Investigation of Kaluza-Klein Compact Star Equation of States

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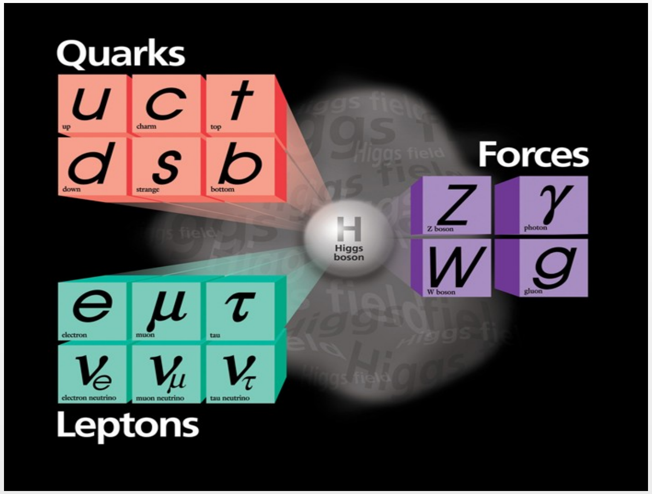
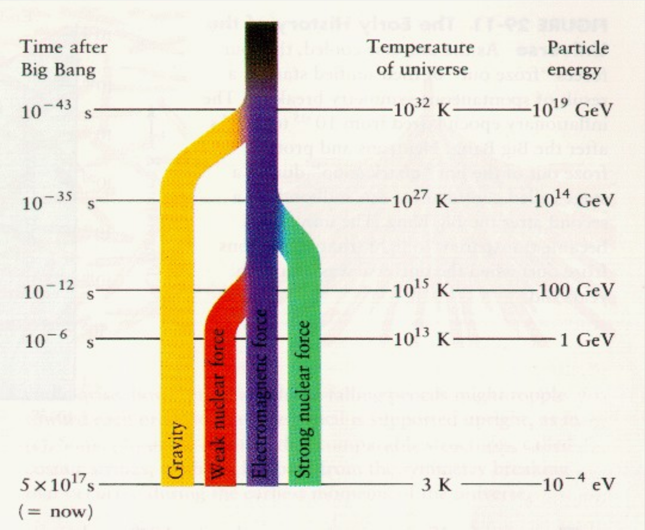
Outline

1. Motivation for Kaluza–Klein theory & stars
2. A special solution in 1+4D space-time
3. Compact Star in Kaluza–Klein World
4. M–R relation via varying R_C for 1+4D compact star
5. Comparison with standard hyperon star models

1. Motivation for Kaluza–Klein Stars

- Standard matter by Standard Model
  - Electromagnetic;
  - Weak and
  - Strong interactions
- Grand Unified Theory...
  - Gravity and QFT are not fitting into the same picture
  - GR locally valid; curved space-time
  - QFT globally valid; Minkowski





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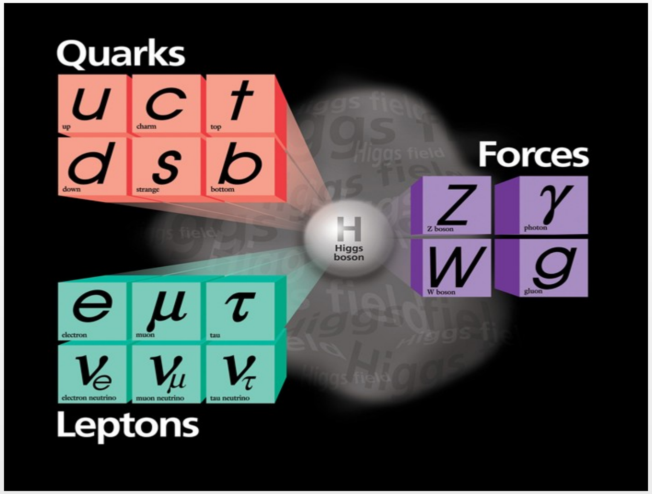

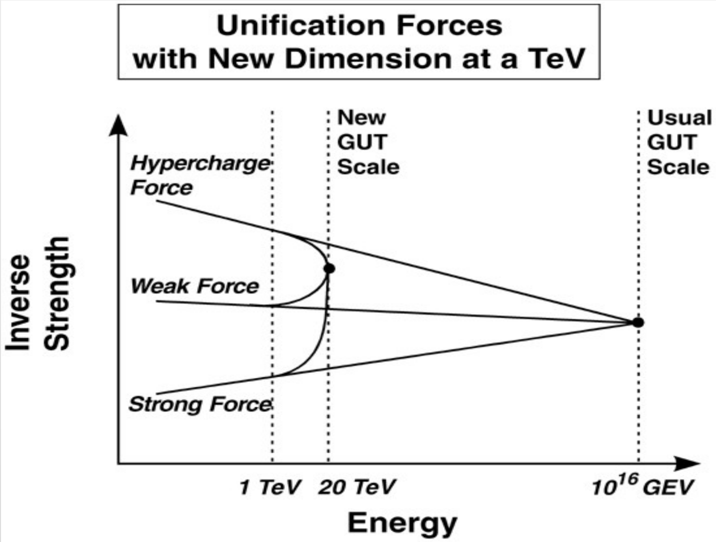

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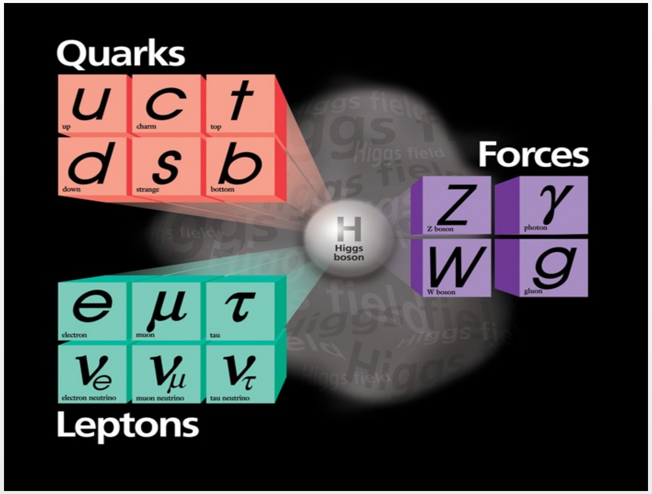

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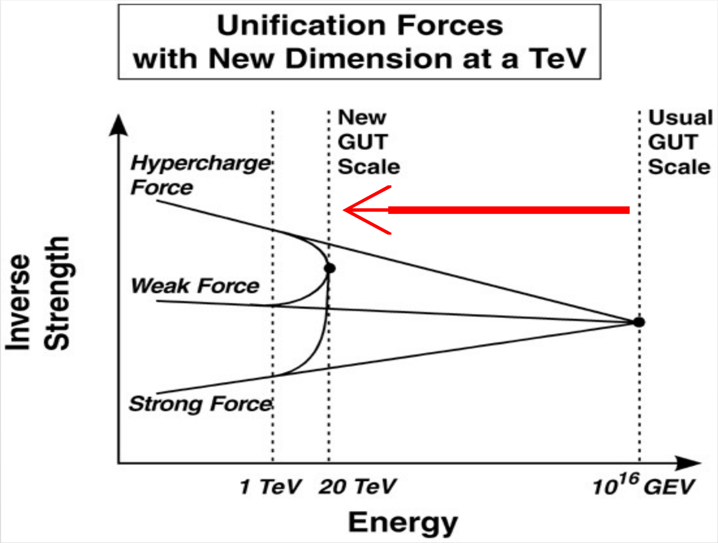
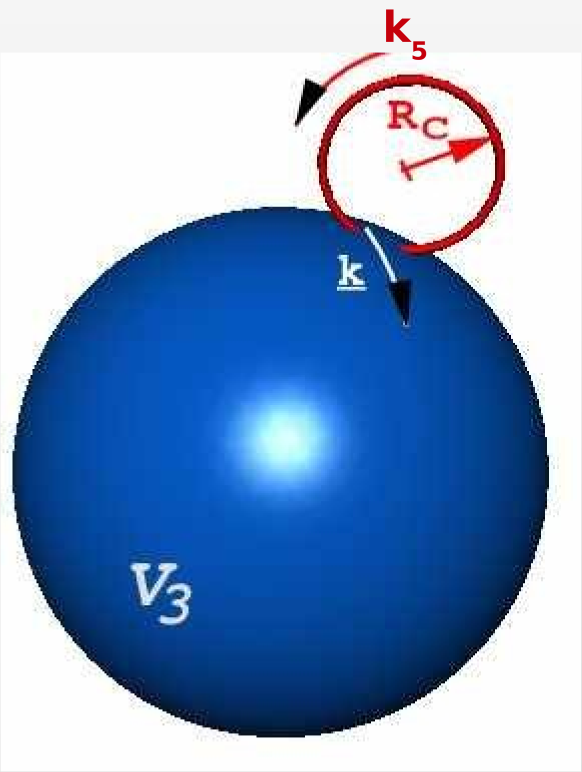


1. Motivation for Kaluza–Klein Stars

- Compact Stars: extremely high energy density →
- Investigation of:
  - Overlap of strong-, electroweak & gravitational forces
  - Compactified extra dimensions
- Extended description of compact stars: introducing a compactified **extra dimension**
- Extra dimensional picture:
  - Excited states of particles: geometrical degrees of freedom
  - e.g. strangeness ~ particle moving in extra dimension



Symmetries in case of a Compact Star

- **Spherical:** invariant under O(3) rotations
- **Static:** elements of metric and $T^{\mu\nu}$ are *t* independent
- **Ideal relativistic fluid:** only diagonal elements, $T^{\mu\nu}{}_{;\nu} = 0$
- **Isotropic**: no explicit φ & θ dependence of the components of $g^{\mu\nu}$ and $T^{\mu\nu}$

- **Extra dimension(s)** need extra assumptions...

[G. G. Barnaföldi, P. Lévai, B. Lukács: „Searching extra dimensions in compact stars" Astron. Nachr. **328**, 809 (2007)]

Assumptions on Kaluza–Klein extra dimension(s)

i. **1+(3+d_c) dimensional space-time:**
dimensions are space-like, except first one: time-like

ii. **GR is the same** as in 1+3 D → 'Equivalence Principle' is unchanged

iii. **All causality postulates are the same** as in 1+3 D (including lightcone structure)

iv. **Extra space-like d_c** dimensions are <u>microscopical</u>

v. **Complete Killing-symmetry** in the d_c-dimensional microscopical subspace

Focus on the simplest $\mathbf{d_c=1}$ case!

[G. G. Barnaföldi, P. Lévai, B. Lukács: J. Phys.: CS. **218** (2010)]

2. A special solution in 1+4D space-time

Metric

i. **Spherical symmetry** → O(3) symmetry

ii. **Static picture** → $g_{\mu 0}=0$ and $g_{\mu\nu,0}=0$

iii. **4D** $g^{\mu\nu}$ is x^5 independent

iv. **Killing transformations** → $g_{01}=0$ and $g_{51}=0$

$$g_{\mu\nu}=\begin{bmatrix} g_{00} & g_{01} & 0 & 0 & g_{05} \\ g_{01} & g_{11} & 0 & 0 & g_{15} \\ 0 & 0 & g_{22} & 0 & 0 \\ 0 & 0 & 0 & g_{22}\sin^2\vartheta & 0 \\ g_{05} & g_{15} & 0 & 0 & g_{55} \end{bmatrix}$$

Coordinates:
$t = x^0$
$r = x^1$
$\vartheta = x^2$
$\varphi = x^3$
$\boldsymbol{\chi = x^5}$

$$g_{\mu\nu}=\mathrm{diag}\left(e^{2\nu},-e^{2\lambda},-r^2,-r^2\sin^2\vartheta,\mathbf{e^{2\Phi}}\right)$$

Radial functions for the metric components: $\nu(r)$, $\lambda(r)$, $\boldsymbol{\Phi(r)}$

Energy-momentum tensor for 1+4D matter

(a) **Energy-momentum tensor** for anisotrop liquid:

$$T_{\mu\nu} := \epsilon u_\mu u_\nu - p\left(g_{\mu\nu} - u_\mu u_\nu + v_\mu v_\nu\right) - p_5 v_\mu v_\nu$$

Liquid is isotrop for 3 dimension: $T^1_1 = T^2_2 = T^3_3 = p$

BUT **pressure** $\boldsymbol{T^5_5 = p_5}$ in the 5th direction is anisotrop, with energy-density:

$$T_{\mu\nu} = \mathrm{diag}\left(\epsilon\, \mathrm{e}^{2\nu}, p\, \mathrm{e}^{2\lambda}, p r^2, p r^2 \sin^2\vartheta, p_5 \mathrm{e}^{2\Phi}\right)$$

(b) Let's construct the $\boldsymbol{R_{\mu\nu}}$ **Ricci tensor** and $\boldsymbol{R}$ **Ricci scalar**:

$$R := R^i_i = R^1_1 = R^2_2 = R^3_3 = R^4_4 = R^5_5$$

[B. Lukács, T. Pacher: KFKI-1985-74, Budapest, Hungary]

Einstein equations in 1+4D space-time

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = -8\pi G\, T_{\mu\nu}$$

Components of the **Einstein equation**:

$$-8\pi G \epsilon = e^{-2\lambda}\left[\Phi'' + \Phi'^2 - \lambda'\Phi' + \frac{2\Phi'}{r} - \frac{2\lambda'}{r} + \frac{1}{r^2}\right] - \frac{1}{r^2}$$

$$-8\pi G p = e^{-2\lambda}\left[\nu'\Phi' - \frac{2\Phi'}{r} - \frac{2\nu'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2}$$

$$-8\pi G p = e^{-2\lambda}\left[-\nu'' - \nu'^2 - + \nu'\lambda' + \Phi'' - \Phi'^2 - \nu'\Phi' + \lambda'\Phi' - \frac{\nu'}{r} + \frac{\lambda'}{r} - \frac{\Phi'}{r}\right]$$

$$-8\pi G\, p_5 = e^{-2\lambda}\left[-\nu'' - \nu'^2 + \nu\lambda - \frac{2\nu'}{r} + \frac{2\lambda'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2}$$

- **Extra variables: p_5, $\Phi(r)$**

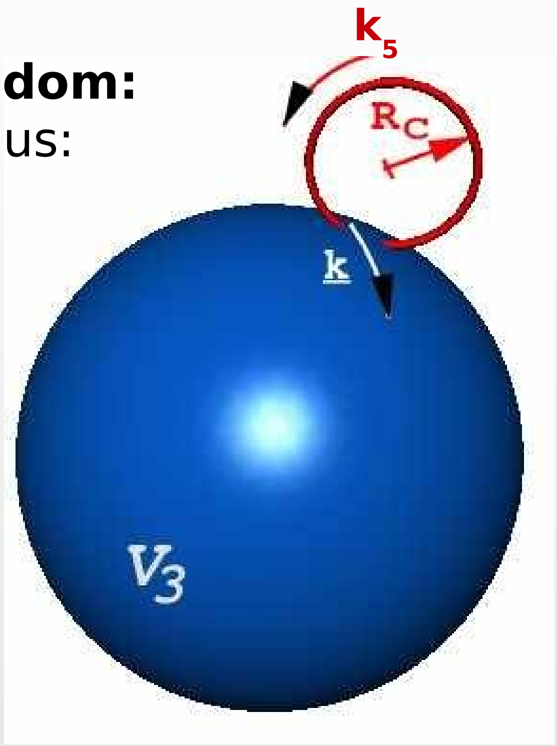
'Observation' of the 5th Dimension

(a) **Strangeness as a new degree of freedom:**
Connection between $\bar{m} = m_s$ and R_c radius:

$$E_5 = \sqrt{\underline{k}^2 + \left(\frac{n}{R_C}\right)^2 + m^2} = \sqrt{\underline{k}^2 + \bar{m}^2}$$

$$\bar{m}^2 = \left(\frac{n}{R_C}\right)^2 + m^2$$

m: light (u, d) quark mass
n: excitation number, n=1
$\bar{m}$ heavy (s) quark mass



(b) **Extra 5thD is compactified** in an **S1 circle** with radius **R_c**
→ periodical boundary condition

$$\psi(x_5) \approx e^{ik_5 \cdot x_5} \quad \text{and} \quad \psi(x_5 + 2\pi R_C) \sim \psi(x_5) \quad \rightarrow \quad k_5 = \frac{n}{R_C}$$

$\boldsymbol{k_5}$: momentum in the 5th direction; $\mathbf{x_5}$: coordinate in the 5th direction; $\boldsymbol{n} \in \mathbb{Z}^+$

Thermodynamics for 1+4D

(a) **Thermodynamical potential for 1+4D Fermion gas**

$$\Omega_5 = -2\frac{V_4}{\beta}\int\frac{\mathrm{d}^4 k}{(4\pi)^4}\left[\ln\left(1+e^{-\beta\left(\sqrt{\underline{k}^2+\bar{m}^2}-\mu\right)}\right)\left(+\mu \leftarrow\rightarrow -\mu\right)\right]$$

$\bar{m}^2 = (n/R_C)^2 + m^2$ **excited mass**

$\int \mathrm{d}k_5 \rightarrow \frac{1}{R_C}\Sigma_n$ discretization

$V_5 = 2\pi R_C V_4$ volume

(b) Thermodynamical potential and its quantities

$$\Omega_5 = \sum_n \Omega_4\left(m^2 + \frac{n^2}{R_C}\right) = \Omega_4(\bar{m})$$

$$p = -\frac{1}{2\pi R_C}\frac{\partial \Omega_5}{\partial V} \qquad p_5 = -\frac{1}{2\pi V}\frac{\Omega_5}{\partial R_C} \qquad \epsilon = \frac{U}{V_4}$$

TOV equations for 1+4D

(1) **'dΦ/dr = 0' special case:**

$$\frac{\mathrm{d}\,\boldsymbol{p}(r)}{\mathrm{d}\,r} = -\frac{[\boldsymbol{p}(r)+\boldsymbol{\epsilon}(r)][\boldsymbol{M}(r)+4\pi r^3\boldsymbol{p}(r)]}{r[r-2\boldsymbol{M}(r)]}$$

$$\boldsymbol{p}_5 = 1-\frac{2\boldsymbol{M}(r)}{r}+\frac{1}{r}\ln\left[1-\frac{2\boldsymbol{M}(r)}{r}\right]-2\boldsymbol{p}$$

3. Compact Star in Kaluza–Klein World

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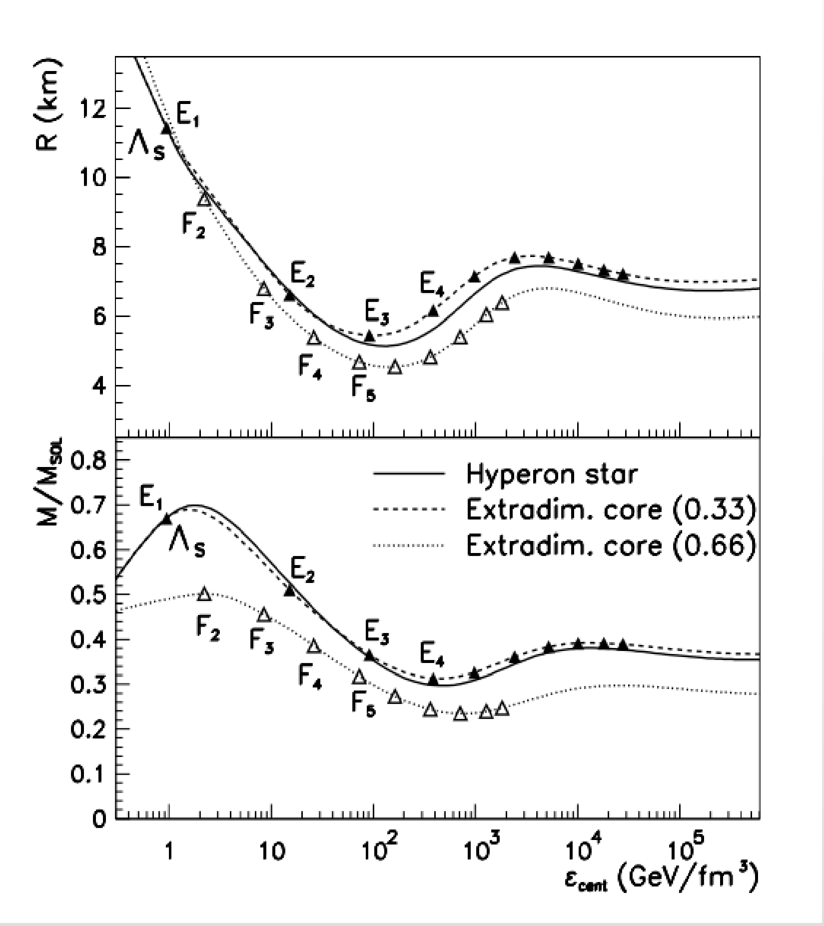

Hyperon Star: n^0, Λ^0, …

vs.

Compact star: with $k_F > \hbar / R_C$

1_C extra dimension

M(R) with n^{th} excited state:

▲ **R_c**=0.33 fm (E_1,E_2,E_3, …)

△ **R_c**=0.66 fm (F_1,F_2,F_3, …)

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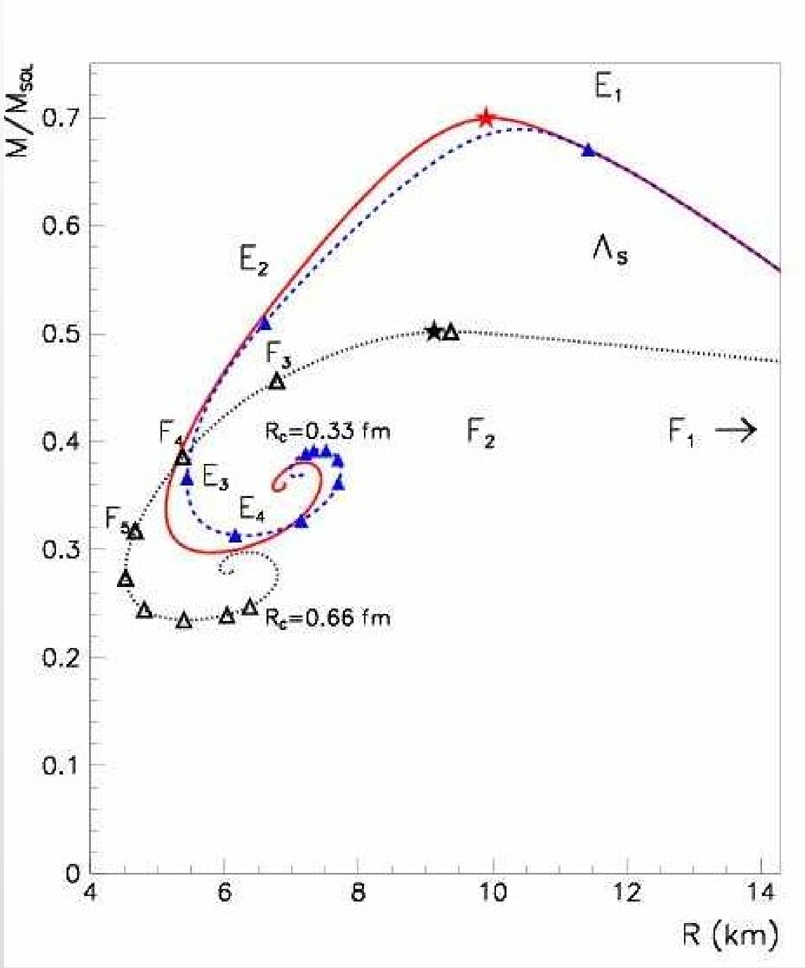

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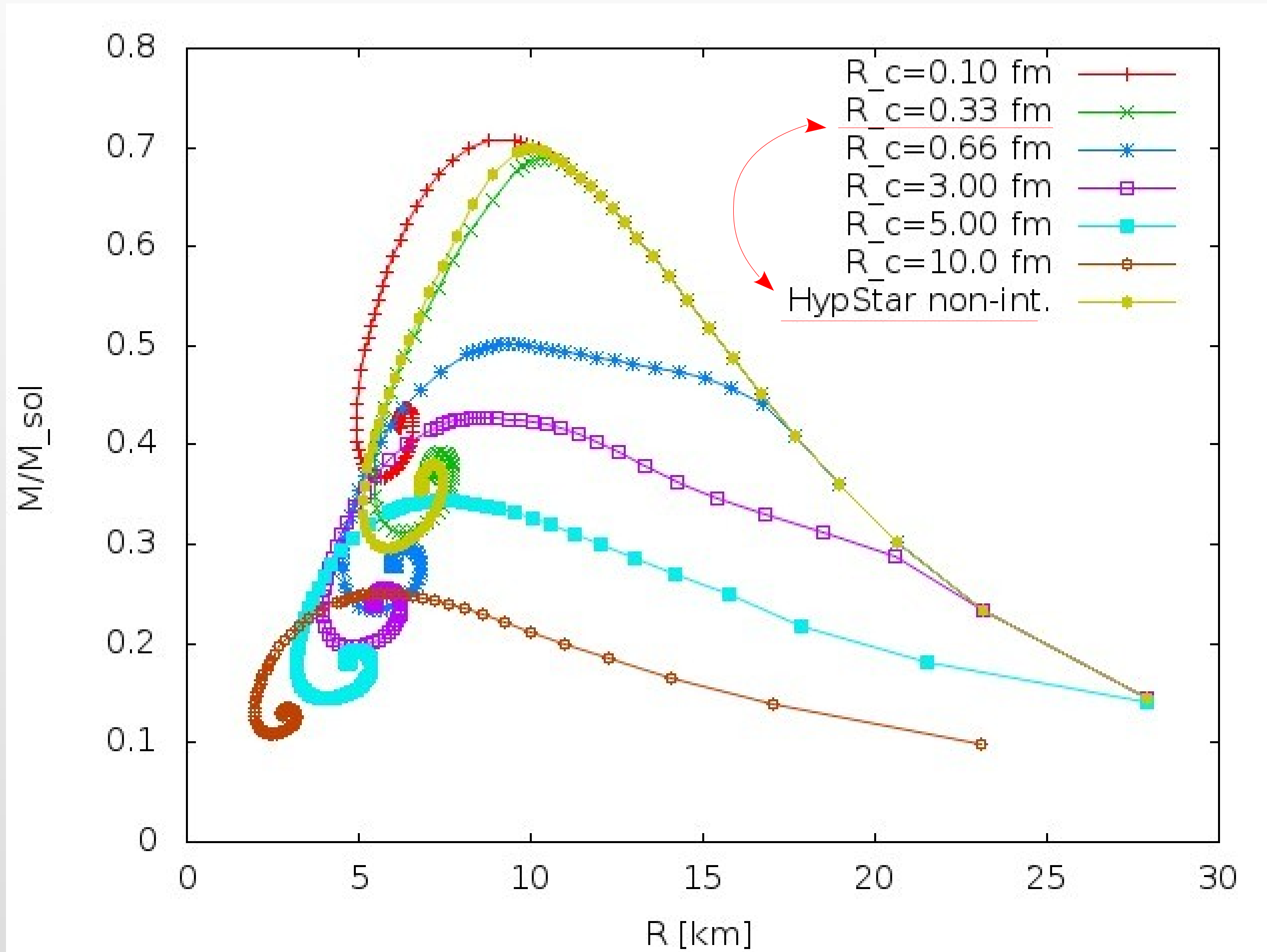
Smooth variation of the compactified D

Change of the R_c:

- excitation with:
- larger the R_c → $\Delta m_n \sim \frac{n}{R_C}$ smaller the excitations to m_n
- similar solutions (shape, stability, etc.)

- **Hyp Star non-interacting** ~ R_c = 0.33 fm



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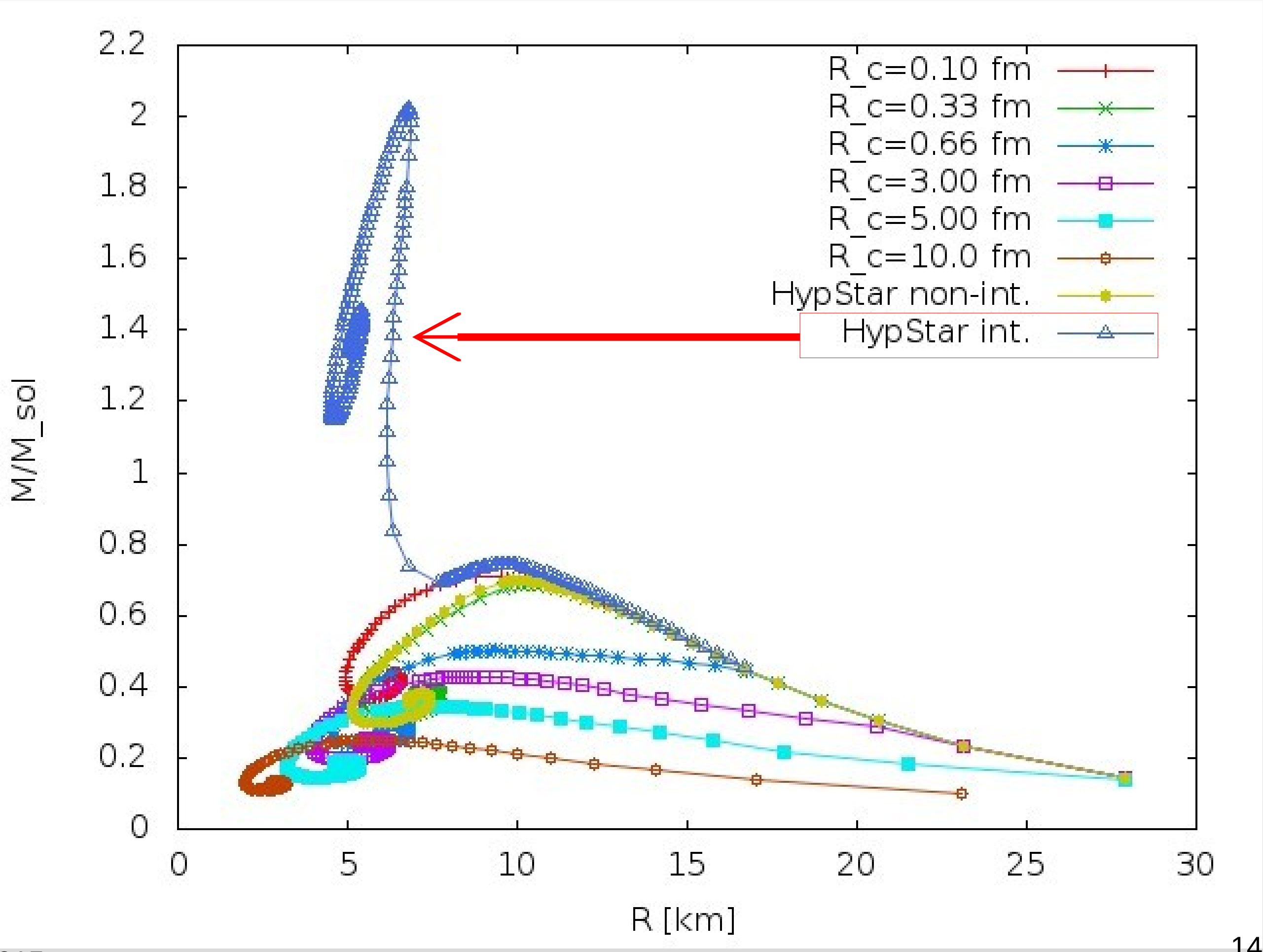
- **Hyp Star interacting matter** → more massive



Summary

- **Compact stars in 1+4D were analyzed:**

- Static, spherical Schwarzschild-like space-time
- TOV-like eqs. with specific, but exact (stable) solution
- Solutions overlap with strange star models if $\mathbf{R_c}$ is set to the mass of strangeness: 10^{-13} cm < $\mathbf{R_c}$ < 10^{-9} cm
- Mass limit: determinated by $\mathbf{R_c}$, D →
- larger $\mathbf{R_c}$ results → decrease in star's mass

- **Extra dimensional fermion stars:**
- For solutions massive enough →
- **Interaction of matter should be treated!**

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Thank You for Your attention!

Backup slides

Smooth variation of the compactified D

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- **Realistic Hyp EoS:** Hyperon star interacting matter
- **Hyp Star interacting**
- **Pure N Star interacting**