Hagedorn mechanism for QCD phase transition

hadrons and quarks in the crossover regime

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Outlines

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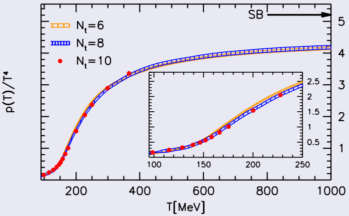
Goal: describe QCD equation of state

- "measurement": Monte Carlo simulations

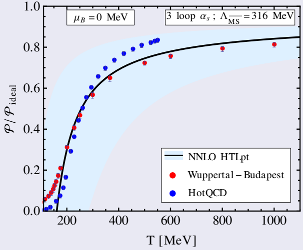
EoS from MC



(Sz. Borsanyi et al, JHEP 1011 (2010) 077)

Goal: describe QCD equation of state

HTL 3-loops

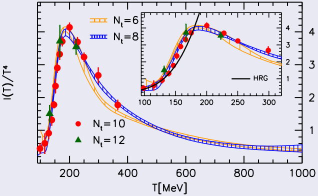


(N. Haque, *et.al.*, JHEP 1405 (2014) 027)

- "measurement": Monte Carlo simulations
- at high T: QGP with 8 gluon + 3 quark; valid for $T \gtrsim 250 - 300\,\mathrm{MeV}$.

Goal: describe QCD equation of state

HRG thermodynamics



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  free hadron resonance gas (HRG) with real masses $T \lesssim 150 - 180\,\mathrm{MeV}$.

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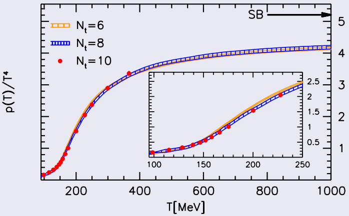

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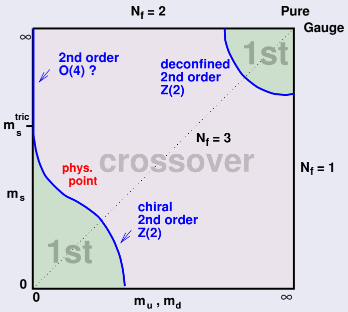
- at low T: hadrons
  free hadron resonance gas (HRG) with real masses $T \lesssim 150 - 180\,\mathrm{MeV}$.
- between: continuous crossover
  "T_c" $= 156$ MeV
  is it a nonperturbative regime?
  or just needs a proper point of view?
  (cf. hadrons are pert. as hadron gas, nonpert. as QCD states)

What drives the the transition?

Columbia plot



Mechanisms of the PT

deconfinement (at $m_{u,d,s} \to \infty$)

- hadrons become unstable
- order parameter: Polyakov-loop

chiral phase transition (at $m_{u,d,s} \to 0$)

- chiral condensate unstable
- order parameter: $\langle \bar{\Psi}\Psi \rangle$

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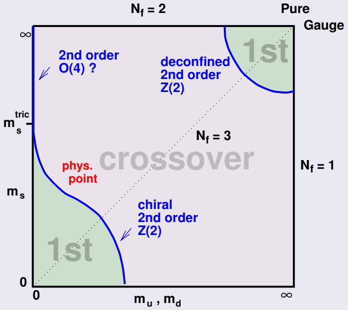

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Hagedorn's mechanism (for crossover?)

- instability because of the large number of available hadronic resonances

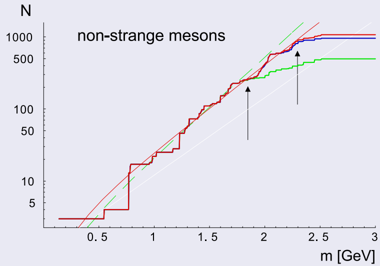
  (R. Hagedorn, Nuovo Cim.Suppl. 3 (1965) 147-186)

- order parameter? heuristically N_{hadr}

Hagedorn's mechanism for phase transition

experimental evidence: exponentially rising energy level density

of hadronic resonances



(W. Broniowski *et.al.*, PRD **70**, 117503 (2004))

Hagedorn-spectrum:

$\varrho_{hadr}(m) \sim (m^2 + m_0^2)^a e^{-m/T_H}$

several fits possible (e.g. $a = 0$)

Hagedorn's mechanism for phase transition

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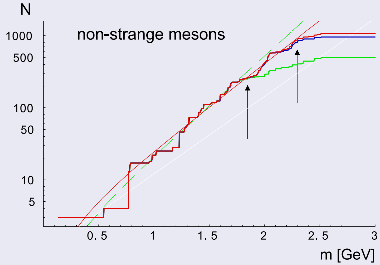

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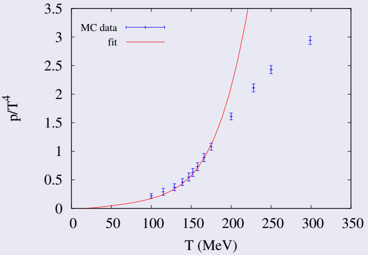
HRG fit for thermodynamics



(AJ. PRD88 (2013) 065012 [arXiv:1306.2660])

Hagedorn-spectrum:

$$\varrho_{hadr}(m) \sim (m^2 + m_0^2)^a e^{-m/T_H}$$

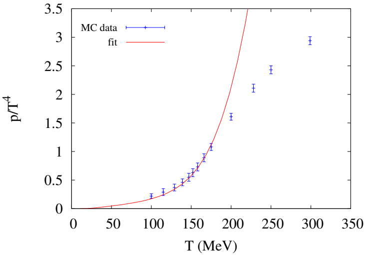
several fits possible (e.g. $a = 0$)

Instability:

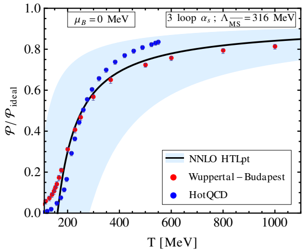
pressure divergent at $T = T_H$

fit: $T_H = 240\,\mathrm{MeV}$

Phase transition?



vs.



Problem: the hadronic pressure is unavoidable there...

- p_{HRG} overshoots the real pressure
- $p_{HRG} \gtrsim p_{pert\ QCD} \quad \Rightarrow \quad F_{HRG} \lesssim F_{pert\ QCD}$, no phase transition

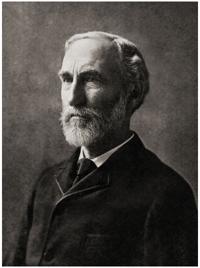
$$\Downarrow$$

hadronic degrees of freedom must disappear from the system!

Is it possible without an abrupt change of ground state?

Outlines

The Gibbs paradox



J.W.Gibbs (1839-1903)

(J.W.Gibbs, 1875-1878; E.T.Jaynes, 1996)

take mixture of two (ideal) gases in volume V

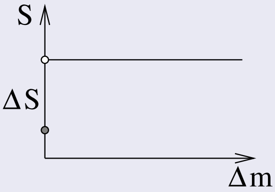
for simplicity $n_1 = n_2 = n$

entropy excess of the mixture (mixing entropy):

$$S = \begin{cases} -nR\log 2 & \text{for different gases} \\ 0 & \text{for indistinguishable particles} \end{cases}$$

In general $S = -nR\log N_{dof}$, where N_{dof} is the number of (distinguishable particle) species.

discontinuous entropy



In quantum system

control parameter: mass difference Δm

$$N_{dof} = \begin{cases} 2 & \text{for } \Delta m \neq 0 \\ 1 & \text{for } \Delta m = 0 \end{cases}$$

Jump in entropy as a function of Δm

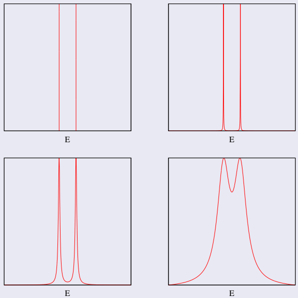
$\Rightarrow$ first order (quantum) phase transition

distinguishable-indistinguishable phase tr.

Gibbs paradox in interacting systems

Observe the mixture of the two gases by a spectrometer!
For interacting gases the spectral lines broaden.
Depending on the line width and mass difference we can have different situations:

spectrum in interacting gases



- in 1st plot clear 2 lines, $N_{dof} = 2$
  4th plot? one broad peak?
- in the overlapping regime common energy states $\Rightarrow$ not possible to divide it into two peaks
- Γ width sets the resolution
  we expect $N_{dof}(\Delta m/\Gamma)$
  $\uparrow$

how can we calculate this function?

Thermodynamics from spectral function

Goal: obtain $p(\varrho)$ function (pressure vs spectrum)

(AJ and T.S. Biro PRDD90 (2014) 9, 094029, AJ. Phys.Rev. D86 (2012) 085007; Phys.Rev. D88 (2013) 065012)

Strategy

- construct a model representing this ϱ
  first a quadratic approximation (cf. HRG)
- calculate thermodynamics from this theory
  energy density $\varepsilon = \frac{1}{Z}\,\mathrm{Tr}\, e^{-\beta H} T_{00}$, use KMS relation

Scalar field case

$$\mathcal{L} = \frac{1}{2}\Phi^*(p)\mathcal{K}(p)\Phi(p), \qquad \varrho = \mathrm{Disc}\, i\mathcal{K}^{-1}$$

defines a consistent field theory

unitary, causal, Lorentz-invariant, E, $\vec{p}$ conserving

Thermodynamics from spectral function II.

Result:

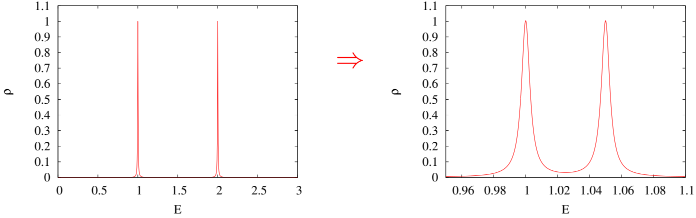
Pressure as a function of the spectral function

$$p = \mp T \int \frac{d^4q}{(2\pi)^4} \frac{\partial \mathcal{K}}{\partial q_0} \ln\left(1 \mp e^{-\beta q_0}\right) \varrho(q)$$

- for free gas mixture $\varrho(p) = \sum_i Z_i \delta(p_0 - E_p)$
  we obtain $P = \sum_i P^{(0)}(m_i)$: sum of partial pressures;
  no dependence on Z_i, while they are nonzero!
- generally nonlinear ϱ dependence
  e.g. P does not depend on the overall normalization of ϱ.

Gibbs paradox for interacting gases

What is going on in the language of spectral functions?



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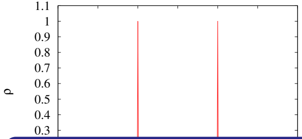

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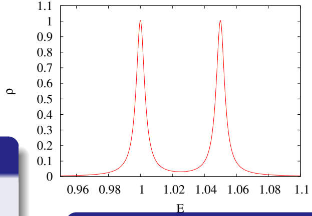
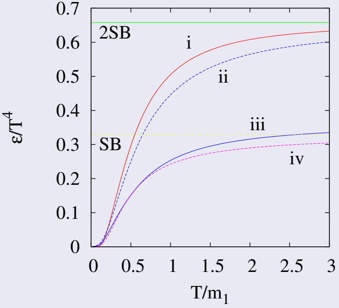


energy density



Curves

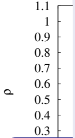
$m_1 = 1$, $m_2 = 2$, $\Gamma = 0$ or 0.2

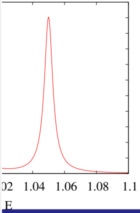
- i. $\Gamma = 0$
- ii. independent, finite Γ
- iii. $\Gamma/\Delta m = 0.2$
- iv. one free particle

characterization: $N_{eff} = \dfrac{P}{P_{free}}$

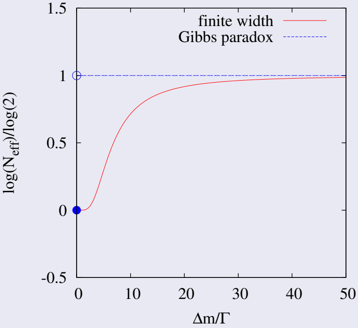
Gibbs paradox for interacting gases

Wha … functions?

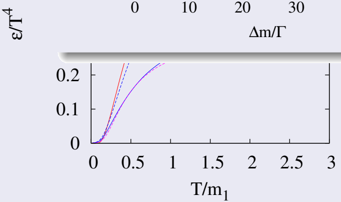




number of dof



energ…



… $= 2$, $\Gamma = 0$ or 0.2

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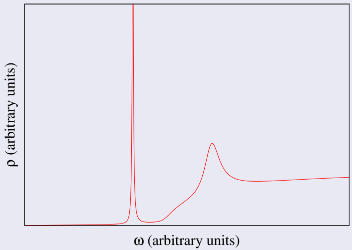
Outlines

QCD case

A typical spectral function



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bound states (peaks)
scattering states (continuum)

We study the effect of merging bound state and scattering states.

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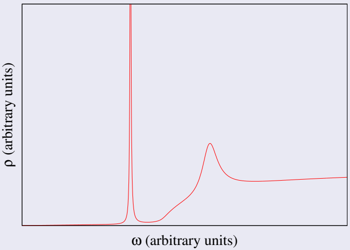


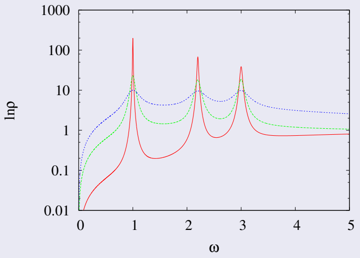
In QFT the spectrum consists of
bound states (peaks)
scattering states (continuum)

We study the effect of merging bound state and scattering states.
Physically: **Melting of bound states!**

Illustrative example

We can examine different realistic spectra:

Three-peak spectra



- Characterize the spectrum with 1st peak width γ

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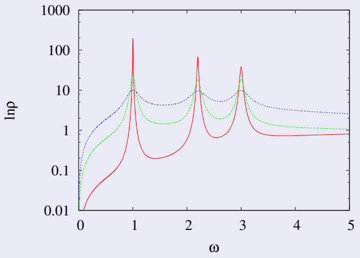
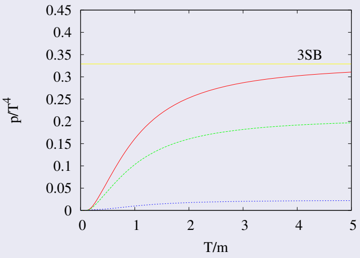


Three-peak pressure



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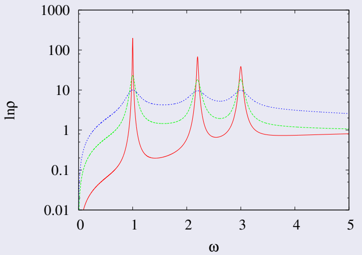

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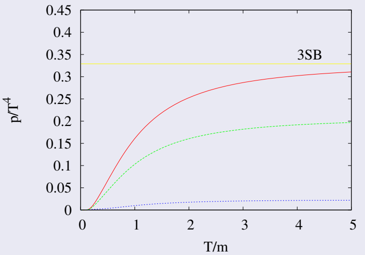
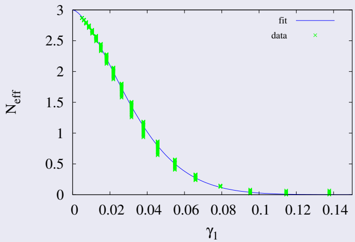


- Characterize the spectrum with 1st peak width γ
- pressure vanishes for $\gamma \to \infty$!
- Observation: P factorizes: $P(T, \gamma) = N_{eff}(\gamma) P_0(T)$.

Effective number of degrees of freedom

Robust result: N_{eff} vs. qp width γ

Effective # of dof



- pressure vanishes $N_{eff}(\gamma) \stackrel{\gamma\to\infty}{\longrightarrow} 0$
- fit function stretched exponential: $N_{eff}(\gamma) = e^{-a\gamma^b}$ (typically $b \sim 1.5-2$)
- treating N_{eff} as an order parameter: crossover transition

Application to QCD

An oversimplified realization of these ideas for QCD

$$P_{hadr}(T) = N_{eff}^{(hadr)} \sum_{n \in \text{hadrons}}^{N} P_0(T, m_n), \qquad \ln N_{eff}^{(hadr)} = -(T/T_0)^b,$$

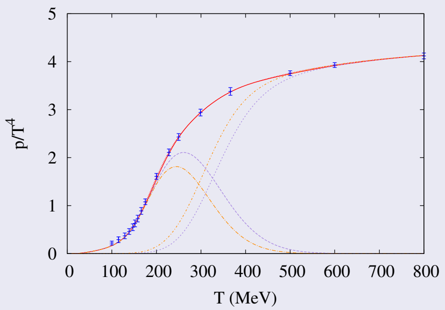
$$P_{QGP}(T) = N_{eff}^{(part)} \sum_{n \in \text{partons}} P_0(T, m_n), \qquad \ln N_{eff}^{(part)} = G_0 - c(N_{eff}^{(hadr)})^d.$$

$P = P_{hadr} + P_{QGP}$ total pressure, P_0 ideal gas pressure

- hadrons: Hagedorn-sp. up to a certain mass ($m \lesssim 3\,\mathrm{GeV}$)
- partons quark and gluon quasiparticles
- $N_{hadr}(\gamma)$ common suppression factor for all hadrons: stretched exponential, and $\gamma \sim T$
- $N_{part}(N_{hadr})$ partonic suppression factor grows with the # of available hadronic resonances.

Matter content of QCD

QCD EoS matter content



- fit parameters: total pressure is well reproduced
- hadrons do not vanish at T_c: they just start to melt there.
- hadrons dominate the pressure until $\sim 2T_c$
- pure QGP only for $T \gtrsim 3T_c$
- different fits yield similar results

Outlines

Conclusions

QCD phase transition at the physical point may be governed by

Hagedorn mechanism with hadron melting

- Hagedorn-mechanism: free pressure of a large amount of hadronic dof dominate the QCD pressure
- melting: Gibbs mechanism for QCD
  - distinguishable-indistinguishable phase transition
  - melting $\equiv$ qp peak merges with the continuum
- result for QCD: hadrons start to melt at $T \sim T_c$, dominate pressure for $T \lesssim 2T_c$ and vanish at $T \sim 3T_c$
- not free-particle-like excitations! (cf. transport, correlations...)
- perturbative crossover? QCD dof & hadrons