A toy model in $M_n(\mathbb{C})$ for selective measurements in QM

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2017.09.01.
arXiv:1707.09821

Abstract

The non-selective and selective measurements of a self-adjoint observable M in quantum mechanics are interpreted as 'jumps' of the state of the measured system into a M-decohered or M-pure state characterized by the spectral projections of M.
One may try to describe the measurement results as asymptotic states of a dynamical process, where the non-unitarity of time evolution arises as an effective description of the interaction with the measuring device.

We present here a two-step effective dynamics:
the first step is the non-selective measurement or M-decoherence, which is known to be described by the linear Lindblad equation, where the generator of the time evolution is the generator of a semigroup of unit preserving completely positive maps.
The second step is a process from the resulted M-decohered state to an M-pure state, which is described by an effective non-linear toy model dynamics: the pure states arise as asymptotic fixed points, their emergent probabilities are the relative volumes of their attractor regions.

Content

Measurements in quantum mechanics

- self-adjoint observable $M = \sum_{m\in\sigma(M)} mP_m \in \mathcal{B}(\mathcal{H})$
- prepared state $\omega\colon \mathcal{B}(\mathcal{H}) \to \mathbb{C}$ of the measured subsystem $\mathcal{B}(\mathcal{H})$
  - non-selective measurement:
    $\omega \mapsto \omega\circ\Phi_M, \quad \Phi_M(A) := \sum_{m\in\sigma(M)} P_m A P_m \in \langle M\rangle'$
    (H): any observable A "jumps" into the commutant $\langle M\rangle' \subset \mathcal{B}(\mathcal{H})$ containing the generated abelian subalgebra $\langle M\rangle$
    (S): an M-decohered repreparation "jump" of the prepared state ω
  - selective measurement: e.g. Stern–Gerlach, double-slit experiments
    $\omega \mapsto \omega\circ\Phi_m, \quad \Phi_m(A) := P_m A P_m/\omega(P_m)$ with probability $\omega(P_m)$
    (H): "jump" into a spectral projecion P_m of M with probability $\omega(P_m)$
    (S): "jump" into an M-pure state with probability $\omega(P_m)$
    probability $\omega(P_m)$ = relative frequency of the spectral outcome m in repeated experiments with identically prepared state ω

- both measurement "jumps" destroy unitary implemented dynamics
(H): $\alpha\colon (\mathbb{R},+) \to \mathrm{Aut}\,\mathcal{B}(\mathcal{H})$, such that $\alpha_t(A) := U_t^* A U_t,\ U_t \in \mathcal{U}(\mathcal{H})$
(S): $\omega_t := \omega\circ\alpha_t,\ t\in\mathbb{R}$
and are not unitary implementable, selective is not even deterministic
- however both Φ_M and Φ_m are completely positive (CP) maps
$\Phi\otimes\mathrm{Id}_n\colon \mathcal{B}(\mathcal{H})\otimes M_n \to \mathcal{B}(\mathcal{H})\otimes M_n$ is positive (linear) $\forall\, n\in\mathbb{N}$

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Connection between CP maps and subsystems in QM

S=subsystem and the E=environment in QM: $\mathcal{B}(\mathcal{H}_S \otimes \mathcal{H}_E) \simeq \mathcal{B}(\mathcal{H}_S) \otimes \mathcal{B}(\mathcal{H}_E)$

- full system $\to$ subsystem
  if $U_t \in \mathcal{U}(\mathcal{H}_S \otimes \mathcal{H}_E), t \in \mathbb{R}$ is a unitary dynamics on the full system then

  $$\mathcal{B}(\mathcal{H}_S) \ni A \mapsto \Phi_t(A) := \mathrm{Tr}_E\left[(\mathbf{1}_S \otimes \rho_E) U_t^* (A \otimes \mathbf{1}_E) U_t\right] \in \mathcal{B}(\mathcal{H}_S)$$

  unit preserving CP map on $\mathcal{B}(\mathcal{H}_S)\ \forall\, t \in \mathbb{R}$
  $\Rightarrow$ one may look for a "CP-dynamics" on the subsystem instead of a unitary one
- subsystem $\to$ extended (= full) system
  If Φ unit preserving σ-weakly continuous CP map on $\mathcal{B}(\mathcal{H}_S) \Rightarrow$
  $\exists\, \mathcal{H}_E$ and V isometry on $\mathcal{H}_S \otimes \mathcal{H}_E$ such that $\forall\, \rho_E \in \mathcal{S}(\mathcal{H}_E)$

  $$\Phi(A) = \mathrm{Tr}_E\left[(\mathbf{1}_S \otimes \rho_E) V^* (A \otimes \mathbf{1}_E) V\right], \quad A \in \mathcal{B}(\mathcal{H}_S)$$

  (V can be made unitary by a ρ_E-dependent further extension of $\mathcal{H}_E$)
  $\Rightarrow$ every CP map on the subsystem comes from a restriction of a isometric/unitary sandwiching on a full system

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Generator of a special CP dynamics: Lindblad operator

- restriction on CP dynamics: special family of CP maps
  - form a semigroup: $\Phi_t \circ \Phi_s = \Phi_{t+s};\, t, s \in \mathbb{R}_+$,
  - has a bounded generator L: $\Phi_t = \exp(tL)$

  latter is not a restriction if $\mathcal{B}(\mathcal{H}) = M_n(\mathbb{C})$
- Theorem (Lindblad; 1976) on the generator of a CP_1 semigroup

  Let $L\colon \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ bounded linear *-map.

  $\Phi_t := \exp(tL) \in \mathrm{CP}_1(\mathcal{B}(\mathcal{H}))_\sigma, t \geq 0 \Leftrightarrow L$ has the form

$$L(A) = i[H, A] + \sum_k V_k^* A V_k - \frac{1}{2}\{V_k^* V_k, A\}, \quad A \in \mathcal{B}(\mathcal{H}),$$

  where $H = H^*;\; V_k,\; \sum_k V_k^* V_k \in \mathcal{B}(\mathcal{H})$.
- Lindblad equation: generalization of the Schrödinger equation
  - $\omega\colon \mathcal{B}(\mathcal{H}) \to \mathbb{C}$ normal state with density matrix ρ: $\omega(A) = \mathrm{Tr}\,(\rho A)$
  - $H \leftrightarrow S$ picture change: $\mathrm{Tr}\,(\hat{L}(\rho) A) := \mathrm{Tr}\,(\rho L(A))$

$$\frac{d\rho}{dt} = \hat{L}(\rho) := -i[H, \rho] + \sum_k V_k \rho V_k^* - \frac{1}{2}\{V_k^* V_k, \rho\}$$

  linear first order differential equation on density matrices

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GP effective one particle state in Bose–Einstein condensation

- Trapped interacting N-boson Hamiltonian in 3D: $\mathcal{H}^{\otimes N}, \mathcal{H} := L^2(\mathbb{R}^3)$

$$\tilde{H}_N = \sum_{j=1}^{N} (-\Delta_{\mathbf{r}_j} + V_{ext}(\mathbf{r}_j)) + \sum_{i<j}^{N} V_N(\mathbf{r}_i - \mathbf{r}_j)$$

  • $0 < V_{ext}(\mathbf{r}) \to \infty, |\mathbf{r}| \to \infty$
  • $0 < V_N(\mathbf{r}) = V_N(|\mathbf{r})| = N^2 V(N|\mathbf{r}|)$
  smooth with compact support and scattering length $a = a_0/N$

- Conjectured effective one-particle description: Gross–Pitaevskii equation (nonlinear, namely cubic) and energy functional in $\mathcal{H}$

$$i\partial_t \varphi(t) = -\Delta\varphi(t) + \sigma|\varphi(t)|^2\varphi(t), \quad \varphi(t) \in \mathcal{H}, \|\varphi\| = 1$$

$$E_{GP}(\varphi) := \int d^3r (|\nabla\varphi(\mathbf{r})|^2 + V_{ext}(\mathbf{r})|\varphi(\mathbf{r})|^2 + 4\pi a_0|\varphi(\mathbf{r})|^4), \ \|\varphi\| = 1$$

- Theorem (Lieb, Seiringer; 2002) on BE-condensation
  Let ψ_N be the ground state of $\tilde{H}_N$ and let $\gamma_N^{(k)}, 1 \le k \le N$ be its k-particle marginal density operator. Let $\sigma := 8\pi N a = 8\pi a_0$ in the GP equation and let φ_{GP} be the minimizer of E_{GP}. Then

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$$\gamma_N^{(k)} \to |\varphi_{GP}\rangle\langle\varphi_{GP}|^{k\otimes}, \quad N \to \infty$$

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GP effective nonlinear dynamics after Bose–Einstein condensation

- N-particle Hamiltonian with trap removed

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- Theorem (Erdős, Schlein, Yau; 2007) on GP-dynamics
  Let $\psi_N(t)$ be the solution of the Schrödinger equation $i\partial_t\psi_N(t) = H_N\psi_N(t)$ with $\tilde{H}_N$ ground state initial condition $\psi_N(0) := \psi_N$ and let $\gamma_N^{(1)}(t)$ be its one-particle marginal density. Then for any $t \geq 0$

$$\gamma_N^{(1)}(t) \to |\varphi(t)\rangle\langle\varphi(t)|, \quad N \to \infty$$

pointwise for compact operators on $\mathcal{H}$, where φ_t solves the GP-equation

$$i\partial_t\varphi(t) = -\Delta\varphi(t) + 8\pi a_0|\varphi(t)|^2\varphi(t)$$

with initial condition $\varphi(0) := \varphi_{GP}$.

Existence of initial state dependent effective dynamics

F = full system: S = subsystem and E = environment

$\mathcal{B}(\mathcal{H}_F) := \mathcal{B}(\mathcal{H}_S \otimes \mathcal{H}_E) \simeq \mathcal{B}(\mathcal{H}_S) \otimes \mathcal{B}(\mathcal{H}_E)$

- initial density matrix (= initial normal state) on $\mathcal{B}(\mathcal{H}_S)$: ρ_0^S
  $\Rightarrow$ compatible initial density matrices on $\mathcal{B}(\mathcal{H}_F)$:

$$\mathrm{Tr}_E^{-1}(\rho_0^S) := \{\rho_0^F \in \mathcal{B}(\mathcal{H}_F)_{+1} \,|\, \mathrm{Tr}_E\,(\rho_0^F) = \rho_0^S\}$$

  inverse image (normal states) of ρ_0^S in $\mathcal{B}(\mathcal{H}_F)_{+1}$
- effective time evolution from the unitary (Hamiltonian) one on $\mathcal{B}(\mathcal{H}_F)_{+1}$

$$\frac{d\rho_0^S}{dt} = -i\,\mathrm{Tr}_E\,[H^F, \rho_0^F]$$

  heavily depends on the initial choice of $\rho_0^F \in \mathrm{Tr}_E^{-1}(\rho_0^S)$
  through the surviving, ρ_0^F-dependent "$\mathcal{H}_S$-blocks"
- given probability distribution on $\mathrm{Tr}_E^{-1}(\rho_0^S)$ $\Rightarrow$
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- effective time evolution from the unitary (Hamiltonian) one on $\mathcal{B}(\mathcal{H}_F)_{+1}$

$$\frac{d\rho_0^S}{dt} = -i\,\mathrm{Tr}_E\,[H^F, \rho_0^F]$$

  heavily depends on the initial choice of $\rho_0^F \in \mathrm{Tr}_E^{-1}(\rho_0^S)$
  through the surviving, ρ_0^F-dependent "$\mathcal{H}_S$-blocks"
- given probability distribution on $\mathrm{Tr}_E^{-1}(\rho_0^S)$ $\Rightarrow$
  given probability distribution of effective (initial) dynamics on ρ_0^S

Two types of effective dynamics in selective measurements (SM)

- Instead of "jumps" try a "very fast" dynamical description of SM:
  SM result should be an asymptotic state of an effective dynamics caused by the interaction of the measured (sub)system with the measuring device
  - no modification of "fundamental" dynamics of quantum theories
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1. CP_1 dynamics with specific Lindblad operators in SM

Describing M-decoherence, that is a non-selective measurement of $M = M^* = \sum_{m \in \sigma(M)} m P_m \in M_n(\mathbb{C})$, one can rely on previous works: Baumgartner, Narnhofer (2008), Weinberg (2016)

- Proposition The set of asymptotic states of a Lindblad evolution

$$\frac{d\rho}{dt} = \hat{L}(\rho) := -i[H, \rho] + \sum_k V_k \rho V_k^* - \frac{1}{2}\{V_k^* V_k, \rho\}.$$

  is equal to $\Phi_M(S_n)$ iff $\{H, V_k, V_k^*\}'' = \langle M \rangle$. In this case any initial state leads to an asymptotic state iff $\{V_k, V_k^*\}'' = \langle M \rangle$, and then

$$S_n \ni \rho_0 \to \rho_\infty := \lim_{t\to\infty} \exp(t\hat{L})(\rho_0) = \Phi_M(\rho_0) := \sum_{m\in\sigma(M)} P_m \rho_0 P_m$$

Proof hint:
• P projection is 'conserved', $P = \exp(tL)(P), t \geq 0$ iff $P \in \{H, V_k, V_k^*\}'$ $\Rightarrow \{H, V_k, V_k^*\}' = \Phi_M(M_n(\mathbb{C})) = \langle M \rangle'$, i.e. the choice $\{H, V_k, V_k^*\}'' = \langle M \rangle$ leads to the required set of possible asymptotic states (the invariant states)
• $\{H, V_k, V_k^*\}'' = \langle M \rangle$ is abelian, hence $\hat{L} = L(-H, V_k^*)$ is a generator of CP_1 maps $\Rightarrow$ $\hat{\Phi}_t, t \geq 0$ are norm one maps

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1. CP_1 dynamics with specific Lindblad operators in SM

Proof hint continued:

• $\hat{L}\colon M_n(\mathbb{C}) \to M_n(\mathbb{C})$ is not self-adjoint (or normal in general wrt the scalar product on $M_n(\mathbb{C})$ given by the trace), but the generalized eigenvalue problem $(\hat{L}-\lambda)^k = 0$ (in Jordan blocks), hence the time evolution can be solved: $\mathrm{Re}\,\lambda \leq 0$ for $k = 1$ and $\mathrm{Re}\,\lambda < 0$ for $k > 1$, because $\hat{\Phi}_t$ is a norm one map

• $\mathrm{Re}\,\lambda = 0 \Leftrightarrow \hat{L}$-eigenmatrix is in $\{V_k, V_k^*\}' \Rightarrow$ nontrivial H-eigenvalues, $\mathrm{Re}\,\lambda = 0, \mathrm{Im}\,\lambda \neq 0$ are excluded iff $\{V_k, V_k^*\}'' = \langle M \rangle$, in that case all initial states lead to asymptotic states, which should be invariant states

2. Nonlinear effective dynamics in selective measurements

Aim: "randomly chosen" nonlinear deterministic dynamics on $S_M := S_{n|\langle M\rangle}$ should result M-pure asymptotic states P_m with probability $p_m := \mathrm{Tr}\,(\rho_0 P_m)$

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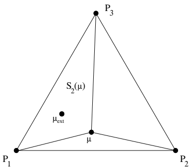
2. Nonlinear toy model dynamics for *M*-purification

- nonlinear dynamics on $S_M = \{\sum_{i=1}^n p_i P_i \,|\, 0 \le p_i \le 1, \sum p_i = 1\}$

$$\begin{aligned} \frac{d\mu}{dt} = F(\mu, \mu_{ext}) &:= f(\mu, \mu_{ext}) - \mu \mathrm{Tr}\, f(\mu, \mu_{ext}), \quad \mu \in S_M \\ f(\mu, \mu_{ext}) &:= a\mu(\lambda\mu - \mu_{ext}) \end{aligned} \tag{1}$$

- $a > 0$ "evolution strength"
- $\lambda \equiv \lambda(\mu, \mu_{ext}) := \max\{\kappa \in [0,1] \,|\, \mu_{ext} - \kappa\mu \ge 0\}$,

that is $\mu_{ext} \equiv \sum_i s_i P_i$ is the convex combintion $\mu_{ext} = \lambda\mu + \sum_{i \ne j} \lambda_i P_i$



- Theorem on the fixpoint structure of the dynamics (1)

If $\mu_{ext} \in S_M$ is chosen uniformly wrt the Lebesgue measure on S_M then the asymptotic state $\mu_\infty := \lim_{t \to \infty} \mu(t)$ of the dynamics (1) on S_M with initial condition $\mu_0 = \sum_{i=1}^n p_i P_i$ is equal to P_i with probability p_i.

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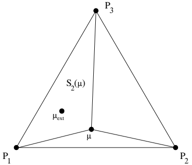

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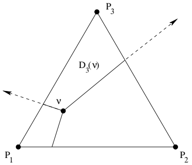
2. Fixpoint structure of the nonlinear toy model dynamics

Proof hint

• use Picard–Lindelöf theorem on first order differential equations on a region with Lipschitz continuity for $\mu_{ext} \equiv \sum_i s_i P_i \in S_M^{int}$

$$\|F(\mu,\mu_{ext}) - F(\tilde{\mu},\mu_{ext})\|_\infty \leq (4+\frac{6}{s_j})\|\mu-\tilde{\mu}\|_\infty,\ \mu,\tilde{\mu} \in D_j(\mu_{ext}) := K_j(\mu_{ext}) \cap S_M,$$

$K_j(\nu)$: affine cone generated by $\nu - P_i, i \neq j$ with base $\nu \in S_M$



$\Rightarrow$ unique integral curves within the domains $D_j(\mu_{ext}), j = 1, \ldots, n$

• for $\mu_{ext} = \lambda\mu + \sum_{i \neq j} \lambda_i P_i$ and $\mu = \sum_i r_i P_i$ the tangent vector is given by $F(\mu,\mu_{ext}) = \sum_{i \neq j} r_i \lambda(\mu - P_i) \in K_j(\mu)$

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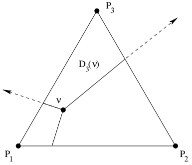

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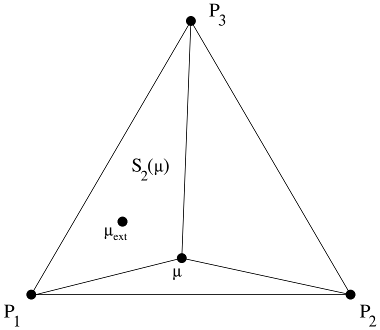
2. Fixpoint structure of the nonlinear toy model dynamics

Proof hint continued

• uniform choice of μ_{ext} within S_M with 'repeated' initial state $\mu_0 = \sum_i p_i P_i$
$\Rightarrow$ probability (= relative frequency in 'repeated experiments') of the asymptotic state P_j is the relative volume of the simplices $S_j(\mu_0)$ and S_M:

$$\frac{V(S_j(\mu_0))}{V(S_M)} \equiv \frac{V(\langle \mu_0, P_1, \ldots, P_{j-1}, P_{j+1}, \ldots, P_n \rangle)}{V(\langle P_1, \ldots, P_n \rangle)} = p_j$$



Closing remarks

- In case of unbounded or continuous spectra $M = \int_{\sigma(M)} m dE(m)$ (e.g. a position operator in $\mathbb{R}^d$) write $\sigma(M) \subseteq \mathbb{R}$ as a partition of finitely many spectral intervals $\Rightarrow$ spectral interval projections generate a finite dimensional unital abelian subalgebra of $\mathcal{B}(\mathcal{H}) \Rightarrow$ finite dimensional aproximations exist
- In case of joint measurements of commuting operators $M^{(1)}, \ldots, M^{(d)}$ (e.g. position operators in $\mathbb{R}^d$) use products of commuting spectral (interval) projections $P^{(1)}_{m_1} P^{(2)}_{m_2} \ldots P^{(d)}_{m_d}, m_i \in \sigma(M^{(i)})$
- Experimental verification of the dynamical nature of measurements: needs slow 'measuring process' and quick switch on/off possibility of the measuring device without disturbing the state of the measured system
  • instead of the outcome distribution at $t = \infty$ from $t = 0$ data given $\mu_0 = \sum p_i P_i$ and uniform μ_{ext} in $S_M \mapsto \mu_\infty = P_i$ with probability p_i a switch-off and immediate switch-on at intermediate time $0 < T < \infty$ $\Rightarrow$ intermediate final distribution of μ_T as initial distribution with new (uniformly chosen) μ_{ext} may lead to a different asymptotic distibution of μ_∞, which is numerically calculable from the nonlinear toy dynamics
- Try one-step dynamics: $d\rho/dt = \hat{L}(\rho) + \tilde{F}(\rho, \mu_{ext})$ with extended $\tilde{F}: S_n \times S_M \to M_n(\mathbb{C})_{sa} \Rightarrow$ two-step dynamics may arise as $a \to 0$

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