{Q} = Q - \frac{\lambda_3}{2} - \frac{\lambda_8}{2\sqrt{3}} = \mathrm{diag}_f(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}) - \mathrm{diag}_c(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$
  - "rotated photon" $= \cos\varphi$ photon $+ \sin\varphi$ gluon

  → no electromagnetic Meissner effect!

  - all quarks carry integer $\tilde{Q}$ charge

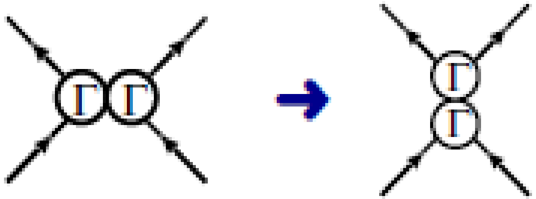
# NJL model for color superconductivity

- "color-current interaction"
  - replace gluon exchange by point interactions:



$$\mathcal{L}_{int}(x) \stackrel{e.g.}{=} -g\,(\bar{q}(x)\,\gamma^{\mu}\lambda_a\,q(x))^2$$

- Fierz transformation
  - identically rewrite particle-antiparticle interactions as particle particle interactions:




$$(\bar{q}\,\Gamma^{(I)}\,q)^2 = \sum_D d_D^I\,(\bar{q}\Gamma^{(D)}C\bar{q}^T)(q^T C\Gamma^{(D)}q)$$

- toy model (two flavors):

$$\mathcal{L}_{int} = H \sum_{A=2,5,7} (\bar{q}\,i\gamma_5\tau_2\lambda_A\,C\bar{q}^T)(q^T C\,i\gamma_5\tau_2\lambda_A\,q)$$

$$(H = \frac{N_c+1}{2N_c}\,g)$$

# Nambu-Gorkov formalism

- interaction Lagrangian:

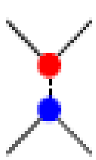
$$\mathcal{L}_{int} = H \sum_{A=2,5,7} (\bar{q}\, i\gamma_5\tau_2\lambda_A\, C\bar{q}^T)(q^T C\, i\gamma_5\tau_2\lambda_A\, q)$$

- artificially double # d.o.f. *(Nambu-Gorkov spinors)*:

$$\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} q \\ C\bar{q}^T \end{pmatrix}, \qquad \bar{\Psi} = \frac{1}{\sqrt{2}} \left(\bar{q}, q^T C\right)$$

→ $$\mathcal{L}_{int} = 4H \sum_{A=2,5,7} \bar{\Psi} \begin{pmatrix} 0 & i\gamma_5\tau_2\lambda_A \\ 0 & 0 \end{pmatrix} \Psi\, \bar{\Psi} \begin{pmatrix} 0 & 0 \\ i\gamma_5\tau_2\lambda_A & 0 \end{pmatrix} \Psi$$

→ vertices:



$$= 4Hi\, \Gamma_A^{\uparrow} \otimes \Gamma_A^{\downarrow} = 4Hi \begin{pmatrix} 0 & i\gamma_5\tau_2\lambda_A \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ i\gamma_5\tau_2\lambda_A & 0 \end{pmatrix}$$

# Nambu-Gorkov propagator

- kinetic term + chemical potential:

$$\begin{aligned}\mathcal{L}_{kin} + \mu\, q^\dagger q &= \bar{q}(i\partial\!\!\!/ + \mu\gamma^0)q \\ &= \frac{1}{2}\left[\,\bar{q}(i\partial\!\!\!/ + \mu\gamma^0)q \;-\; q^T C(i\overleftarrow{\partial\!\!\!/} + \mu\gamma^0)C\bar{q}^T\,\right] \\ &= \bar{\Psi}\begin{pmatrix} i\partial\!\!\!/ + \mu\gamma^0 & 0 \\ 0 & -i\overleftarrow{\partial\!\!\!/} - \mu\gamma^0 \end{pmatrix}\Psi \\ &= \bar{\Psi}(x)\, S_0^{-1}(x)\, \Psi(x)\end{aligned}$$

➔ inverse bare propagator in momentum space:

$$S_0^{-1}(p) \;=\; \begin{pmatrix} p\!\!\!/ + \mu\gamma^0 & 0 \\ 0 & p\!\!\!/ - \mu\gamma^0 \end{pmatrix}$$

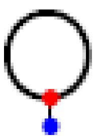
# Selfconsistency problem

- dressed propagator (Hartree approximation):


$$iS(p) \;=\; iS_0(p) \;+\; iS_0(p)\,(-i\Sigma)\,iS(p)$$

$$\Leftrightarrow\; S^{-1}(p) \;=\; S_0^{-1}(p) \;-\; \Sigma$$

- self-energy:

$-i\Sigma =$  $= 4iH \sum_A \Big\{ \;\Gamma_A^{\uparrow} \int \frac{d^4k}{(2\pi)^4}\,(-\tfrac{1}{2})\mathrm{Tr}[\Gamma_A^{\downarrow}\, iS(k)] + \Gamma_A^{\downarrow} \int \frac{d^4k}{(2\pi)^4}\,(-\tfrac{1}{2})\mathrm{Tr}[\Gamma_A^{\uparrow}\, iS(k)] \Big\}$

➜ selfconsistency problem!

# Gap equation

- selfconsistency problem: $S^{-1} = S_0^{-1} - \Sigma[S]$
- ansatz:

$$\Sigma = \begin{pmatrix} 0 & -\Delta\,\gamma_5\tau_2\lambda_2 \\ \Delta^*\,\gamma_5\tau_2\lambda_2 & 0 \end{pmatrix} \Rightarrow S^{-1} = \begin{pmatrix} \not p + \mu\gamma^0 & \Delta\,\gamma_5\tau_2\lambda_2 \\ -\Delta^*\,\gamma_5\tau_2\lambda_2 & \not p - \mu\gamma^0 \end{pmatrix}$$

- strategy:

  invert $S^{-1}$ ➜ calculate $\Sigma[S]$ ➜ compare with ansatz

- result:

$$\Delta = 16H\,\Delta\,i \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{k_0^2 - \omega_-^2} + \frac{1}{k_0^2 - \omega_+^2} \right)$$

"gap equation"

quasiparticle dispersion laws: $\omega_\mp = \sqrt{(|\vec{k}| \mp \mu)^2 + |\Delta|^2}$

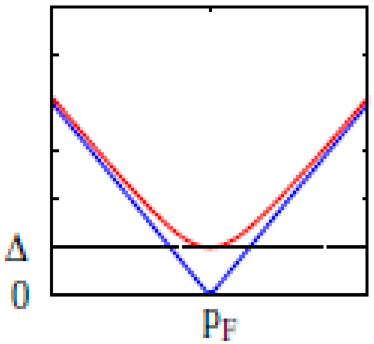
# Propagator

- dressed propagator: $S = \begin{pmatrix} \not{p} + \mu\gamma^0 & \Delta\,\gamma_5\tau_2\lambda_2 \\ -\Delta^*\,\gamma_5\tau_2\lambda_2 & \not{p} - \mu\gamma^0 \end{pmatrix}^{-1}$
  - dimension: $2 \times 4 \times N_f \times N_c$
    - ➜ $48 \times 48$ matrix for $N_f = 2, N_c = 3$
  - inversion straight forward, but some work required ...
- diagonalization:

$$S(p^0,\vec{p}) = U(\vec{p}) \begin{pmatrix} \frac{1}{p^0-\lambda_1(\vec{p})} & & \\ & \ddots & \\ & & \frac{1}{p^0-\lambda_{48}(\vec{p})} \end{pmatrix} U^\dagger(\vec{p})\,\gamma^0$$

  - $U(\vec{p})$ = unitary matrix, does not depend on $p^0$ !

# Dispersion relations




- 48 eigenvalues

  $=$ 24 quasiparticle dispersion relations:

  - $\omega_-(\vec{p}) = \sqrt{(|\vec{p}| - \mu)^2 + |\Delta|^2}$ (8-fold) red and green quarks
  - $\omega_+(\vec{p}) = \sqrt{(|\vec{p}| + \mu)^2 + |\Delta|^2}$ (8-fold) " antiquarks
  - $\epsilon_-(\vec{p}) = \big|\, |\vec{p}| - \mu \,\big|$ (4-fold) blue quarks
  - $\epsilon_+(\vec{p}) = \big|\, |\vec{p}| + \mu \,\big|$ (4-fold) " antiquarks

- $+$ 24 quasiholes: $-\omega_\mp(\vec{p}),\ -\epsilon_\mp(\vec{p})$

# Dispersion relations (CFL)

- 72 eigenvalues

  = 36 quasiparticle dispersion relations:

| | | |
|---|---|---|
| • $\omega_{8,-}(\vec{p}) = \sqrt{(|\vec{p}| - \mu)^2 + |\Delta|^2}$ | (16-fold) | quark octet × spin |
| • $\omega_{8,+}(\vec{p}) = \sqrt{(|\vec{p}| + \mu)^2 + |\Delta|^2}$ | (16-fold) | antiquark " |
| • $\omega_{1,-}(\vec{p}) = \sqrt{(|\vec{p}| - \mu)^2 + |2\Delta|^2}$ | (2-fold) | quark singlet × spin |
| • $\omega_{1,+}(\vec{p}) = \sqrt{(|\vec{p}| + \mu)^2 + |2\Delta|^2}$ | (2-fold) | antiquark " |

- + 36 quasiholes: $-\omega_{8,\mp}(\vec{p}),\ -\omega_{1,\mp}(\vec{p})$

# Gap equation: solutions

- gap equation: $\Delta = 16H\,\Delta\, i\int \frac{d^4k}{(2\pi)^4}\left(\frac{1}{k_0^2-\omega_-^2}+\frac{1}{k_0^2-\omega_+^2}\right)$

# Gap equation: solutions

- gap equation: $\Delta = 16H\,\Delta \int \frac{d^3k}{(2\pi)^3}\, T\sum_n \left(\frac{1}{\omega_n^2+\omega_-^2} + \frac{1}{\omega_n^2+\omega_+^2}\right)$
  - $\omega_n = (2n+1)\pi T$ fermionic Matsubara frequencies

# Gap equation: solutions

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  - $\omega_n = (2n+1)\pi T$ fermionic Matsubara frequencies
- turning out the sum, $T \to 0$:

$$\Delta = \frac{4H}{\pi^2}\,\Delta \int k^2\,dk \left\{\frac{1}{\omega_-(k)} + \frac{1}{\omega_+(k)}\right\}$$

- solutions:
  - trivial solution: $\Delta = 0$
  - other solutions? $\int k^2\,dk \left\{\frac{1}{\omega_-(k)} + \frac{1}{\omega_+(k)}\right\} \stackrel{!}{=} \frac{\pi^2}{4H}$

  $\Delta \to 0 \quad \Rightarrow \quad \frac{1}{\omega_-(k)} \to \frac{1}{|k-\mu|} \quad \Rightarrow \quad \int \cdots \to \infty$

  ➜ nontrivial solutions always exist for $H > 0$!

# Thermodynamic potential

- back to our Lagrangian:

$$\hat{\mathcal{L}} = \bar{q}(i\partial\!\!\!/ + \mu\gamma^0)q + H \sum_{A=2,5,7} (q^T C \gamma_5 \tau_2 \lambda_A q)^\dagger (q^T C \gamma_5 \tau_2 \lambda_A q)$$

- linearize $\mathcal{L}_{int}$ around $\Delta = -2H\langle q^T C \gamma_5 \tau_2 \lambda_A q \rangle$

  and use Nambu-Gorkov spinors to get :

$$\mathcal{L}_{MF} = \bar{\Psi} \begin{pmatrix} i\partial\!\!\!/ + \mu\gamma^0 & \Delta \gamma_5 \tau_2 \lambda_2 \\ -\Delta^* \gamma_5 \tau_2 \lambda_2 & -i\overleftarrow{\partial\!\!\!/} - \mu\gamma^0 \end{pmatrix} \Psi - \frac{|\Delta|^2}{4H} \equiv \bar{\Psi} S^{-1} \Psi - \mathcal{V}$$

- thermodynamic potential:

$$\Omega(T,\mu) = -T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \mathrm{Tr} \ln \left( \frac{1}{T} S^{-1}(i\omega_n, \vec{k}) \right) + \mathcal{V}$$

  - $\ln A = \ln\left((1 - (1 - A)\right) = \sum_{n=1}^{\infty} \frac{1}{n}(1 - A)^n$
  - useful formula: $\mathrm{Tr}\ \ln A = \ln \mathrm{Det} A$

# Thermodynamic potential

- result after Matsubara summation:

$$
\begin{aligned}
\Omega(T,\mu) \;=\; -\int \frac{d^3p}{(2\pi)^3} \Big\{ \;\; & 8\left( \frac{\omega_-}{2} + T\,\ln(1+e^{-\omega_-/T}) \right. \\
& \left. +\frac{\omega_+}{2} + T\,\ln(1+e^{-\omega_+/T}) \right) \\
& +4\left( \frac{\epsilon_-}{2} + T\,\ln(1+e^{-\epsilon_-/T}) \right. \\
& \left. +\frac{\epsilon_+}{2} + T\,\ln(1+e^{-\epsilon_+/T}) \right) \Big\} \\
+ \; \frac{|\Delta|^2}{4H} &
\end{aligned}
$$

# Thermodynamic quantities

- standard thermodynamic relations:
  - pressure: $p = -\Omega$
  - density: $n = -\frac{\partial\Omega}{\partial\mu}$
  - entropy density: $s = -\frac{\partial\Omega}{\partial T}$
  - energy density: $\varepsilon = -p + Ts + \mu n$

# Minima

- thermodynamic potential:

$$\Omega(T,\mu) \;=\; -T\sum_n \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2}\,\mathrm{Tr}\,\ln\left(\frac{1}{T}\,S^{-1}(i\omega_n,\vec{k})\right) \;+\; \frac{|\Delta|^2}{4H}$$

- stable solutions = minima

$$\rightarrow\; \frac{\partial\Omega}{\partial\Delta^*} \;=\; -T\sum_n \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2}\,\mathrm{Tr}\left(S\,\frac{\partial S^{-1}}{\partial\Delta^*}\right) \;+\; \frac{\Delta}{4H} \stackrel{!}{=} 0$$

- inverse propagator:

$$S^{-1}(p) = \begin{pmatrix} \not{p}+\mu\gamma^0 & \Delta\,\gamma_5\tau_2\lambda_2 \\ -\Delta^*\,\gamma_5\tau_2\lambda_2 & \not{p}-\mu\gamma^0 \end{pmatrix}$$

$$\Rightarrow\quad \frac{\partial S^{-1}}{\partial\Delta^*} \;=\; \begin{pmatrix} 0 & 0 \\ -\gamma_5\tau_2\lambda_2 & 0 \end{pmatrix} \;=\; i\Gamma_2^{\downarrow}$$

$$\rightarrow\; \Delta \;=\; 4H\,T\sum_n \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2}\,\mathrm{Tr}\left[\,iS\,\Gamma_2^{\downarrow}\,\right]$$ gap equation!

# Condensation energy

- free energy gain: $\delta\Omega = \Omega(\Delta) - \Omega(0)$
- simplifications: neglect antiparticles, $T = 0$

➜ $\Omega(\Delta) \; = \; -\frac{1}{2\pi^2} \int p^2 \, dp \, \sqrt{(p-\mu)^2 + |\Delta|^2} \; + \; \frac{|\Delta|^2}{4H}$

- gap equation:

$$\frac{\partial\Omega}{\partial\Delta^*} \; = \; -\frac{1}{4\pi^2} \int p^2 \, dp \, \frac{\Delta}{\sqrt{(p-\mu)^2+|\Delta|^2}} \; + \; \frac{\Delta}{4H} \; = \; 0$$

➜ $\Omega(\Delta) \; = \; -\frac{1}{2\pi^2} \int p^2 \, dp \, \frac{(p-\mu)^2 + \frac{1}{2}|\Delta|^2}{\sqrt{(p-\mu)^2+|\Delta|^2}}$

- integrand strongly peaked at $|\vec{p}| = \mu$ ➜ $\int p^2 \, dp \approx \mu^2 \int dp$
- Taylor expansion of the remaining integral in $\Delta$

➜ $-\delta\Omega \propto \mu^2 \Delta^2$

# CFL pairing in the bag model

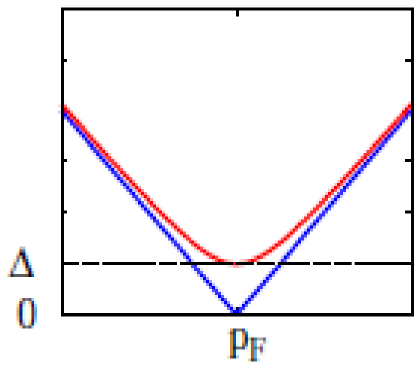
- bag-model pressure for unpaired quark matter at $T = 0$:
  - $\Omega_{BM}(\mu, \mu_Q) \;=\; \frac{3}{\pi^2} \sum_f \int_0^{p_F^f} dp\, p^2 \left(\sqrt{p^2 + m_f^2} - \mu_f\right) \;+\; B$
  - $\mu_u = \mu + \frac{2}{3}\mu_Q, \quad \mu_d = \mu_s = \mu - \frac{1}{3}\mu_Q, \quad p_F^f = \sqrt{\mu_f^2 - m_f^2}$
- effects of BCS pairing:
  - equalize Fermi momenta
  - pairing energy (expressed through the gap)
- CFL phase:
  - $\Omega_{BM}^{CFL}(\mu) \;=\; \frac{3}{\pi^2} \sum_f \int_0^{p_F^{common}} dp\, p^2 \left(\sqrt{p^2 + m_f^2} - \mu\right) \;-\; \frac{3\Delta^2\mu^2}{\pi^2} \;+\; B$
  - $p_F^{common} = 2\mu - \sqrt{\mu^2 + \frac{m_s^2}{3}}$ (for $m_u = m_d = 0$)

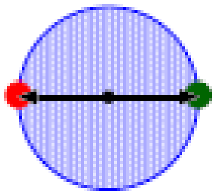
➜ parameters: masses, $B$, $\Delta$

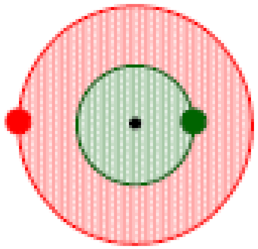
# Realistic masses

- realistic quark masses: $M_u, M_d \ll M_s < \infty$
  - ➜ unequal Fermi momenta, $p_F^a = \sqrt{\mu^2 - M_a^2}$
- recall argument about Cooper instability:
  - pairing close to the Fermi surface (no free energy cost for pair creation)
- Cooper pairs in BCS theory:
  - opposite momenta
- unequal Fermi momenta: $p_F^{a,b} = \bar{p}_F \pm \delta p_F$

free energy





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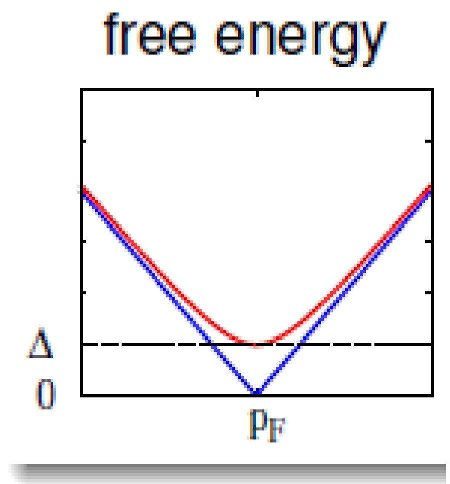


- Cooper pairs in BCS theory:
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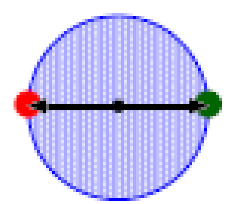
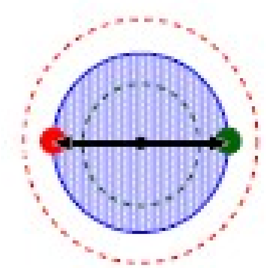


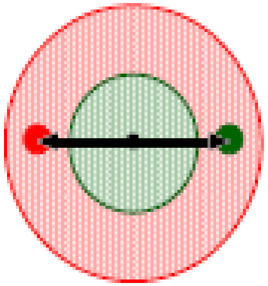
- unequal Fermi momenta: $p_F^{a,b} = \bar{p}_F \pm \delta p_F$
  - BCS pairing favored if $E_{binding} > E_{pair\ creation}$
  - approximately: $\frac{\Delta}{\sqrt{2}} \gtrsim \delta p_F$



# Which phase is favored ?

- precondition for standard BCS pairing:

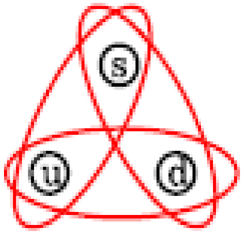
$$|p_F^a - p_F^b| \lesssim \sqrt{2}\Delta_{ab}$$



  - Fermi momenta: $p_F = \sqrt{\mu^2 - M^2}$
  - realistic quark masses: $M_s \gg M_d \approx M_u$

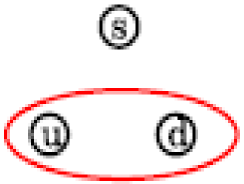
- case 1: *high densities*

$$\mu \gg M_s \;\rightarrow\; p_F^s \approx p_F^d \approx p_F^u \;\rightarrow\; \text{CFL}$$




- case 2: *moderate densities*

$$M_s \gtrsim \mu \gg M_{u,d} \;\rightarrow\; p_F^s \ll p_F^d \approx p_F^u \;\rightarrow\; \text{2SC}$$



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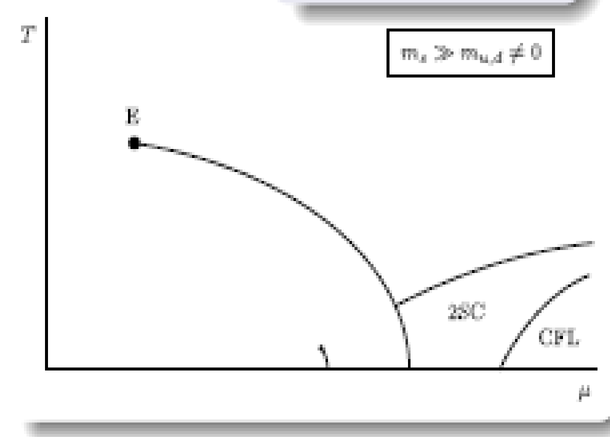
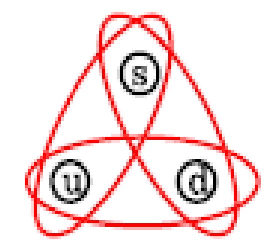
  - realistic quark masses: $M_s \gg M_d \approx M_u$

K. Rajagopal (1999)




- case 1: *high densities*

$$\mu \gg M_s \;\rightarrow\; p_F^s \approx p_F^d \approx p_F^u \;\rightarrow\; \text{CFL}$$

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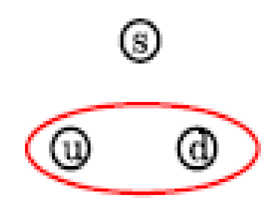


# 3-flavor NJL model

- Lagrangian: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq}$
  - free part: $\mathcal{L}_0 = \bar{q}(i\partial\!\!\!/ - \hat{m})q$ , $\hat{m} = diag_f(m_u, m_d, m_s)$
  - quark-antiquark interaction (as used earlier):

$$\mathcal{L}_{\bar{q}q} = G\left\{(\bar{q}\tau^a q)^2 + (\bar{q}i\gamma_5\tau^a q)^2\right\}$$
$$-K\left\{\det_f\left(\bar{q}(1+\gamma_5)q\right) + \det_f\left(\bar{q}(1-\gamma_5)q\right)\right\}$$

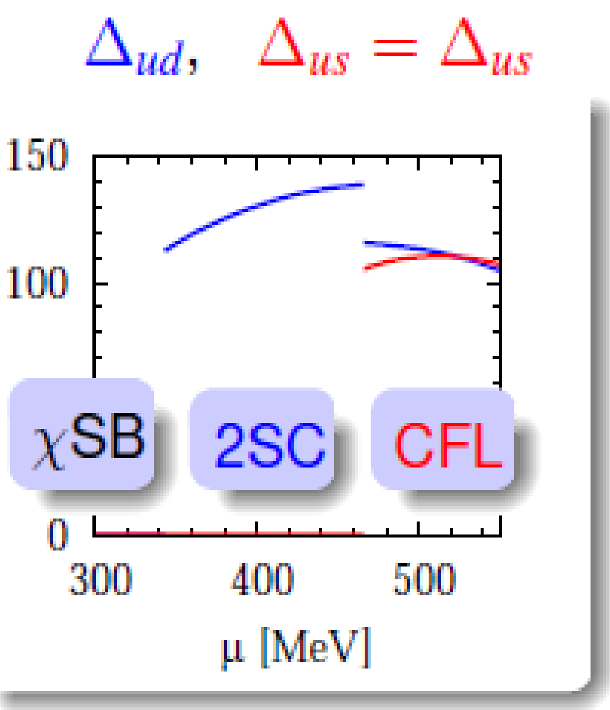
  - quark-quark interaction:

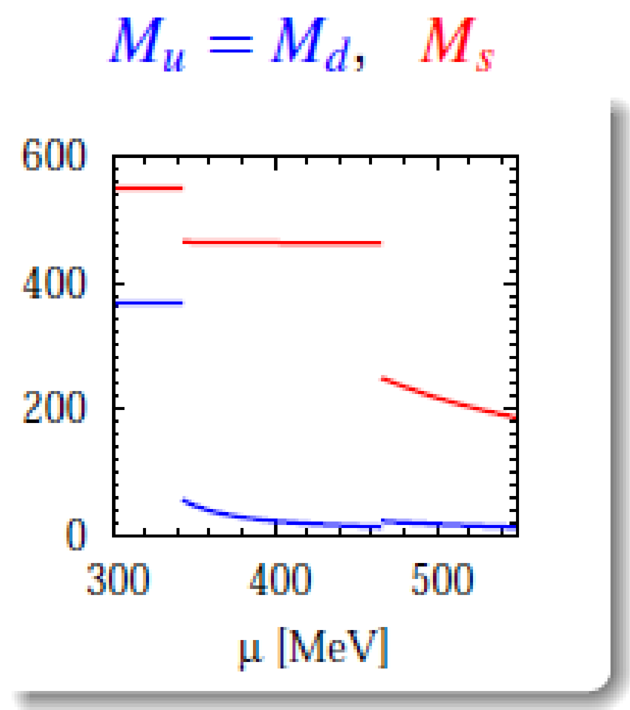
$$\mathcal{L}_{qq} = H\,(\bar{q}\,i\gamma_5\tau_A\lambda_{A'}\,C\bar{q}^T)(q^T C\,i\gamma_5\tau_A\lambda_{A'}\,q)$$

- mean-field approximation:
  - $\bar{q}q$-condensates: $\langle\bar{u}u\rangle$, $\langle\bar{d}d\rangle$, $\langle\bar{s}s\rangle$ $\leftrightarrow$ *dynamical masses*
  - $qq$-condensates: $\langle ud\rangle$, $\langle us\rangle$, $\langle ds\rangle$ $\leftrightarrow$ *diquark gaps*

# Results for T=0

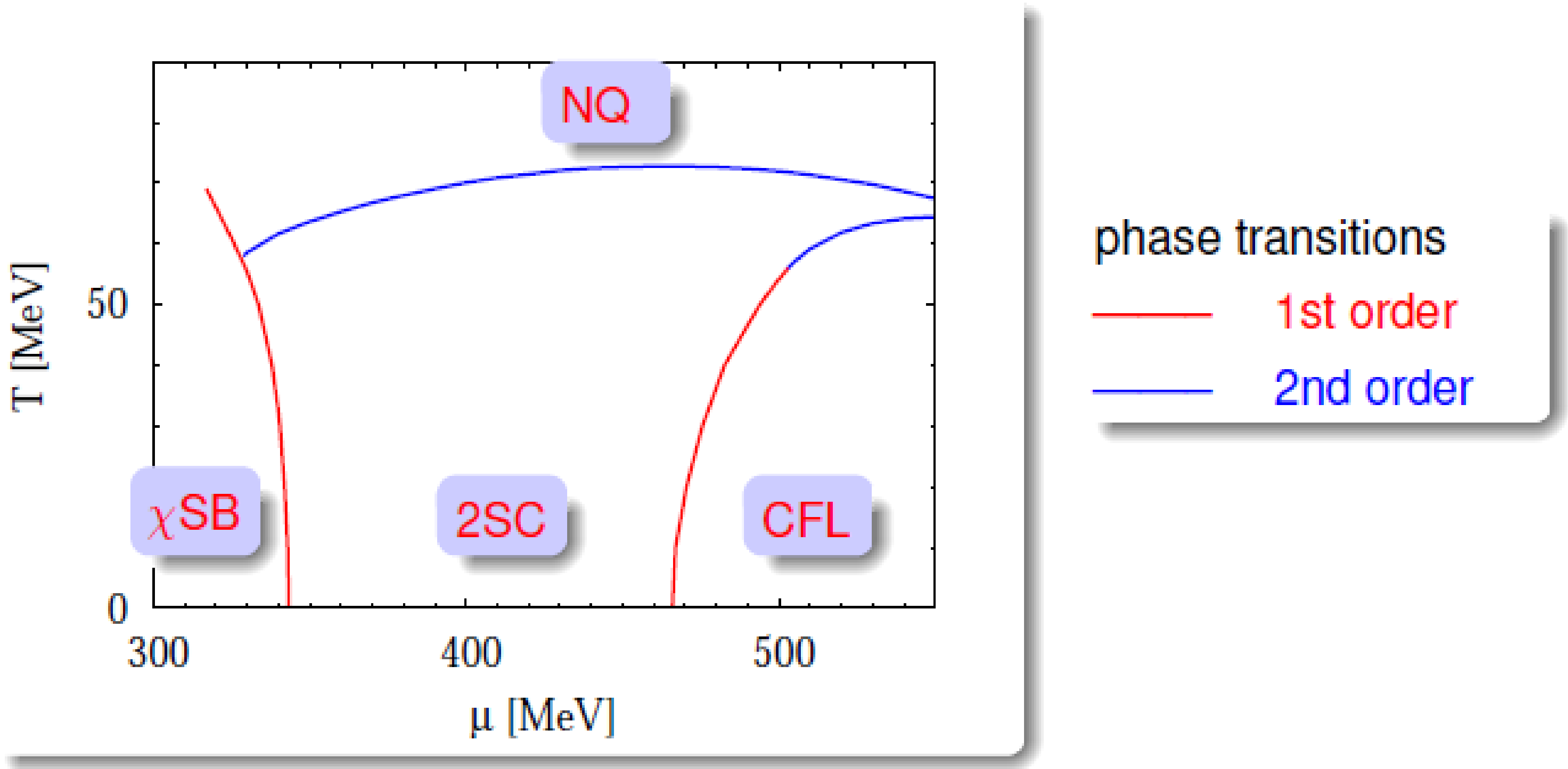
- "realistic" parameters
- isospin symmetry




→ strong interdependencies between dynamical masses and diquark gaps

# Phase diagram




S. Ruester et al. Phys. Rev. D 72 (2005) 034004
D. Blaschke et al. Phys. Rev. D 72 (2005) 065020

# Exploring hybrid star matter at NICA

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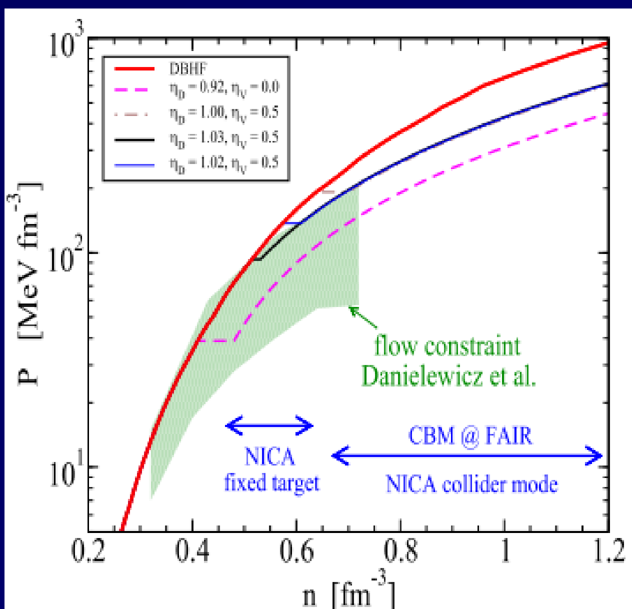
(2) Joint Institute for Nuclear Research, Dubna
(3) Department of Physics, San Diego State University, USA

JINR DUBNA

NICA

## Heavy-Ion Collisions

## Compact Stars




- stiff EoS (at flow limit)

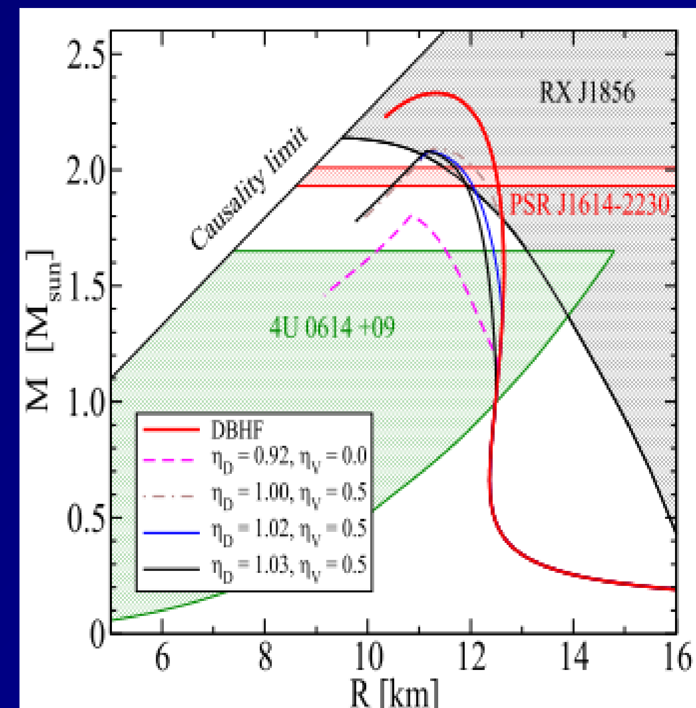
- low $n_{crit}$ (at NICA fixT)

- soft EoS (dashed line)

- high $M_{max}$ (J1614-2230)

- low $M_{onset}$ (all NS hybrid)

- excluded (J1614-2230)



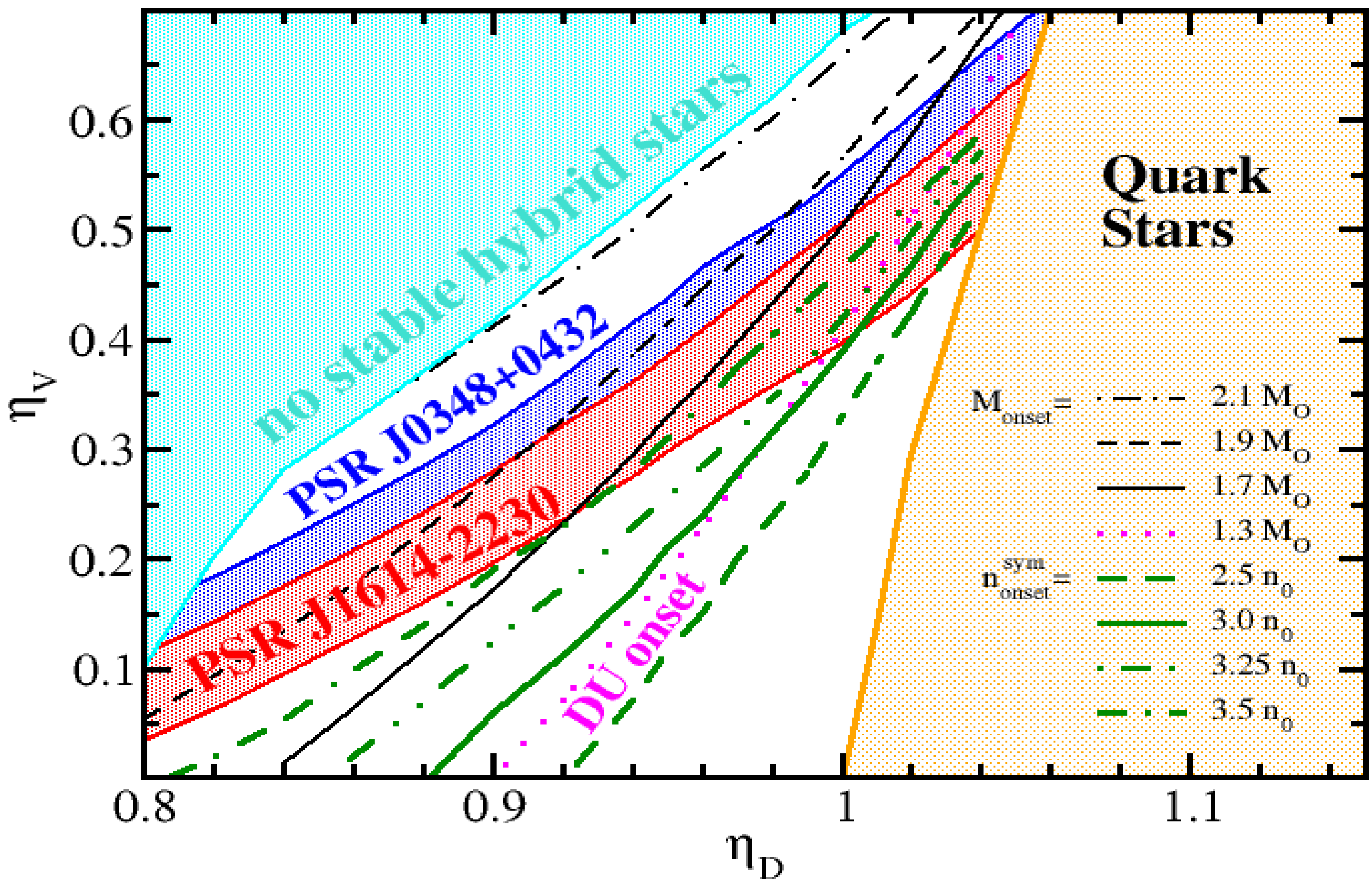

**Proposal:**

1. Measure transverse and elliptic flow for a wide range of energies (densities) at NICA and perform Danielewicz's flow data analysis ---> constrain stiffness of high density EoS
2. Provide lower bound for onset of mixed phase ---> constrain QM onset in hybrid stars

„The CBM Physics Book“, Springer LNP 841 (2011), pp.158-181

NICA White Paper, http://theor.jinr.ru → BLTP TWikipages

# Quark matter in $2M_{sun}$ neutron stars?
# → only color superconducting + vector int.




T. Klahn et al., PRD 88 (2013) 085001; arxiv:1307.696

# Backup slides