Bound Entanglement and the Partial Transpose: Basics

Zoltán Zimborás

University College London

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Partial transposed states

- Given a bipartite Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, and a density matrix ρ on it, then ρ^{T_B} can only have negative eigenvalues if ρ is entangled:

$$\rho^{T_B} = \sum_i p_i (\rho_{A,i} \otimes \rho_{B,i})^{T_B} = \sum_i p_i \rho_{A,i} \otimes \rho_{B,i}^T.$$

- What about entangled states? Let us first consider pure states.
- Consider a pure state $\rho = |\psi\rangle\langle\psi|$ in a bipartite system $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ with Schmidt decomposition $|\psi\rangle = \sum_k \sqrt{\lambda_k} |\phi_k^A\rangle |\phi_k^B\rangle$,

$$\rho = \sum_{k,\ell} \sqrt{\lambda_i \lambda_j} |\phi_k^A\rangle |\phi_k^B\rangle \langle \phi_\ell^A| \langle \phi_\ell^B|,$$

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the eigenvectors and eigenvalues are

$$|\phi_k^A\rangle |\phi_k^B\rangle \;, \quad \lambda_k$$

$$|\phi_k^A\rangle |\phi_\ell^B\rangle \;, \quad \pm\sqrt{\lambda_k \lambda_\ell}$$

- There are NPT states. All entangled pure states are NPT.

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Partial transposed states

- By continuity this, of course, also means that there exist NPT mixed states, and one can define $\mathcal{N}(\rho) = (\|\rho\|_1 - 1)/2$ and $\mathcal{E}(\rho) = \log(\|\rho\|_1)$.
- For a two-qubit mixed state, one can fairly easily obtain that:

$$C(\rho) \geq 2N(\rho) \geq \sqrt{[1 - C(\rho)]^2 + C(\rho)} - [1 - C(\rho)].$$

- All entangled two-qubit states are NPT.

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Partial transposed Werner states

- From the beginning of entanglement theory it has been useful to consider examples of entangled states with high symmetry, e.g. the Werner states $\rho \in \mathcal{B}(\mathbb{C}^d \otimes \mathbb{C}^d)$

$$[\rho, U \otimes U] = 0, \quad \forall U \in U(d), \Rightarrow \rho = \alpha \mathbb{1} + \beta F = \frac{2p_s}{d^2+d} P_+ + \frac{2(1-p_s)}{d^2-d} P_-$$

- They are invariant with respect to unitary twirling

$$\rho \to \int_{U(n)} \mathrm{d}U \, (U \otimes U) \, \rho \, (U^\dagger \otimes U^\dagger)$$

- Any state can be brought by twirling to a Werner state.
- One can easily obtain the NPT and the entanglement regions: $p_s < 1/2$.

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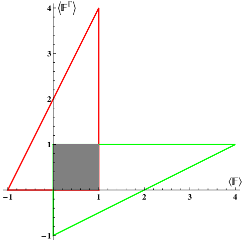
Partial transposed $O \otimes O$ states

- Also $O \otimes O$-symmetric states were considered

$$[\rho, O \otimes O] = 0, \ \ \forall O \in O(d), \ \Rightarrow \rho = \alpha \mathbb{1} + \beta F + \gamma \widetilde{F}$$

They have richer structure than Werner states, but they are still treatable



- One can define the :

$$\rho \to \int_{O(n)} \mathrm{d}O \, O^{\otimes t} \rho (O^\dagger)^{\otimes t}$$

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- Some work has been done on more general $G \otimes G$-invariant, but their structure becomes quickly more complicated, and not available for general dimensions.

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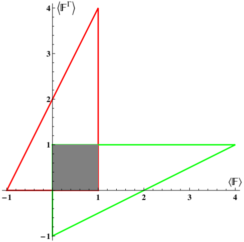

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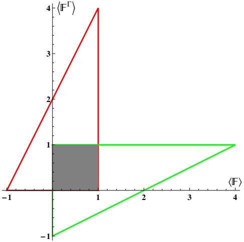

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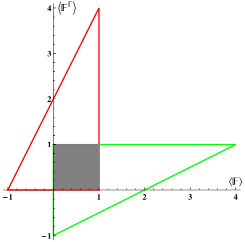

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Partial transposed states and distillability

- Given a bipartite state ρ, considering the the partial transpose of n identical copies of the state $(\rho^{T_B})^{\otimes n} = (\rho^{\otimes n})^{T_B}$. Then the necessary and sufficient condition of distillability (and n-copy distillability) is that there exists a Schmidt-rank-two vector $|\psi\rangle$ such that

$$\langle\psi|(\rho^{T_B})^{\otimes n}|\psi\rangle < 0.$$

- Thus in order to be distillable, the state should be NPT.
- Does there exist non-distillable entangled states? Yes:

$$\rho = \sum_{i=1}^{4} \lambda_i |\psi_i\rangle\langle\psi_i|,$$

with $\lambda = \left(\frac{3257}{6884}, \frac{450}{1721}, \frac{450}{1721}, \frac{27}{6884}\right)$, and the eigenvectors $|\psi_i\rangle \in \mathbb{C}^3 \otimes \mathbb{C}^3$ are given by

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\ ,\ |\psi_2\rangle = \frac{a}{12}(|01\rangle + |10\rangle) + \frac{1}{60}|02\rangle - \frac{3}{10}|21,\rangle$$

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(T. Vértesi, N. Brunner, Nature Comm. 5, 5297 (2014))

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PPT entangled $USp \otimes USp$ states

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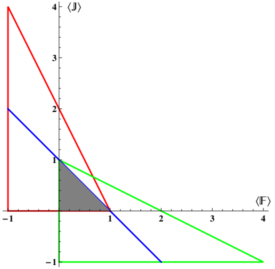

- The NPT and entangled regions do not coincide.
- First systematic generation of bound entangled states.

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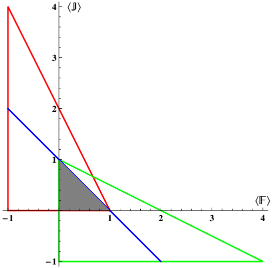

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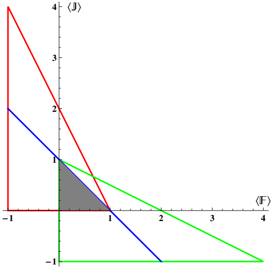
- First systematic generation of bound entangled states.

PPT entangled $USp \otimes USp$ states

- $USp \otimes USp$-invariant states:

$$[\rho, S \otimes S] = 0, \quad \forall S \in USp(2d), \; \Rightarrow \rho = \alpha \mathbb{1} + \beta F + \gamma \mathbb{J}$$



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NPT bound entangled states?

- Does there exist NPT bound entangled states?
- Or in other words: does there exists states, for which there is exists a vector $|\varphi\rangle$ with
$$\langle\varphi|\rho^{T_B}|\varphi\rangle < 0,$$
but for all n and all Schmidt-rank-two vectors $|\psi\rangle$
$$\langle\psi|(\rho^{T_B})^{\otimes n}|\psi\rangle \geq 0.$$
- If there exist NPT bound entangled states, then there exist NPT bound entangled Werner states.
- For all n, there exist entangled but non-n-distillable Werner states.

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