Functional Renormalization Group Approach to Nuclear Matter

Péter Pósfay
Eotvos Lorand University, Wigner RCP of the HAS
Supervisors: Gergely G. Barnafölid, Antal Jakovác

Outline

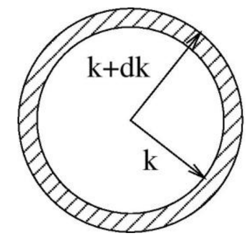
- Short Introduction to FRG
  - Local Potential Approximation (LPA)
- FRG at Finite temperature
- Solving FRG equations at finite temperature
  - Semi finite temperature approximation
- Numerical Solution
  - Toy model
- Walecka-Type model

Motivation for using FRG

- FRG is a general method for finding the effective action of a system.
  - RG idea: gradual momentum integration
  - If a theory is defined at **high energy** scale it is possible to calculate **low energy** effective quantities which includes **quantum fluctuations**.
  - Investigation of phase transitions
- Using FRG methods at finite temperature it is possible to calculate equation of state which include quantum effects.
  - Go beyond mean-field approximation
  - Find tools for FRG calculations suited for Compact Stars

Introduction to FRG-I

- Generating Functional+ Regulator
  - The regualtos acts as a mass term and suppresses fluctuations below scale *k*
  - gradual momentum integration



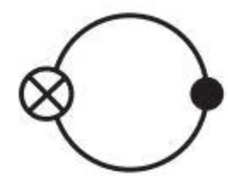
$$Z_k[J] = \int \left(\prod_a d\Psi_a\right) e^{-S[\Psi]-\frac{1}{2}R_{k,ab}\Psi_a\Psi_b+\Psi_a J_a}$$

- The effective Action is the Legenrdre-transform of the Schwinger functional:

$$\Gamma_k[\psi] = \sup_J (\psi_a J_a - W[J]) - \frac{1}{2}R_{k,ab}\psi_a\psi_b$$

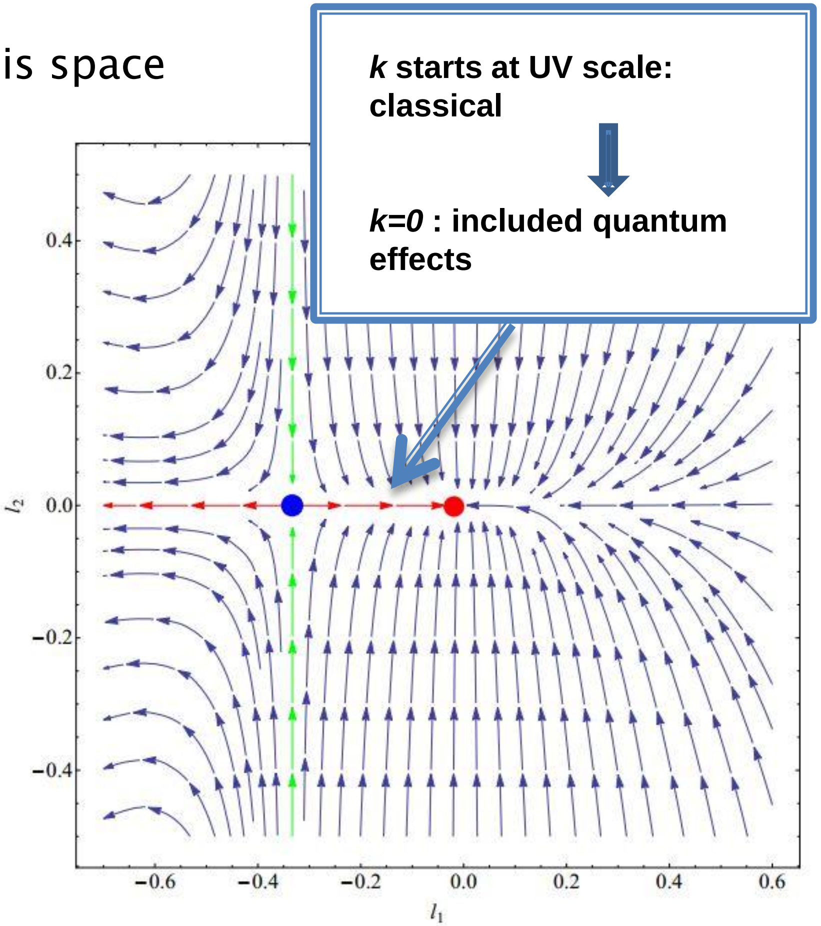
- The scale-dependece of the effective action is given by the Wetterich-equation:

$$\partial_k \Gamma_k = \frac{1}{2} Str\left[(\partial_k R_k)\left(\Gamma_k^{(1,1)} + R_k\right)^{-1}\right]$$



Introduction to FRG-II

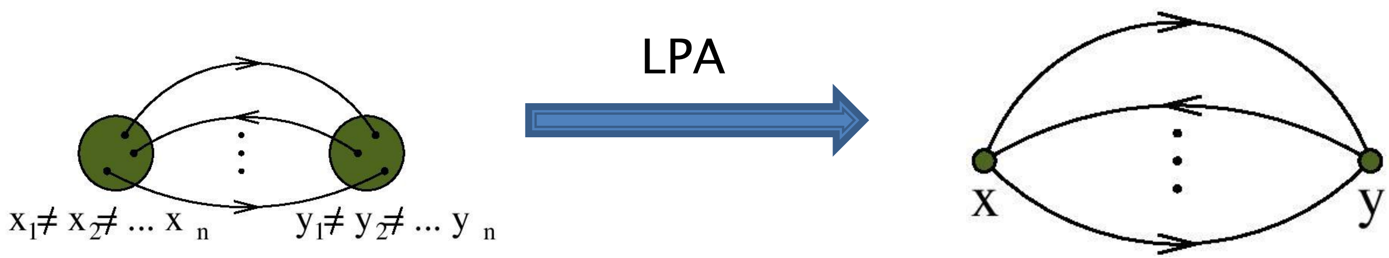
- The scale dependent coupling constants in the effective action defines theory space
  - Each point in this space is a different initial conditon for the Wetterich-equation
  - Wetterich-equation defines a flow in this space
- We define our theory at UV scale k_{UV}.
- Integrating out the Wetterich-equation from k_{UV} to $k=0$, gives the IR scale effective action which includes all quantum fluctuations.
- At fininte temperature this process yields an EoS which contains quantum fluctuations



Solving Wetterich-equation in LPA

- The Wetteric-equation is **exact**, but
  - it is too complicated to solve directly, because we have to use all possible operators in the effective action.
  - For practical purposes one have to use some kind of truncation

- Local potential approximation (LPA):
  - LPA is based on the assumption that the contribution of these two diagrams are close.

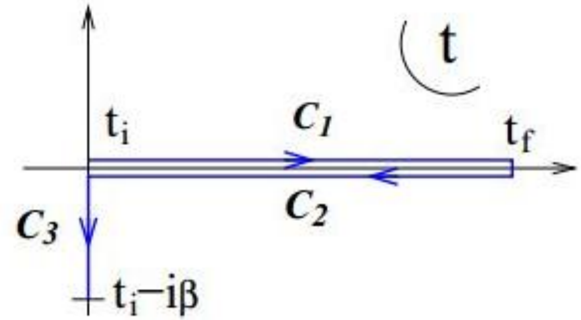


- The LPA ansatz for the effective action:

$$\Gamma_k[\psi] = \int d^4x \left[\frac{1}{2}\psi_i K_{k,ij}\psi_j + U_k(\psi)\right]$$

FRG in LPA at finte temperature

- At finite temperature the path integral needs to extend for imaginary time.



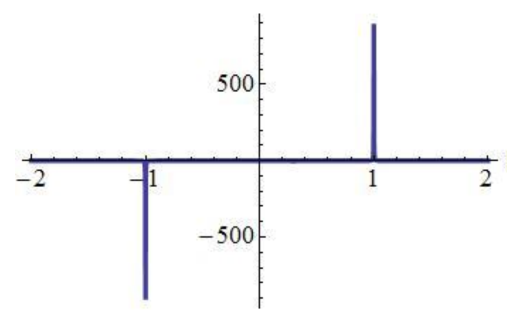
- Since the regulator term is **time-independent**, the Wetterich-equation takes the following form in LPA:

$$\partial_k U = -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4}(\partial_k R_{ij})G_{ij}(p) \quad \Rightarrow \quad \partial_k U = -\frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\,\partial_k R_{ij}(\mathbf{p})\left(\frac{1}{2}+n_{\alpha_i}(p_0)\right)\varrho_{ij}(p)$$

  - Where the Fermi-Dirac/Bose-Einstein distribution is denoted by

$$n_\alpha(\omega) = \frac{\alpha}{e^{\beta\omega}-\alpha}$$

  - and $\varrho_{ij}(p)$ is the spectral function of the system.



Solving FRG-equations numerically

- In the LPA approximation the aim is to determine the scale-dependence of the effective potential U.
- The initial condintion: U function is given at k_{UV}
- For one scalar field at $T=0$, the Wetterich-equation for the effective potential is:

$$\frac{\partial}{\partial k} U_k(\phi) = \frac{k^4}{12\pi^2} \frac{1}{\sqrt{k^2 + \frac{\partial^2 U_k(\phi)}{\partial \phi^2}}}$$

- Methods for numerically solving this equation
  - Newton-Raphson (more widely used)
  - Runge-Kutta type methods (problems with instability)
  - Taylor expansion of the equation and compare the coefficients

Solving FRG-equations at finte T

- For fermionic fields at finite temperature the **Fermi-Dirac distribution** the Newton-Raphson method is non-convergent.
  - Derivatives of Fermi-Dirac distribution at low temperature does not behave well

$$\frac{\partial}{\partial k}U_k(\phi) = \frac{k^4}{12\pi^2}\frac{n_\alpha(p)}{\sqrt{k^2 + \frac{\partial^2 U_k(\phi)}{\partial \phi^2}}}$$

- Modified version of the **Dormand-Price Method** (adaptive Runge-Kutta type)
  - We have to deal with the instabilities in these **explicite methods.**

Semi Finite Temperature Approximation

- The basic idea:
  - If the running of $U_k(\phi)$ is given, Wetterich equation is just an integral
  - Approximate the running of $U_k(\phi)$
- Possible applications:
  - Low temperature approximations of EoS (Compact Stars!)
  - Investigation of relevant parameters in the running of the potential
- LPA for bosonic field at finite temperature

$$\frac{\partial}{\partial k}U_k(\phi) = \frac{k^4}{12\pi^2}\frac{1}{\sqrt{k^2 + \frac{\partial^2 U_k(\phi)}{\partial \phi^2}}}$$

Solve at *T=0*

$$\Rightarrow$$

$$\frac{\partial}{\partial k}U_k(\phi) = \frac{k^4}{12\pi^2}\frac{n_\alpha(p)}{\sqrt{k^2 + \frac{\partial^2 U_k(\phi)}{\partial \phi^2}}}$$

Using the *T=0* solution this is an integral with parameters *T*, *μ*

Toy model

$$\Gamma_k = \underbrace{\bar{\psi}(\not{p} - m}_{\text{Fermionic part}} \underset{\substack{\downarrow \\ \text{Yukawa Coupling}}}{- g_\sigma \sigma)\psi} + \frac{1}{2}(\underbrace{\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2}_{\text{Bosonic part}} + \underset{\substack{\downarrow \\ \text{Running potential}}}{U_k(\sigma)})$$

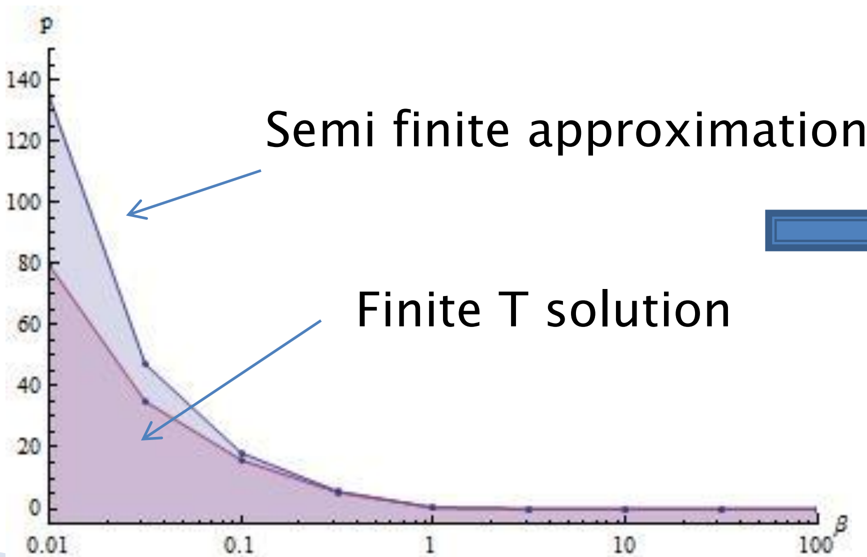
- The Wetterich-equation on Finite temperature in LPA

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left(\underbrace{\frac{2n_b(\omega_\sigma)+1}{\omega_\sigma}}_{\text{Bosonic part}} - \underbrace{8\frac{1-n_f(\omega-\mu)-n_f(\omega+\mu)}{\omega}}_{\text{Fermionic part}}\right)$$

$$\omega_\sigma = \sqrt{k^2 + \frac{\partial^2 U_k}{\partial \sigma^2}} \qquad \omega = \sqrt{k^2 + (g_\sigma \sigma)^2}$$

Properties of the toy model

- FRG equations numerically solveable
  - very similar to the walecka-type models (difference in chemical potential)
  - Ideal to test the semi finite temperature approximation
    - Results: low temperatures: very good approximation
- Compact stars: very good approximation



Enough to solve FRG equations at T=0

The effective potential corresponds to the Grand potential:

$$\phi = \epsilon - Ts - \mu n = -p$$

Walecka-type model

$$\Gamma_k = \bar{\psi}\left(\not{\partial} - g_\sigma(\sigma + i\gamma_5\tau_j\pi^j) - g_\omega\not{\omega}\right)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + U(\sigma,\pi)$$

LPA + Mean Field Approximation to the ω-meson

- Wetterich-equation is very similar to the toy-model

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left(\underbrace{\frac{2n_b(\omega_\sigma)+1}{\omega_\sigma} + 3\frac{2n_b(\omega_\pi)+1}{\omega_\pi}} - 8\underbrace{\frac{1-n_f(\omega-\mu)-n_f(\omega+\mu}{\omega}}\right)$$

$$\omega_\sigma = \sqrt{k^2 + \frac{\partial^2 U_k}{\partial\sigma^2}}$$

$$\partial_k\Gamma_k = \frac{k^4}{12\pi^2}(bosonic - fermionic)$$

$$\omega_\pi = \sqrt{k^2 + \frac{\partial U_k}{\partial\sigma}}$$

- Running of ω

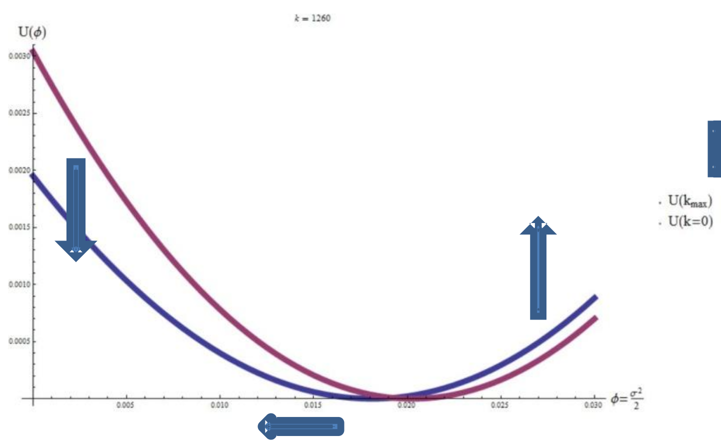
$$\omega = \sqrt{k^2 + (g_\sigma\sigma)^2}$$

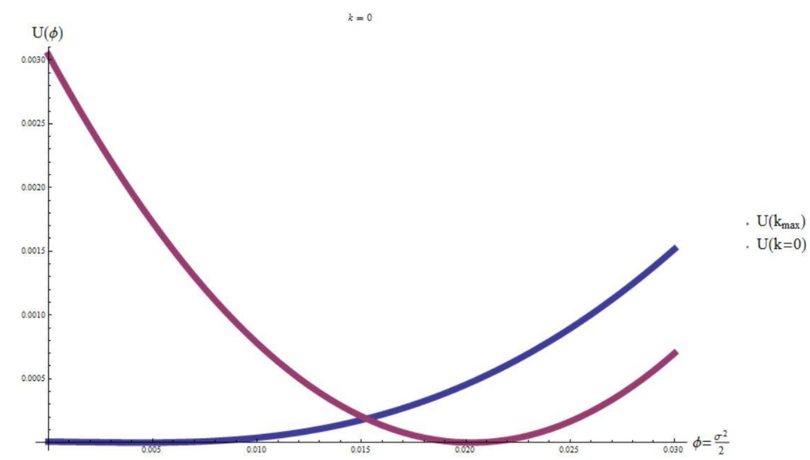
$$\partial_k\omega_{0,k} = -\frac{2g_\omega k^4}{3\pi^2 m_\omega^2}\frac{\partial}{\partial\mu}\left(\frac{n_f(\omega-\mu)+n_f(\omega+\mu)}{\omega}\right)$$

Numerical Solution

- Set U to reproduce vacuum expectation value at *k=0, vev=93MeV*
  - *k=1.3 GeV*
  - *m=1.2GeV²*
  - *λ=7.4*

$$U(\phi) = -m\phi + \lambda\phi^2 \qquad \phi = \frac{\sigma^2}{2}$$





Maxwell construction

Conclusions

- Motivation:
  - Exploring methods to go beyond mean field approximation
  - Quantum fluctuations can be calculated in FRG
- Goal:
  - Scaleable equation of state for compact stars
- Status:
  - In case of compact stars: semi finite temperature approximation can be used

Thank you for your attention!