Event-by-event anisotropies in hydro

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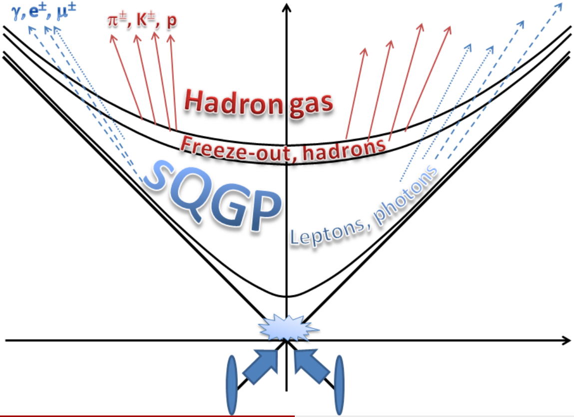
Balaton workshop, Tihany, 14 July 2015

July 14, 2015

Hydrodynamics in high energy physics

- Strongly interacting QGP discovered at RHIC & created at LHC
- A hot, expanding, strongly interacting, perfect fluid
- Hadrons created at the "chemical" freeze-out
- Hadron distributions decouple at "kinetic" freeze-out

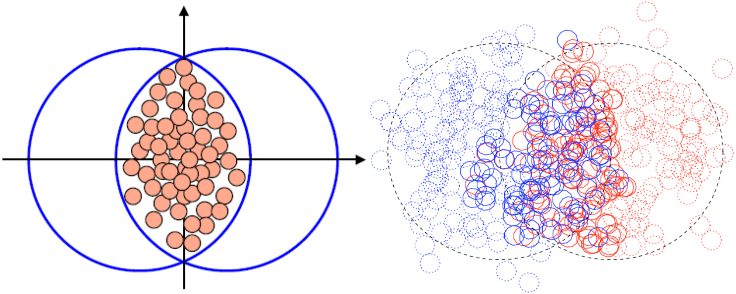


Known solutions of relativistic hydrodynamics

- Many solve the hydro equations numerically
- *Exact, analytic solutions* are important to connect initial and final state
- Famous 1+1D solutions: Landau, Hwa, Bjorken
- Many new 1+1D solutions, few 1+3D, with spherical/axial symmetry
- First truly 3D relativistic solution
  Csörgő, Csernai, Hama, Kodama, Heavy Ion Phys. **A21**, 73 (2004)
- Assumes ellipsoidal symmetry via scaling variable
$$s = \frac{x^2}{X^2} + \frac{y^2}{Y^2} + \frac{z^2}{Z^2}$$
- X, Y, Z: time dependent axes of expaning ellipsoid
- Thermodynamical quantities depend only on s
- *Describes hadron data*
  Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010)
- *Describes photon & lepton data*
  Csanád, Májer, Central Eur. J. Phys. **10** (2012)
  Csanád, Krizsán, Central Eur.J.Phys. **12** (2014)

Higher order anisotropies?

- Elliptic-like shape $\Rightarrow$ anisotropic particle production
- Finite number of nucleons $\rightarrow$ higher order anisotropies!



- Final state anisotropy characterized by $v_n = \langle \cos n\phi \rangle$
- *Exact solutions handling this?*
- *Viscous effects on the time evolution of the anisotropies?*
- *Mixing of anisotropies in flow and in coordinate space?*

Generalization of elliptic symmetry

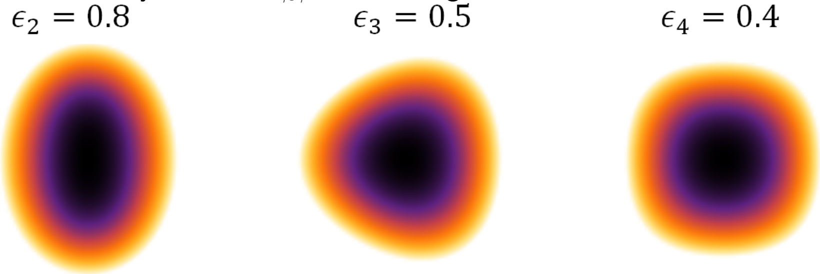
- *How to generalize* the ellipsoidal scaling variable of $s = \frac{x^2}{X^2} + \frac{y^2}{Y^2} + \frac{z^2}{Z^2}$?
- Redefine it via

$$\frac{1}{R^2} = \frac{1}{X^2} + \frac{1}{Y^2} \text{ and } \epsilon = \frac{X^2 + Y^2}{X^2 - Y^2} \Rightarrow s = \frac{r^2}{R^2}\left(1 + \epsilon \cos(2\phi)\right)$$

- Generalize: *N-pole symm.* in transverse plane

$$s = \frac{r^N}{R^N}\left(1 + \epsilon_N \cos(N\phi)\right)$$

- ϵ_1 defines only a shift, $\epsilon_{2,3,\dots}$ interesting



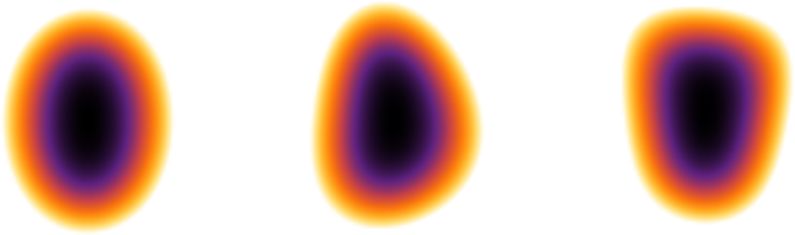
Multipole symmetries combined

- *Multiple symmetries* can be combined:

$$s = \sum_N \frac{r^N}{R^N} \left(1 + \epsilon_N \cos(N(\phi - \psi_N))\right)$$

- Aligned by Nth order reaction planes ψ_N
- Again, $\epsilon_1 = 0$ can be assumed
- R defines time dependent scale: expansion
- Basically any shape can be described, via a "multipole expansion"

$\epsilon_2 = 0.8, \epsilon_3 = 0, \epsilon_4 = 0$ $\quad$ $\epsilon_2 = 0.8, \epsilon_3 = 0.5, \epsilon_4 = 0$ $\quad$ $\epsilon_2 = 0.8, \epsilon_3 = 0.5, \epsilon_4 = 0.4$



New solutions of hydrodynamics

- *New solutions* with multipole symmetries
  Csanád, Szabó, Phys.Rev. C90 **5** (2014) 054911
  based on Csörgő, Csernai, Hama, Kodama, Heavy Ion Phys. **A21**, 73 (2004)

$$s = \sum_N \frac{r^N}{R^N}\left(1+\epsilon_N \cos(N(\phi-\psi_N))\right) + \frac{z^N}{Z^N}$$

$$u^\mu = \gamma\left(1, \frac{\dot{R}}{R} r\cos\phi, \frac{\dot{R}}{R} r\sin\phi, \frac{\dot{R}}{R} z\right)$$

$$T = T_f\left(\frac{\tau_f}{\tau}\right)^{3/\kappa}\frac{1}{\nu(s)}$$

- *Observed higher order harmonics*: Maxwell-Jüttner type source function

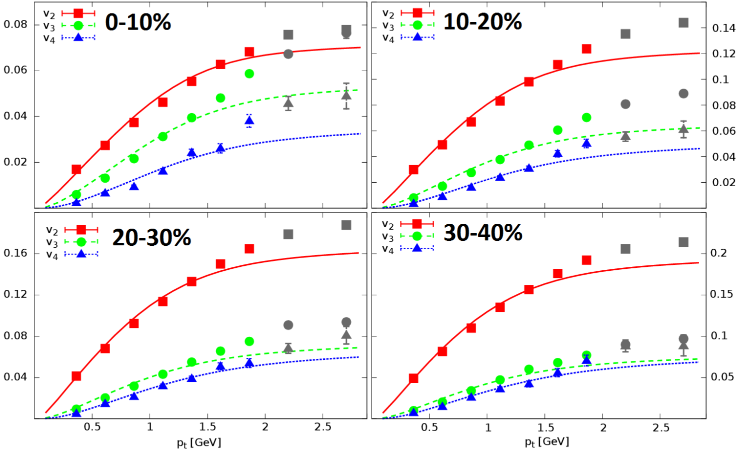
$$S(x,p) \propto \exp\left[-\frac{p_\mu u^\mu(x)}{T(x)}\right]\delta(\tau-\tau_f)\frac{p_\mu u^\mu}{u^0}$$

- Momentum distribution $N(p)$ and anisotropies $v_n(p_t)$:

$$N(p) = \int S(x,p) d^4x \text{ and } v_n(p_t) = \langle\cos(n\alpha)\rangle_{N(p)}$$

Comparison to PHENIX anisotropy coefficients

- PHENIX measured v_2, v_3 and v_4 in various centrality classes
  PHENIX Coll., Phys. Rev. Lett. **107** (2011) 252301
- Fitted parameters: ϵ_N and transverse flow u_t



Multipole velocity field?

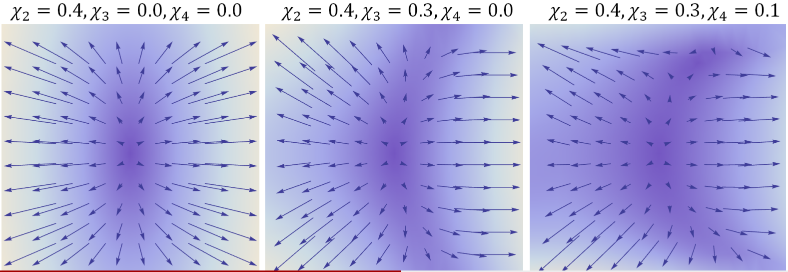
- No analytic solutions with multipole flow!
- *Buda-Lund model*: hydro final state parametrization

  Csanád, Csörgő, Lörstad, NPA742 (2004) 80, Csörgő, Lörstad, PRC54 (1996) 1390
- Add multipole densities (just as previously), add *multipole flow*!

  Csanád *et al.*, arXiv:1504.07932

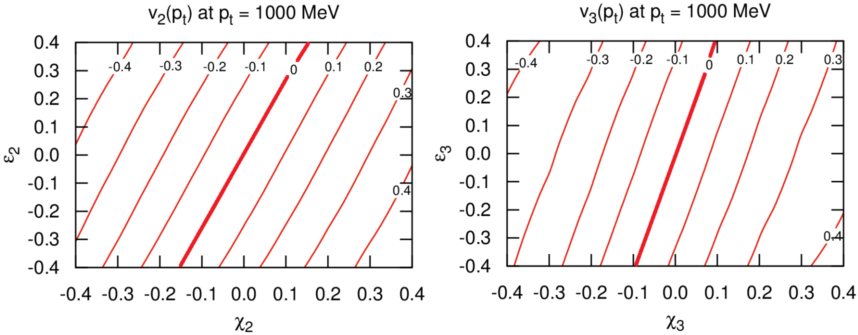
$$u^\mu = (\gamma, \partial_x\Phi, \partial_y\Phi, \partial_z\Phi) \text{ with } \Phi = \frac{r^2}{2H}\left(1 + \sum_n \chi_n \cos(n\phi)\right)$$

- H is Hubble-coefficient like (take $N = 1$ with $\chi_1 = 0$)



Anisotropy mixing in flow

- *Flow- and density anizotropies mix* in v_n
- Both ϵ_n (spatial anisotropy) and χ_n (flow anisotropy) determine v_n

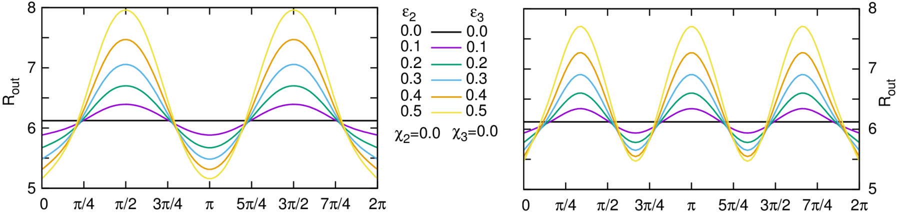


- Measurement of v_n does not directly relate to final spatial or flow anisotropy

Azimuthally sensitive HBT

- Bose-Einstein correlation radii depend on momentum angle
- If measured w.r.t. the second order event plane: $\cos 2\phi$ oscillation
- If measured w.r.t. the third order event plane: $\cos 3\phi$ oscillation

  PHENIX Coll., Phys.Rev.Lett. **112** (2014) 222301
- Their oscillation reveals source geometry as well

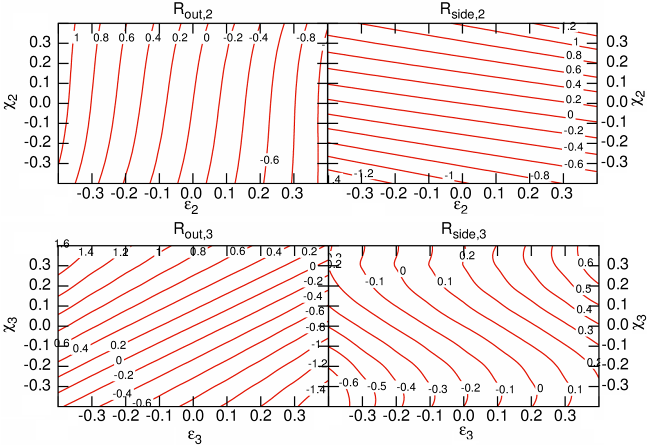


- Quantify the 2nd and 3rd order oscillations w.r.t to the given event plane:

$$R^2_{\mathrm{out,side}} = R^2_{\mathrm{out,side,0}} + R^2_{\mathrm{out,side,2}} \cos 2\phi$$
$$R^2_{\mathrm{out,side}} = R^2_{\mathrm{out,side,0}} + R^2_{\mathrm{out,side,3}} \cos 3\phi$$

Anisotropy mixing in asHBT

- Flow anisotropies (χ) and spatial anisotropies (ϵ) mix



- R_{out} and R_{side} tell about χ and ϵ together

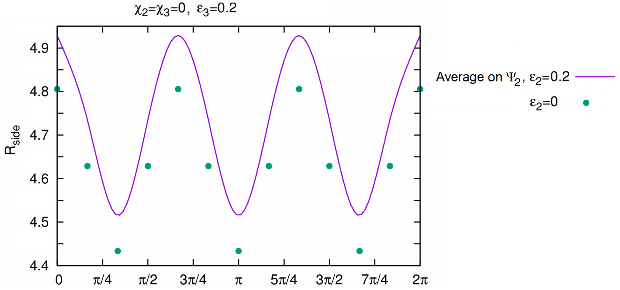
Mixing of 2nd and 3rd order anisotropies?

- Random relative orientation of the event planes
- Rotating to the 2nd or 3rd order event plane gives:

$$s = \frac{r^2}{R^2}\left(1 + \epsilon_2 \cos(2\phi) + \epsilon_3 \cos(3\phi - \Delta\Psi_{2,3})\right)$$

$$s = \frac{r^2}{R^2}\left(1 + \epsilon_2 \cos(2\phi + \Delta\Psi_{2,3}) + \epsilon_3 \cos(3\phi)\right)$$

- Averaging on $\Delta\Psi_{2,3}$ removes one of the oscillations? Almost.



Time evolution of the anisotropies

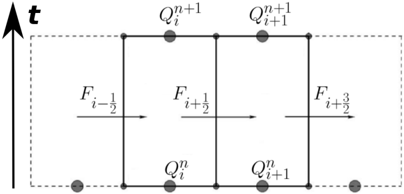
- Solution described in the beinning of the talk: anisotropies constant!
- What about initial conditions with pressure gradients?
- Does the value of sound speed play a role?
  Note exact solution with temperature dependent EoS
  Csanád, Nagy, Lökös, Eur.Phys.J. **A48** (2012) 173
- What is the effect of viscosity? Does it "wash out" anisotropies?
- No exact solutions handling these questions!
- Our work: start from initial conditions "near" known solutions
  Csanád *et al.*, arXiv:1504.07932
- Measure anisotropies as $\varepsilon_n = \langle \cos(n\varphi) \rangle_{n,e,v}$ in every time-step
- Relation between input ϵ_n (in scale variable s) and resulting ε_n:

$$\varepsilon_1 = \frac{(\epsilon_2 + \epsilon_4)\epsilon_3}{2 + \sum_n \epsilon_n^2}, \qquad \varepsilon_3 = \frac{-\epsilon_3}{2 + \sum_n \epsilon_n^2},$$

$$\varepsilon_2 = \frac{-\epsilon_2 + \epsilon_2\epsilon_4}{2 + \sum_n \epsilon_n^2}, \qquad \varepsilon_4 = \frac{-\epsilon_4 + \frac{1}{2}\epsilon_2^2}{2 + \sum_n \epsilon_n^2}$$

Our numerical hydro

- Convective 1+2D form: $\partial_t Q + \partial_x F(Q) + \partial_y G(Q) = 0$
- Q values in the cells, intercell fluxes needed



- Method: multistage GFORCE fluxes

  E. F. Toro and V. A. Titarev, J. Comp. Phys. **216**, 403 (2006)

- Iterated intercell Q and flux

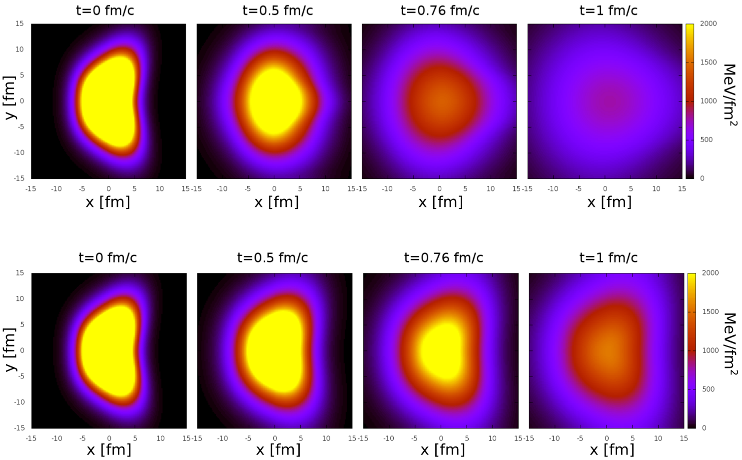
$$Q_{i+\frac{1}{2}}^{(l)} = \frac{1}{2}\left[Q_i^{(l)} + Q_{i+1}^{(l)}\right] - \frac{1}{2}\frac{\Delta t}{\Delta x}\left[F_{i+1}^{(l)} - F_i^{(l)}\right]$$

$$F_{i+\frac{1}{2}}^{(l)} = \frac{1}{4}\left[F_{i+1}^{(l)} + 2F_M^{(l)} + F_i^{(l)} - \frac{\Delta x}{\Delta t}\left(Q_{i+1}^{(l)} - Q_i^{(l)}\right)\right]$$

- Iterate this e.g. 8 times within one timestep

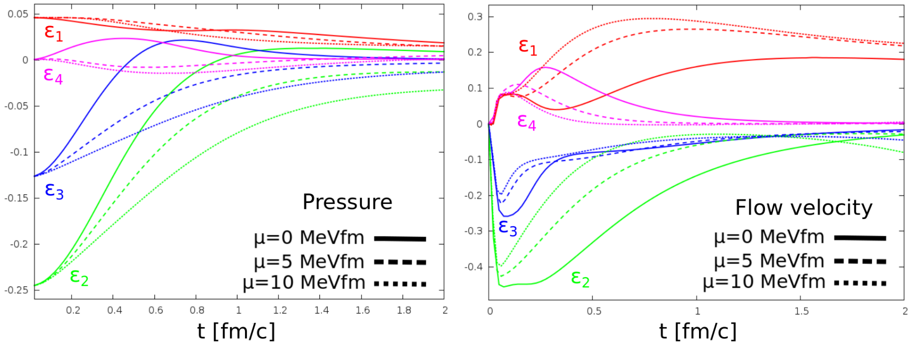
Effect of viscosity in nonrel hydro

- Pressure evolution without and with viscosity ($\mu = 10$ MeV fm):



Effect of viscosity in nonrel hydro

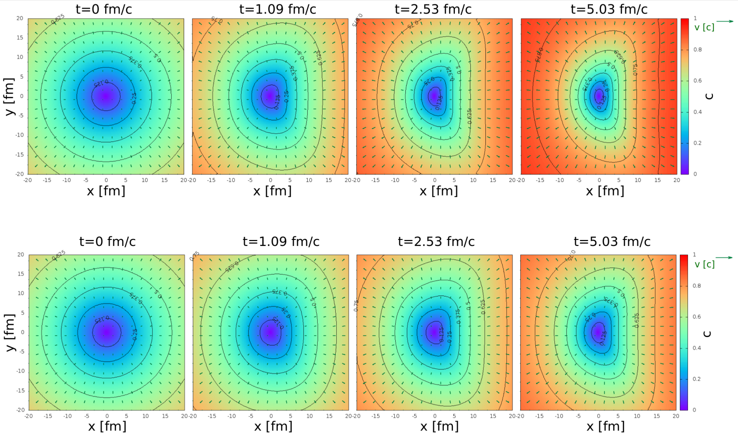
- Time evolution of the anisotropies:



- Pressure: viscosity keeps anisotropies there
- Flow: viscosity "washes out" anisotropies

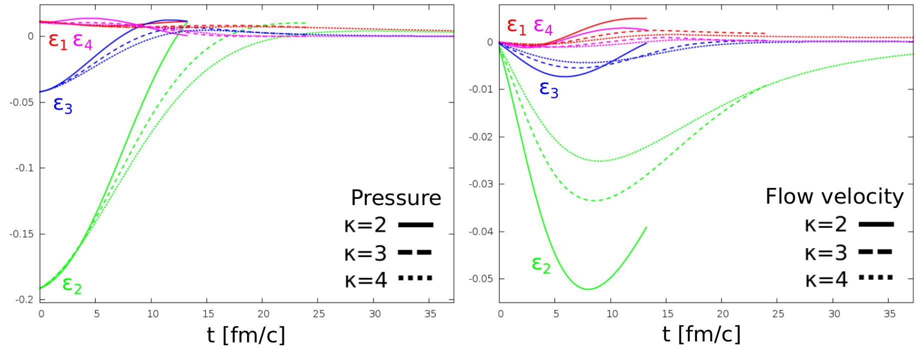
Effect of speed of sound in relativistic hydro

- Flow evolution with $c_s^2 = 0.5$ and $c_s^2 = 0.25$



Effect of speed of sound in relativistic hydro

- Here we indicate the freeze-out time as well
- Time evolution of the anisotropies:



- Pressure: soft EoS ⇒ slow isotropisation
- Flow: soft EoS ⇒ smaller anisotropies

Summary

- Medium of high energy collisions: hydro expansion
- Higher order anisotropies measured
- Arise due to fluctuating initial conditions
- *First analytic solutions* to describe v_n's
- *Anisotropy mixing* in multipole Buda-Lund model
- *Time evolution of the anisotripies* in a numerical framework

Thank you for your attention!

And let me invite you to the 15th Zimanyi School in Budapest

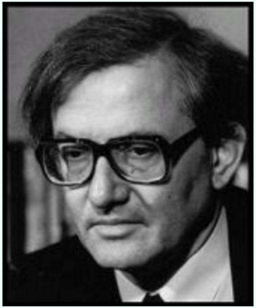
ZIMÁNYI SCHOOL'15

Arnold Gross: Lexicon

15. Zimányi

WINTER SCHOOL ON HEAVY ION PHYSICS

Dec. 7. - Dec. 11., Budapest, Hungary



József Zimányi (1931 - 2006)

http://zimanyischool.kfki.hu/15/

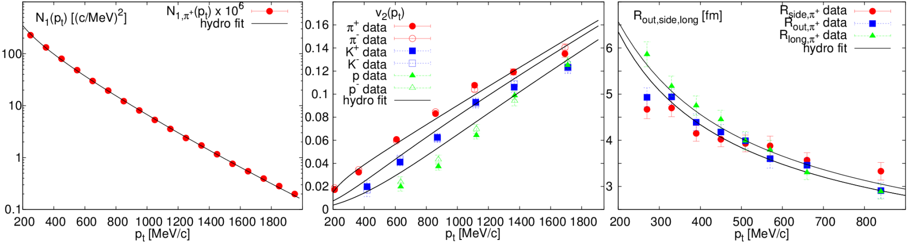
Soft hadron creation in A+A via hydro

- Take first exact, analytic and truly 3D relativistic solution

  Csörgő, Csernai, Hama *et al.*, Heavy Ion Phys. **A21**, 73 (2004), nucl-th/0306004
- Calculate observables for identified hadrons
  - Transverse momentum distribution $N_1(p_t)$
  - Azimuthal asymmetry $v_2(p_t)$
  - Bose-Einstein correlation radii $R_{out,side.long}(p_t)$
- Compared to data successfully (RHIC shown, LHC done as well)

  Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010), arXiv:0909.4842

  Data: PHENIX Coll., PRC**69**034909(2004), PRL**91**182301(2003), PRL**93**152302(2004)

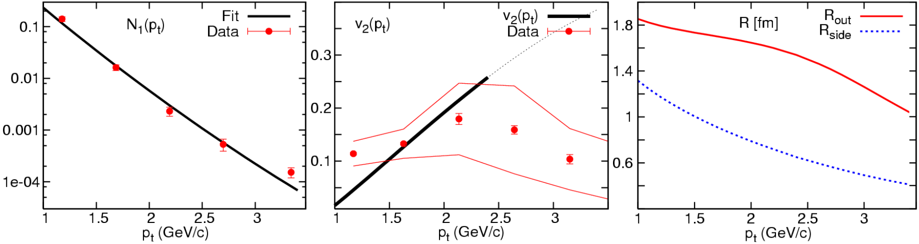


Penetrating probes: photons and leptons

- Photons and leptons are created throughout the evolution
- *Their distribution reveals information about the EoS!*
- Compared to PHENIX data (spectra and flow) successfully
- Predicted photon HBT radii

Csanád, Májer, Central Eur. J. Phys. **10** (2012), arXiv:1101.1279

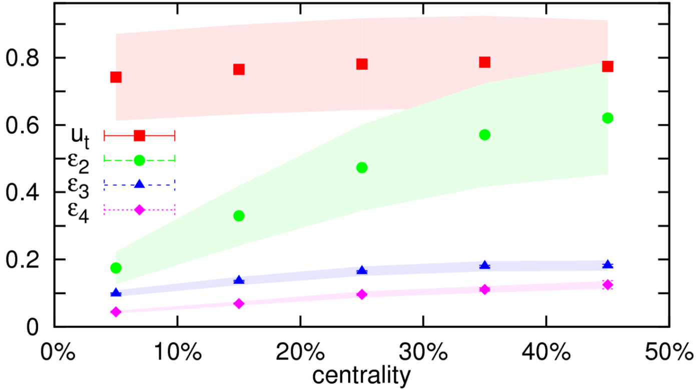
Data: PHENIX Collaboration, arXiv:0804.4168 and arxiv:1105.4126



- Average EoS: $c_s = 0.36 \pm 0.02_{stat} \pm 0.04_{syst}$ (i.e. $\kappa = 7.7$)
- Compatible with soft dilepton data as well

Comparison to PHENIX anisotropy coefficients

- *Successful fit*, see details in arXiv:1405.3877
- Transverse flow u_t: minor dependence on centrality
- Strongly influenced by temperature gradient
- ϵ_N increased for peripheral collisions



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