Cold quarks stars from hot lattice QCD

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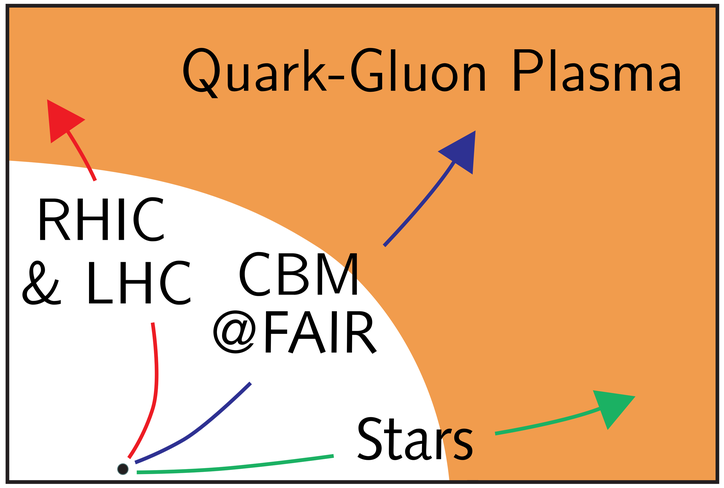
1. hot lattice QCD and quasiparticles
2. quasiparticle model: going to $\mu > 0$
3. cold quark stars



CJT formalism

- effective action

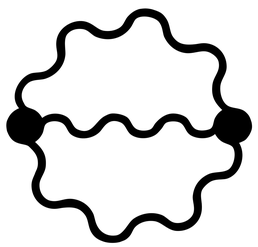
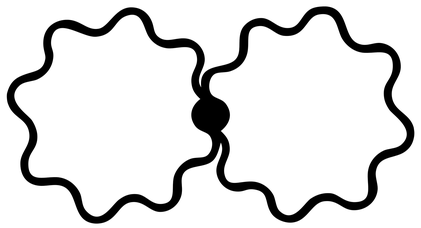
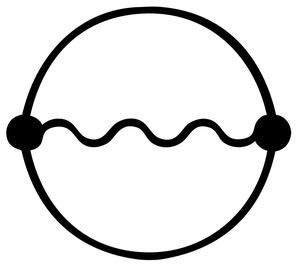
$$\Gamma[D,S] = I - \frac{1}{2}\left\{\mathrm{Tr}\left[\ln D^{-1}\right] + \mathrm{Tr}\left[D_0^{-1}D - 1\right]\right\}$$
$$+ \quad \left\{\mathrm{Tr}\left[\ln S^{-1}\right] + \mathrm{Tr}\left[S_0^{-1}S - 1\right]\right\} \; + \; \Gamma_2[D,S]$$

- translation-invariant systems, no broken symmetries

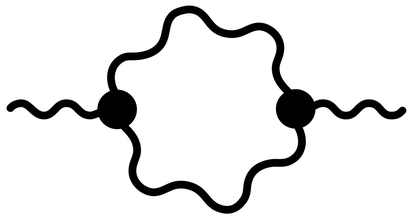
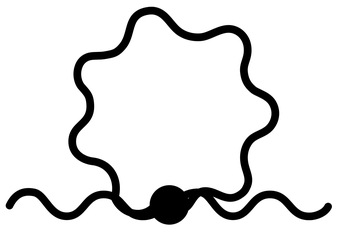
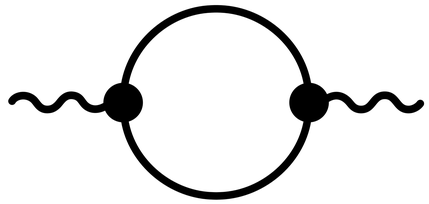
$$\frac{\Omega}{V} = \mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4} n_{\mathrm{B}}(\omega)\,\mathrm{Im}\big(\ln D^{-1} - \Pi D\big)$$
$$+ \; 2\,\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4} n_{\mathrm{F}}(\omega)\,\mathrm{Im}\big(\ln S^{-1} - \Sigma S\big) - \frac{T}{V}\Gamma_2$$

2-loop QCD thermodynamics

- truncate Γ_2 at 2-loop order

$\Gamma_2 = \frac{1}{12}$  $+ \frac{1}{8}$  $- \frac{1}{2}$ 

$\longrightarrow$ self-energies of 1-loop order

$\Pi = \frac{1}{2}$  $+ \frac{1}{2}$  $-$ 

- gauge invariance: hard thermal loops (HTL)

Blaizot, Iancu, Rebhan: PRD'01
RS, Bluhm, Kämpfer: JPPNP'09

Entropy

- stationarity of Ω

$$s := -\frac{1}{V}\left.\frac{\partial\Omega}{\partial T}\right|_{\mu} = -\frac{1}{V}\left(\left.\frac{\partial\Omega}{\partial T}\right|_{\text{expl.}} + \underbrace{\frac{\delta\Omega}{\delta D}}_{0}\frac{\partial D}{\partial T}\right)_{\mu}$$

$$= s_{g,\mathrm{T}} + s_{g,\mathrm{L}} + s_{q,\mathrm{Pt}} + s_{q,\mathrm{Pl}} + s' \qquad s' = 0$$

Vanderheyden, Baym: JSP'98

- e.g. gluons:

$$s_{g,\mathrm{T}} \sim \int_{\mathrm{d}^4k} \frac{\partial n_{\mathrm{B}}}{\partial T}\left\{\underbrace{\pi\varepsilon(\omega)\Theta\left(-\mathrm{Re}D_{\mathrm{T}}^{-1}\right)}_{\text{qp contribution}} + \underbrace{\mathrm{Re}D_{\mathrm{T}}\mathrm{Im}\Pi_{\mathrm{T}} - \arctan\left(\frac{\mathrm{Im}\Pi_{\mathrm{T}}}{\mathrm{Re}D_{\mathrm{T}}^{-1}}\right)}_{\text{damping terms}}\right\}$$

Pressure

- self-consistent formulation of the pressure

$$p = -\frac{\Omega}{V} := \sum_i p_i - B \qquad p_i \sim \int_{\mathrm{d}^4k} n_{\mathrm{B/F}} \left\{ \mathrm{qp} + \mathrm{damping} \right\}$$

$$\frac{\partial B}{\partial T} := \sum_i \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} \qquad \left(\frac{\partial B}{\partial \mu} = \sum \frac{\cdot\cdot}{\cdot\cdot} \frac{\cdot\cdot}{\partial \mu} \right)$$

- entropy density

$$s = \frac{\partial p}{\partial T} = \sum_i s_i + \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} - \frac{\partial B}{\partial T} = \sum_i s_i$$

- net quark density

$$n_q = \frac{\partial p}{\partial \mu} \sim \int_{\mathrm{d}^4k} \left(\frac{\partial n_{\mathrm{F}}}{\partial \mu} + \frac{\partial n_{\mathrm{F}}^A}{\partial \mu} \right) \left\{ \mathrm{qp} + \mathrm{damping} \right\}$$

$\rightarrow$ first parameter: B_0

Effective coupling

- fundamental parameter

$$g^2(x^2) = \frac{16\pi^2}{\beta_0 \ln(x^2)} \left(1 - \frac{4\beta_1}{\beta_0^2} \frac{\ln[\ln(x^2)]}{\ln(x^2)}\right)$$

- running coupling g^2 $\qquad x = \frac{\bar{\mu}}{\Lambda_{\mathrm{QCD}}}$

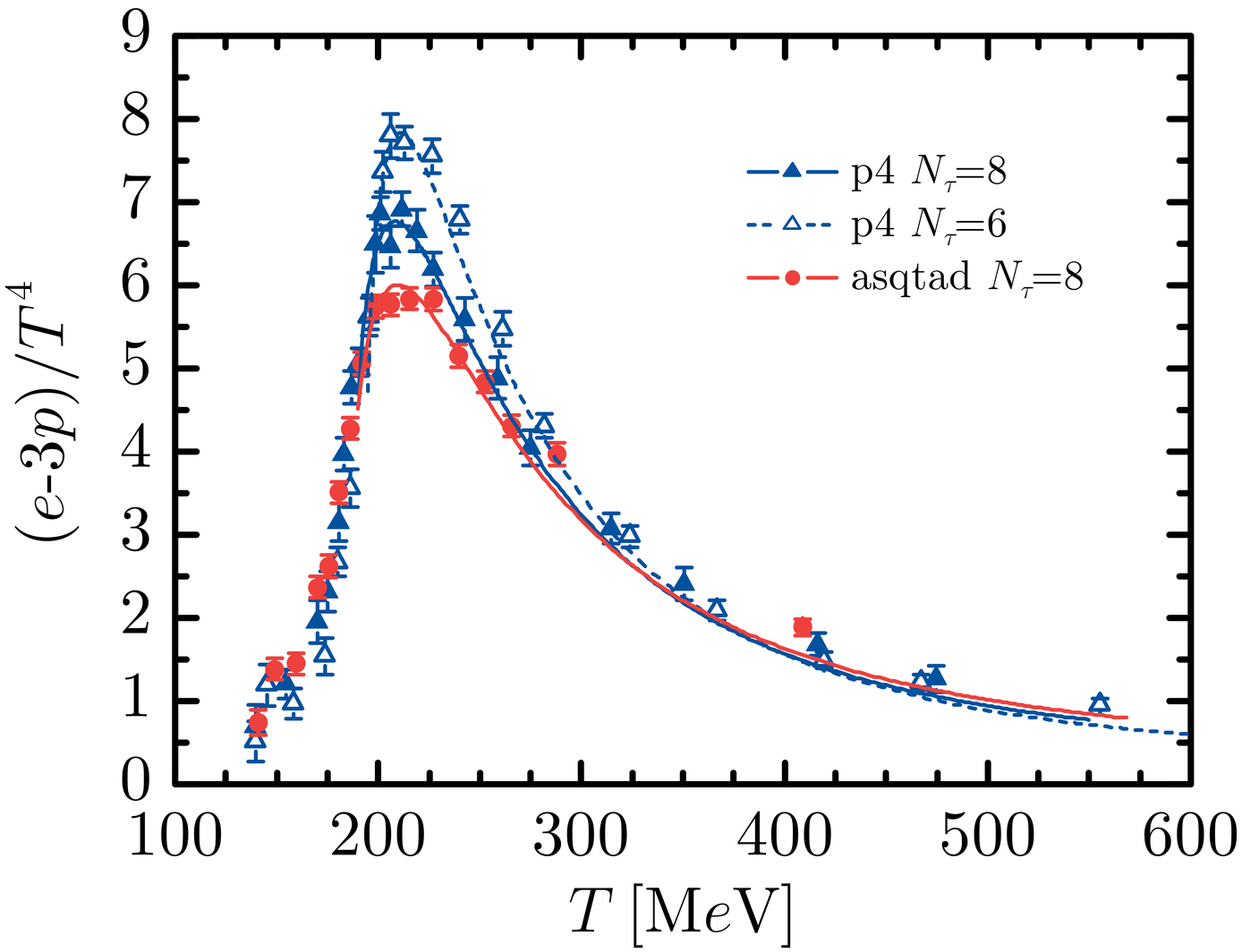
$$T > T_c,\ \mu = 0 \qquad \bar{\mu} \sim T$$

- effective coupling G^2 $\qquad x = \frac{(T\text{-}T_s)}{\lambda_{\mathrm{QCD}}}$

Bluhm, Kämpfer, RS, Seipt: EPJC'07

At μ=0

- quasiparticle model (QPM) fit to lattice results



Bazavov et al.: PRD '09

$$p_{\mathrm{QPM}}(T_c) \stackrel{!}{=} p_{latt}(T_c)$$

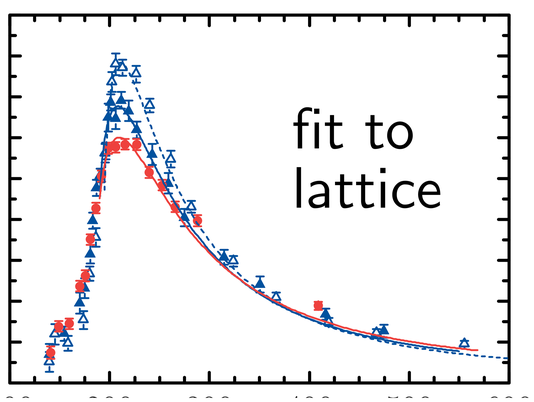
$$\longrightarrow \lambda_{\mathrm{QCD}}, T_s, B_0$$

$$\longrightarrow G^2(T, \mu = 0)$$

state variables s, n, p, $(e\text{-}3p)$, ...

effective coupling G^2

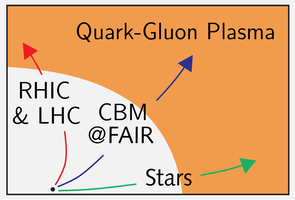
@ $\mu = 0$:



@ $\mu \neq 0$:

??

Into the T-μ-plane



- $\mu > 0$: stationary potential, self-consistent model
  $\longrightarrow$ impose Maxwell's relation

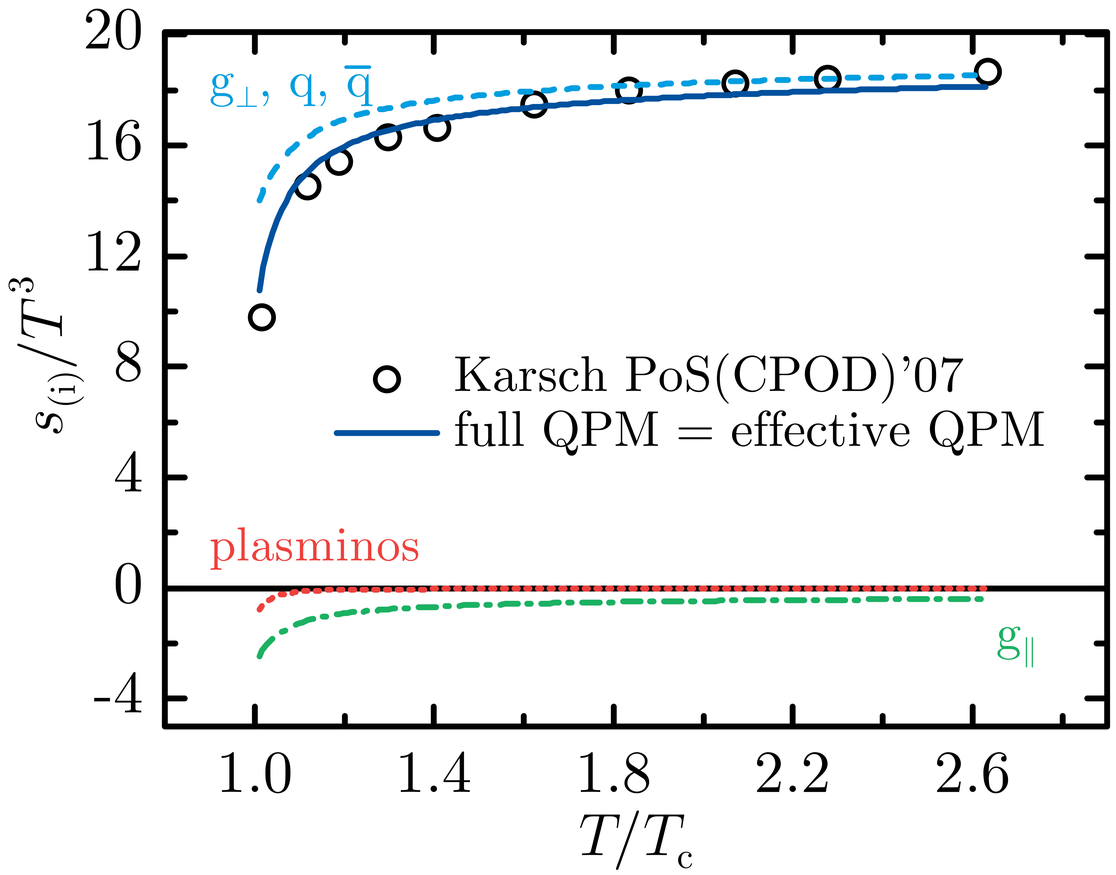
$$\frac{\partial s}{\partial \mu} = \frac{\partial n}{\partial T} \qquad \longrightarrow \qquad a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b$$

Peshier, Kämpfer, Soff: PRC'00, PRD'02

- quasilinear PDE for $G^2(T, \mu \neq 0)$:
  $\longrightarrow$ method of characteristics + boundary values
  $\longrightarrow$ T-μ-plane accessible

Effective quasiparticle model

- full Hard Thermal Loops



- effective quasiparticles
  - no collective excitations
  - no damping
  - asymptotic dispersion relation

$\longrightarrow$ crossing characteristics

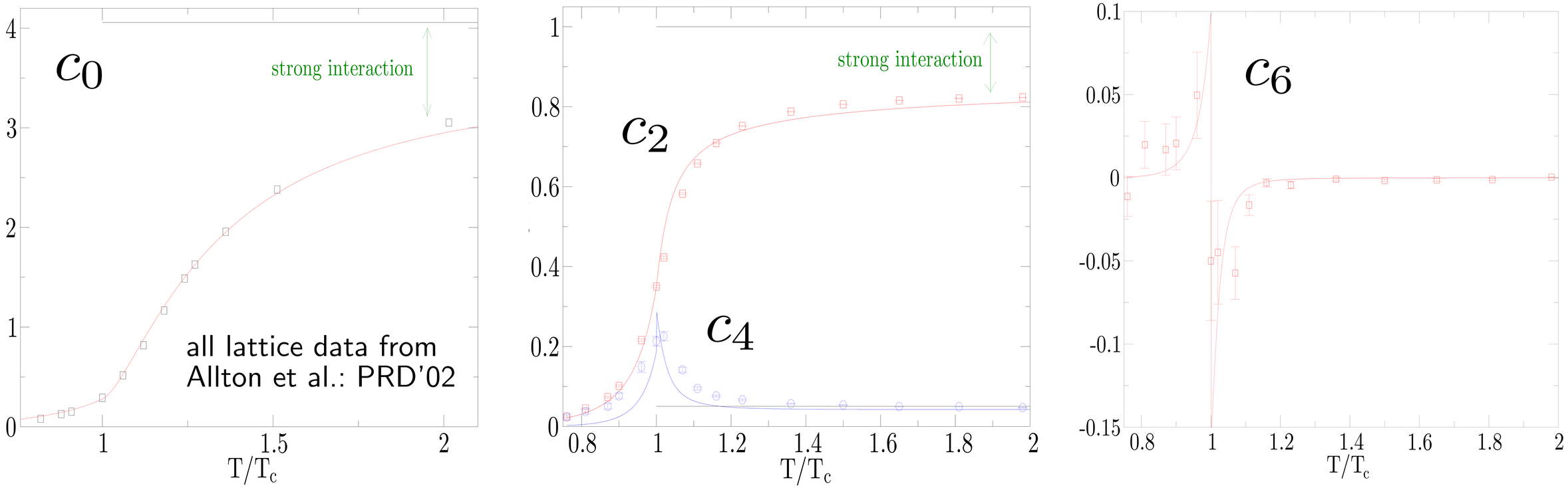
however: thermodynamic results similar

RS, Bluhm, Kämpfer: EPJ ST'08

Small chemical potential

- test with $p(T, \mu \gtrsim 0)$ lattice data

$$p = T^4 \sum_n c_n(T) \left(\frac{\mu}{T}\right)^n \qquad c_n(T) = \frac{1}{n!} \frac{\partial^n (p/T^4)}{\partial (\mu/T)^n}$$



Bluhm, Kämpfer, Soff: PLB'05

- application in hydro @ RHIC successful

Bluhm, Kämpfer, RS, Seipt, Heinz: PRC'07

Isospin asymmetric QPM

- five chemical potentials

$$\mu_u,\, \mu_d,\, \mu_s \quad + \quad \mu_e,\, \mu_\mu$$

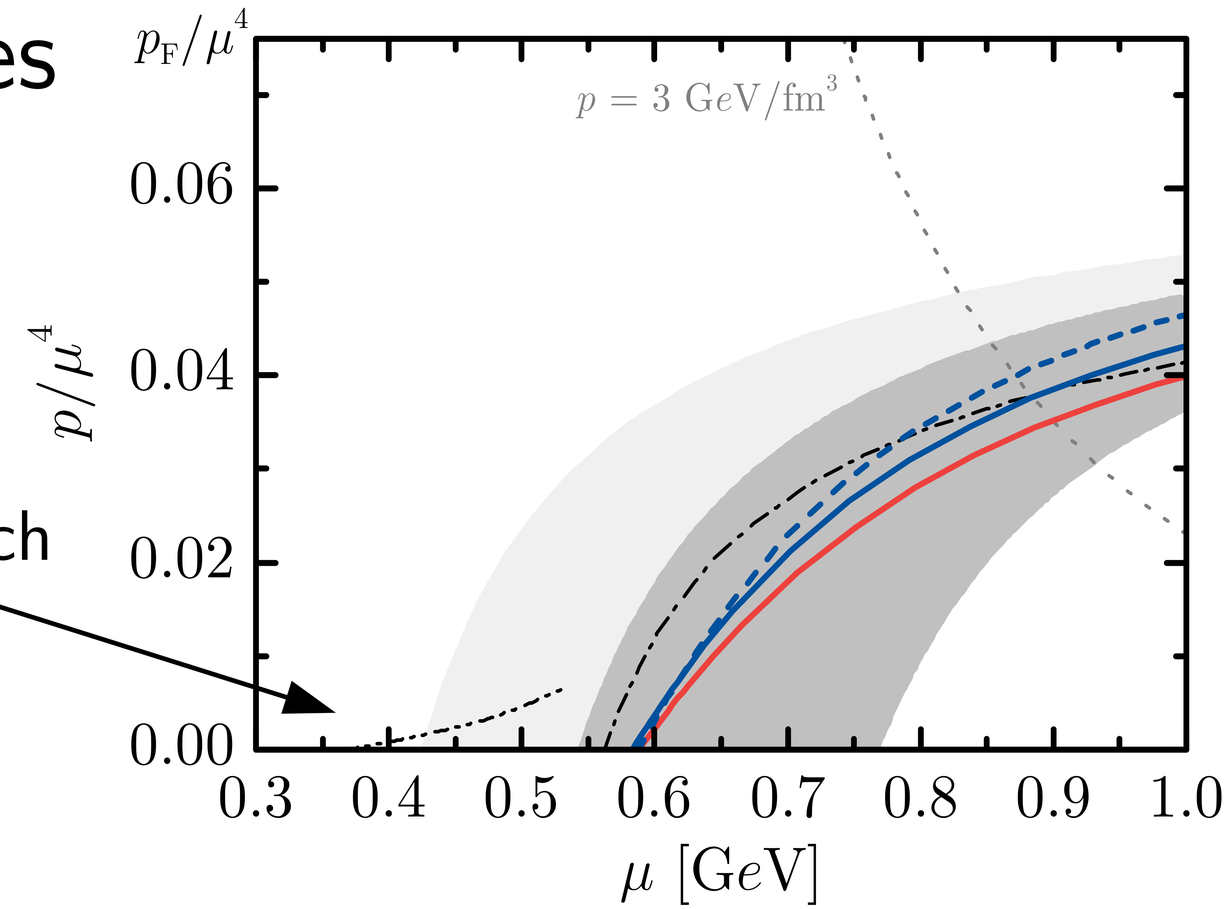
- four side conditions
  - β equilibrium (e.g. $n \leftrightarrow p^+ + e^- + \bar{\nu}_e$; $\mu_d = \mu_u + \mu_e$)
  - equilibrium in strangeness changing decays (e.g. $\Lambda \leftrightarrow p^+ + \pi^-$; $\mu_s = \mu_d$)
  - muon decay (e.g. $\mu^- \leftrightarrow e^- + \bar{\nu}_e + \nu_\mu$; $\mu_\mu = \mu_e$)
  - electric neutrality

$\longrightarrow$ only one independent chemical potential $\mu = \mu_u$

Schäfer, Wilczek: PRD'99

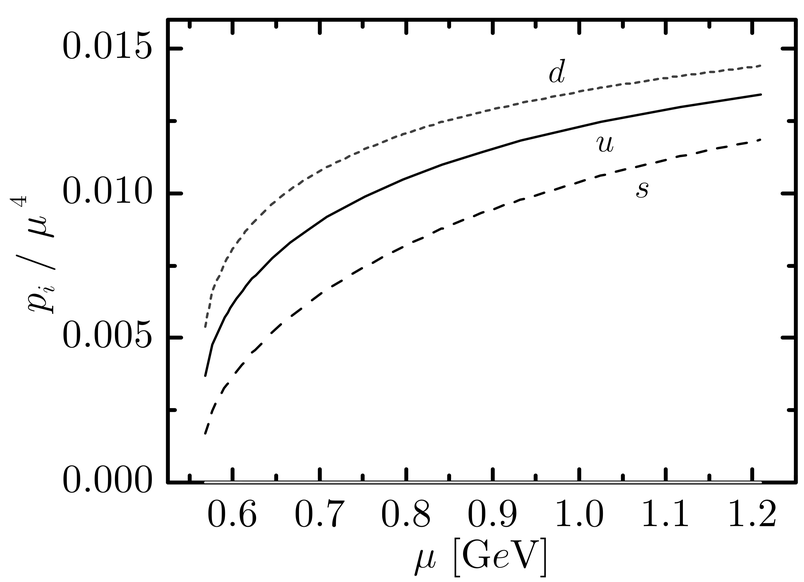
At T=0

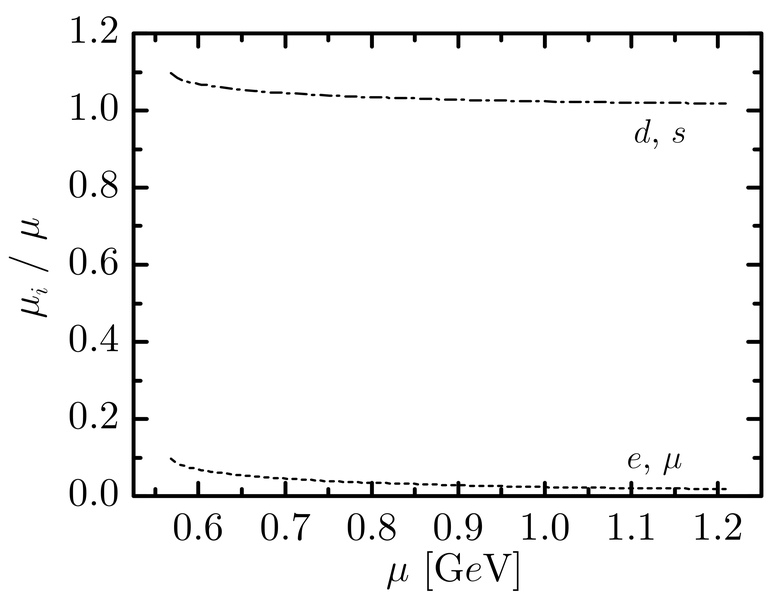
- thermodynamic quantities well within perturbative predictions
  (Andersen, Strickland: PRD'02
  Fraga et al.: NPA'02)



hybrid approach possible

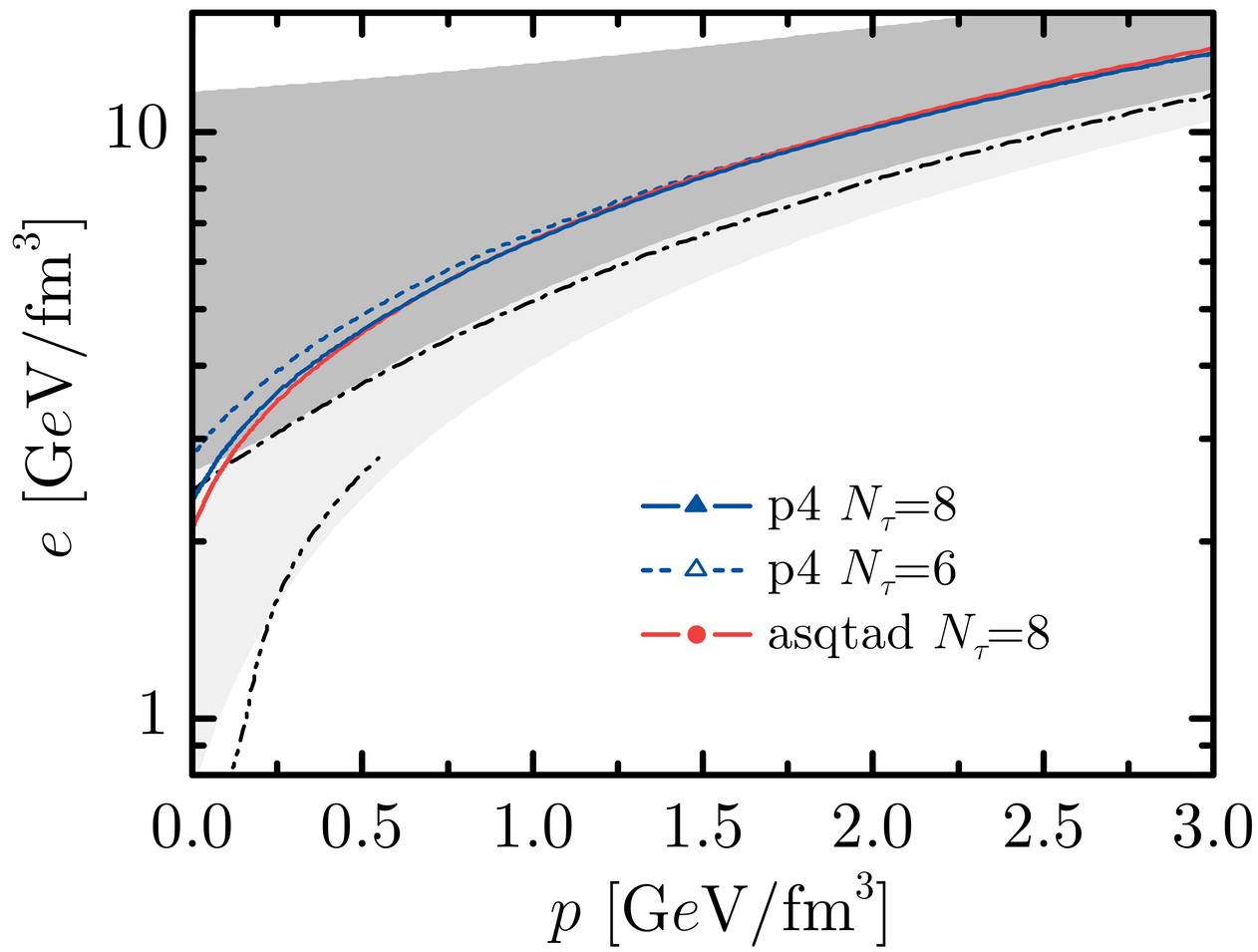
- individual contributions





At T=0

- EOS: narrow range for all actions
  - vacuum energy density dep. on lattice spacing
  - asymptotics governed by lattice action



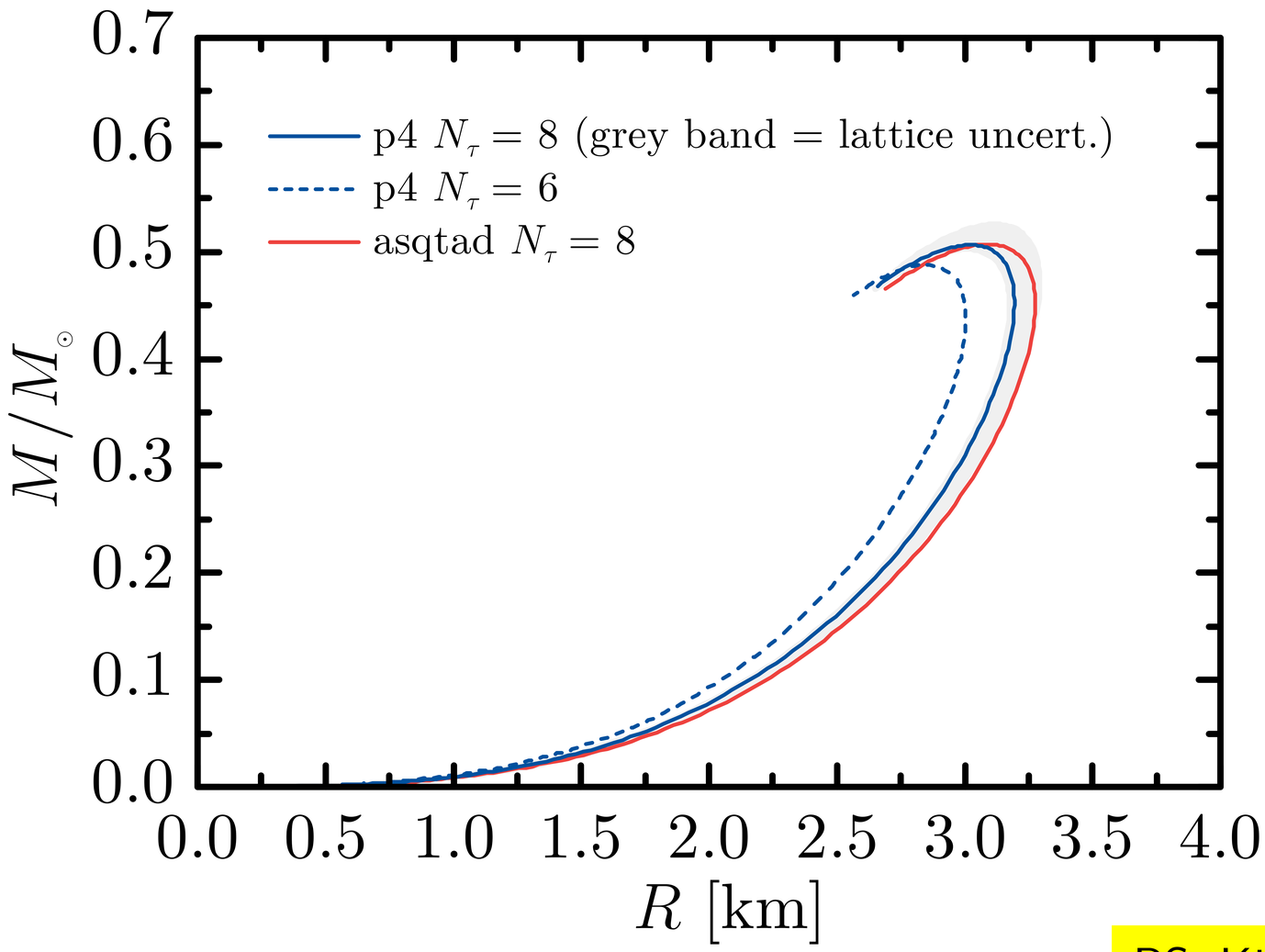
- good parametrization

$$e = v_s^{-2} p + e_0$$

$3.3 - 3.6$ $\quad$ $(375 - 395\mathrm{MeV})^4$

Pure quark stars

- solutions of TOV equations



RS, Kämpfer: submitted to PRC

$\longrightarrow$ rather small and light

reason: scaling $M, R \sim e_0^{-1/2}$

Scaled TOV equations

- introduce scaled quantities

$$p = \bar{p} e_0$$

$$m = \bar{m} (G_N e_0)^{-1/2} G_N^{-1} \qquad \text{where} \qquad e = v_s^{-2} p + e_0$$

$$r = \bar{r} (G_N e_0)^{-1/2}$$

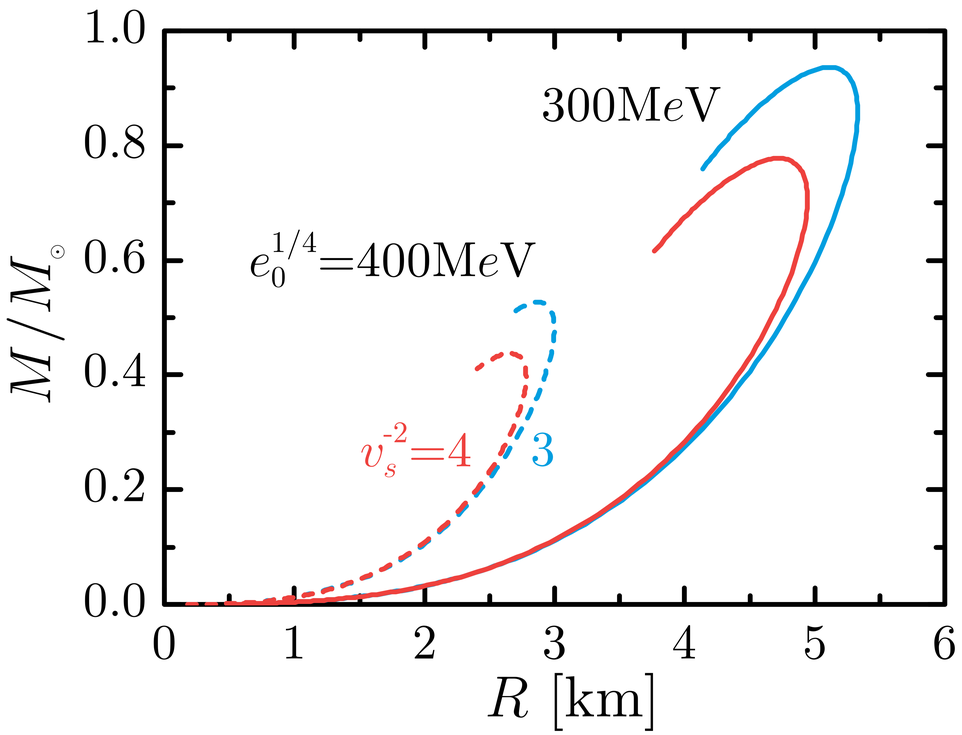
$\rightarrow$ scaled TOV equations for $\bar{p}$, $\bar{m}$ and $\bar{r}$

$\rightarrow$ independent of e_0

- observables

$$M = m(p{=}0) \sim e_0^{-1/2}$$

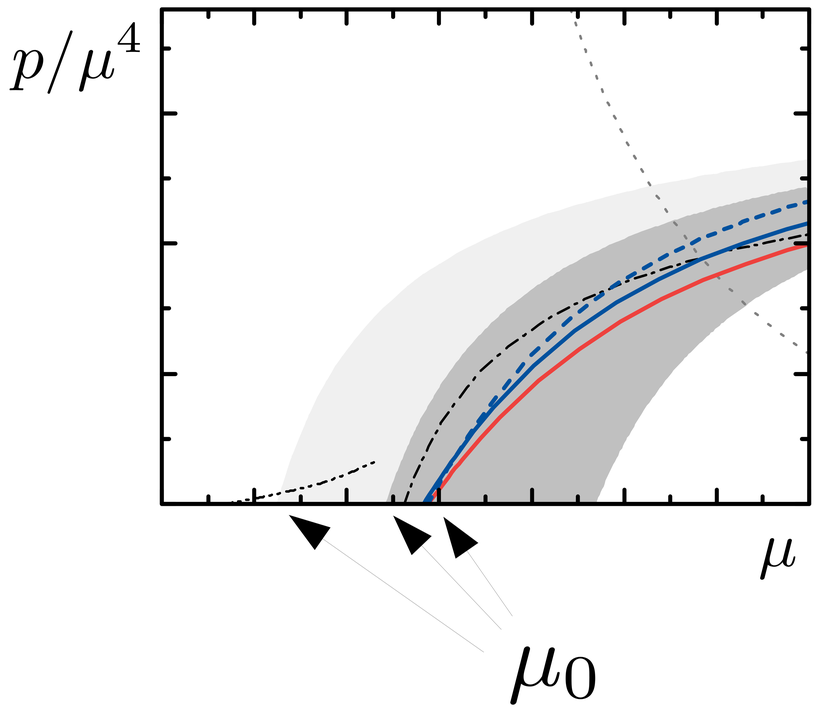
$$R = r(p{=}0) \sim e_0^{-1/2}$$



Binding energy

- binding energy of the star

$$BE = M - m_{\mathrm{q}} N_{\mathrm{q}}$$



- for scaled quantities

$$\overline{BE} = 4\pi \int \mathrm{d}\bar{r}\, \bar{r}^2 \left[\left(\frac{m_{\mathrm{q}}}{\mu_0} \right)^2 \frac{\bar{p}}{(1\text{-}2\bar{m}/\bar{r})^{\text{-}1/2}} \left(\frac{1+v_s^2}{v_s^2} + 1 \right)^{\frac{1}{1+v_s^2}} - v_s^{\text{-}2}\bar{p} - 1 \right]$$

independent of e_0
shrinks with increasing μ_0

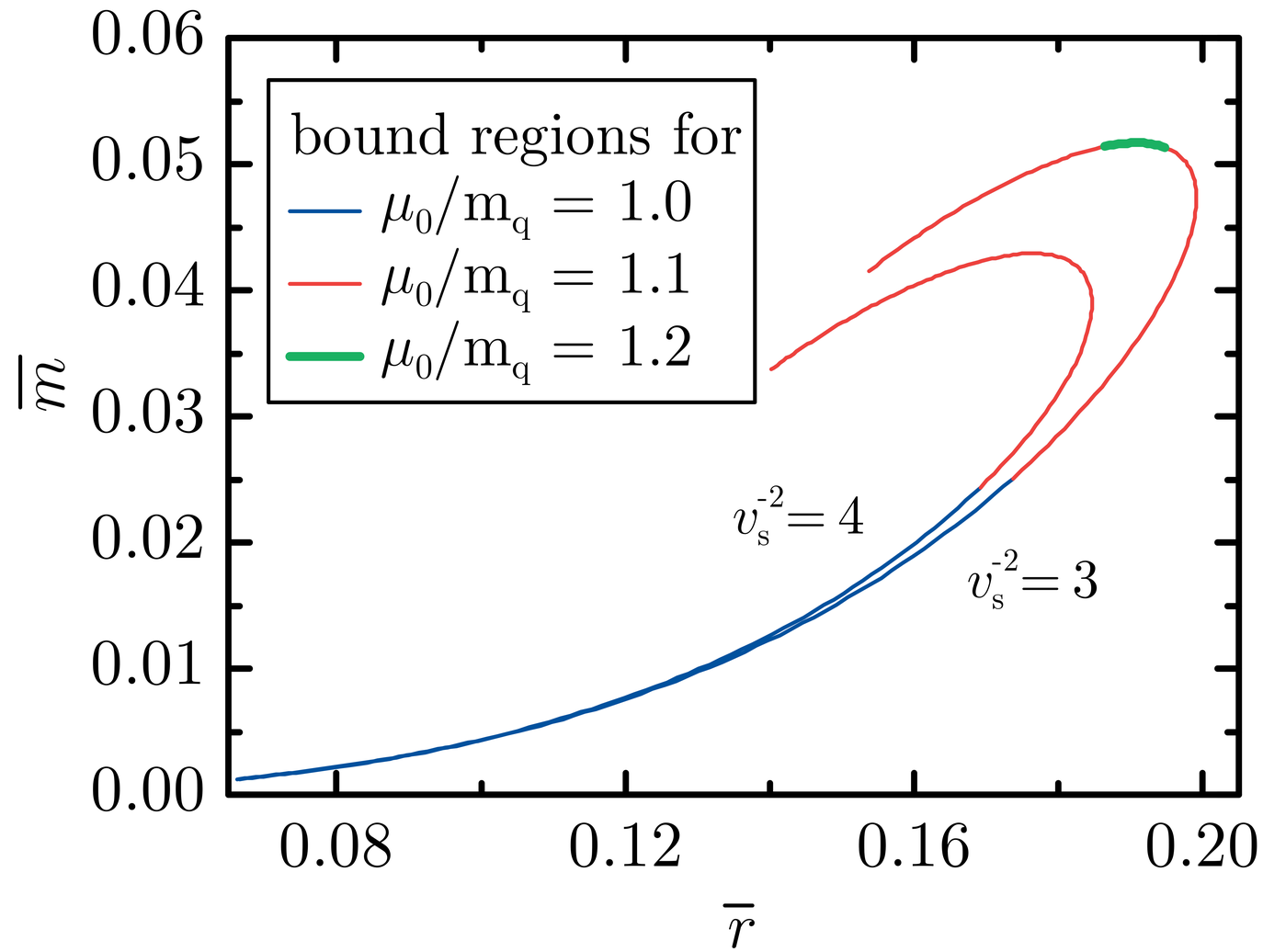
Systematic of binding energies

- binding if

$$\frac{\mu_0}{m_\mathrm{q}} \sim 1.0...1.2$$

- quasiparticle model + lattice

$$\frac{\mu_0}{m_\mathrm{q}} \sim 1.85$$



- stable quark stars $\nexists$ within the model (existence as meta-stable objects possible)

Summary & Outlook

- quasiparticle model follows from actual QCD
- ℓQCD results mapped to large μ, even $T{=}0$
- EOS for quark stars similar for all actions
- quark stars with rather smaller radii + masses
  however: unbound

- outlook: hybrid stars
  full HTL quasiparticle model with
  Landau damping and collective modes
  EOS for FAIR/CBM

Quarks stars

- static, spherical stellar objects

$$\frac{dp}{dr} = -G\frac{(e+p)(m+4\pi r^3 p)}{r^2(1-\frac{2m}{r}G)}$$

$$\frac{dm}{dr} = 4\pi r^2 e,$$

TOV equations

$$e = e(p)$$

- EOS of the quark-gluon plasma

  $\longrightarrow$ from quasiparticles

Tolman-Oppenheimer-Volkov equations

- for the special parametrization $e = v_s^{-2} p + e_0$

$$\frac{dp}{dr} = -G_N \frac{([1 + v_s^{-2}]p + e_0)(m + 4\pi r^3 p)}{r^2(1 - \frac{2m}{r} G_N)}$$

$$\frac{dm}{dr} = 4\pi r^2 (v_s^{\mathrm{i}\ 2} p + e_0)$$

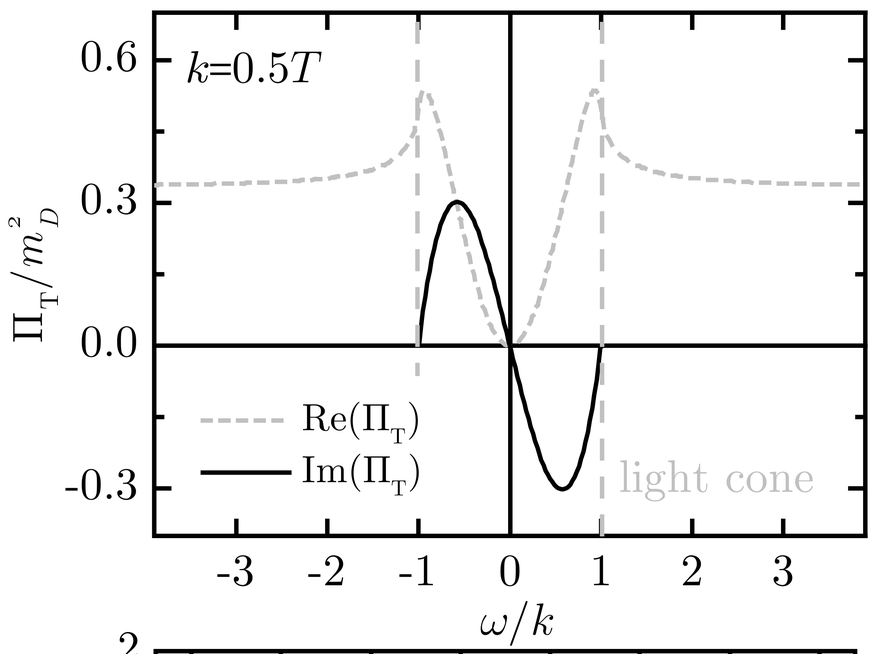
- scaled TOV equations:

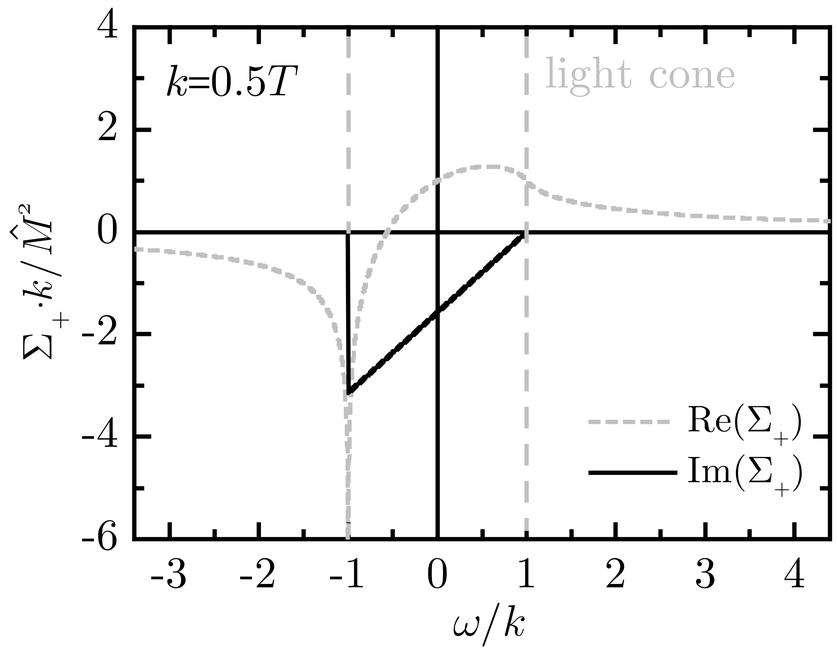
$$\frac{\partial \bar{p}}{\partial \bar{r}} = -\frac{\left(\left[1 + v_s^{-2}\right] \bar{p} + 1\right)\left(\bar{m} + 4\pi \bar{r}^3 \bar{p}\right)}{\bar{r}^2 \left(1 - \frac{2\bar{m}}{\bar{r}}\right)}$$

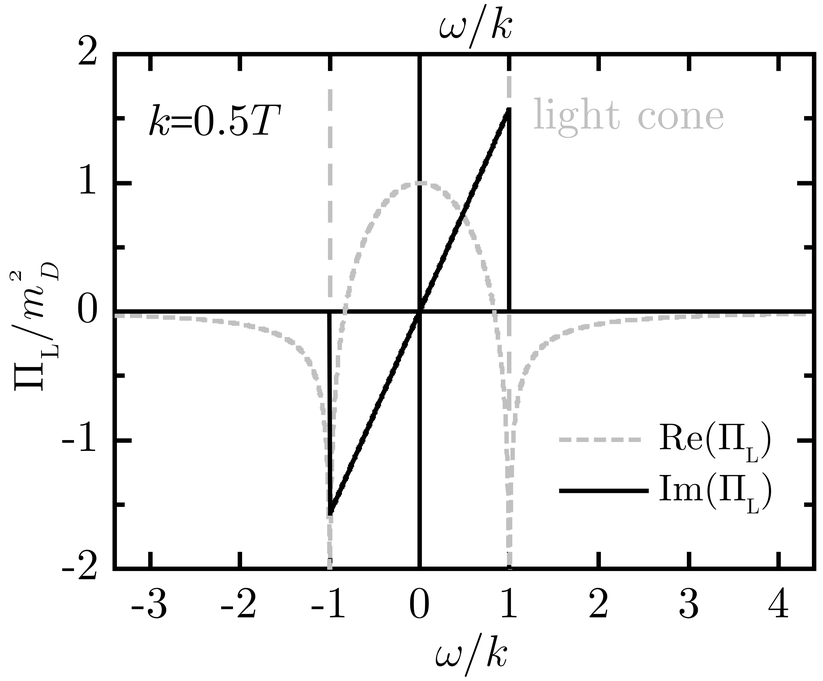
$$\frac{\partial \bar{m}}{\partial \bar{r}} = 4\pi \bar{r}^2 \left(v_s^{-2} \bar{p} + 1\right)$$

HTL self-energies

- $\mathrm{Im}\,\Pi \neq 0$ below the lightcone (solid lines)







$\longrightarrow$ Landau damping