Comparing different models of pulsar timing noise

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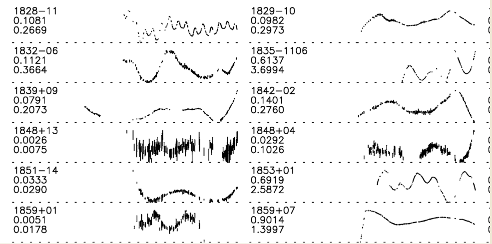
Motivation

- The signal from pulsars is highly stable, but variations do exist in the time-of-arrivals, often referred to as *timing-noise*
- Variations are thought to be intrinsic to the pulsar and tell us there is unmodelled physics
- Understanding the cause of timing-noise may help us to infer properties of the neutron star interior

- There is a lot of variation in the observed timing-noise, but a few show highly periodic variations $\sim 1 - 10$ yrs



Hobbs, Lyne & Kramer (2010): *An analysis of the timing irregularities for 366 pulsars*

- Multiple models exist to explain timing noise
- We require a quantitative way to determine which models the data supports

Periodic modulations: B1828-11

- Demonstrates periodic modulations at 500 days
- Harmonics at 250 and 1000 days
- Correlated changes in the timing observations and the beam-shape
- Explanation from Stairs (2000): Pulsar is precessing

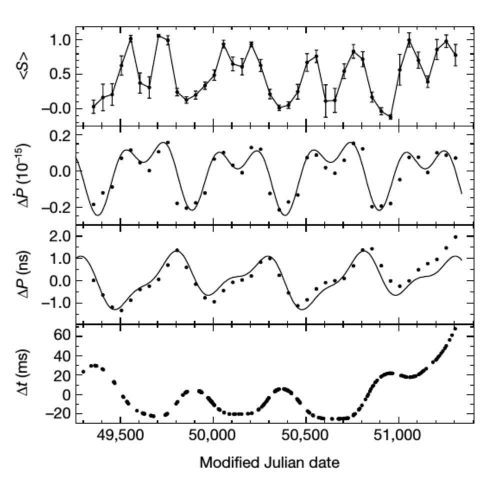
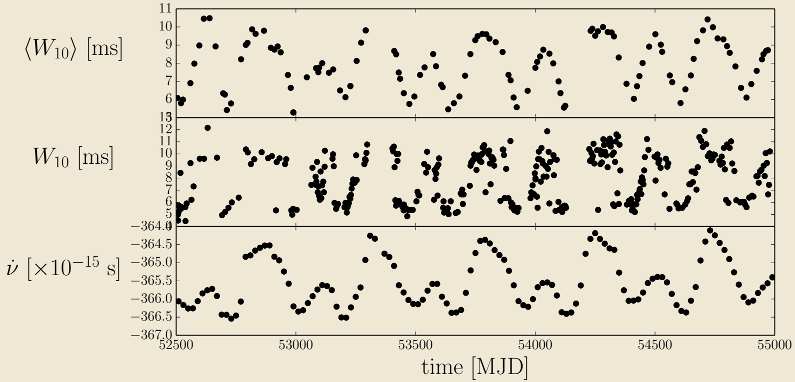


Figure: Fig. 2 from Stairs et al. (2000): *Evidence for Free Precession in a Pulsar*

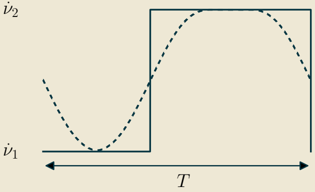
Lyne et al. (2010) revisited the data looked at W_{10} (the beam-width) which is not *not* time-averaged.

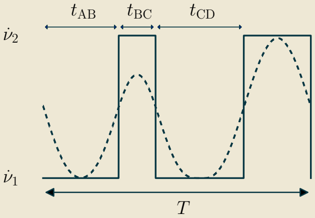


Data courtesy of Lyne at al. (2010): *Switched Magnetospheric Regulation of Pulsar Spin-Down*

Model 2: Switching

- Lyne et al. (2010): the magnetosphere undergoes periodic switching between two states
- The smooth modulation in the spin-down is due to time-averaging of this underlying spin-down model
- To explain the *double-peak*, Perera (2015) suggested four times were required





We would like to quantify how well the two models fit the data. To do this we will use Bayes theorem:

$$P(\mathcal{M}|\mathbf{y}_{\rm obs}) = P(\mathbf{y}_{\rm obs}|\mathcal{M})\frac{P(\mathcal{M})}{P(\mathbf{y}_{\rm obs})}.$$

The odds ratio:

$$\mathcal{O} = \frac{P(\mathcal{M}_A|\mathbf{y}_{\rm obs})}{P(\mathcal{M}_B|\mathbf{y}_{\rm obs})} = \frac{P(\mathbf{y}_{\rm obs}|\mathcal{M}_A)}{P(\mathbf{y}_{\rm obs}|\mathcal{M}_B)}\frac{P(\mathcal{M}_A)}{P(\mathcal{M}_B)}.$$

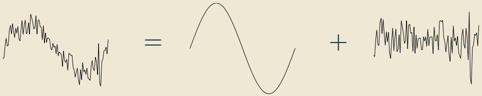
If we have no preference for one model or the other then set

$$\frac{P(\mathcal{M}_A)}{P(\mathcal{M}_B)} = 1.$$

Bayesian data analysis: Likelihood

For a signal in noise:

$$y^{\text{obs}}(t_i|\mathcal{M}_j, \boldsymbol{\theta}, \sigma) = f(t_i|\mathcal{M}_j, \boldsymbol{\theta}) + n(t_i, \sigma)$$



If the noise is stationary and can be described by a normal distribution:

$$y^{\text{obs}}(t_i|\mathcal{M}_j, \boldsymbol{\theta}, \sigma) - f(t_i|\mathcal{M}_j, \boldsymbol{\theta}) \sim N(0, \sigma)$$

Then the likelihood for a single data point is:

$$\mathcal{L}(y_i^{\text{obs}}|\mathcal{M}_j, \boldsymbol{\theta}, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{\frac{-\left(f(t_i|\mathcal{M}_j, \boldsymbol{\theta}) - y_i\right)^2}{2\sigma^2}\right\}$$

and the likelihood for all the data is:

$$\mathcal{L}(\mathbf{y}_{\text{obs}}|\mathcal{M}_j, \boldsymbol{\theta}, \sigma) = \prod_i^N \mathcal{L}(y_i^{\text{obs}}|\mathcal{M}_j, \boldsymbol{\theta}, \sigma)$$

First we use Markov chain Monte Carlo methods to fit the model to the data and find the posterior distribution

$$p(\boldsymbol{\theta}, \sigma | \mathbf{y}_{\text{obs}}, \mathcal{M}) \propto \mathcal{L}(\mathbf{y}_{\text{obs}} | \boldsymbol{\theta}, \sigma, \mathcal{M}) \pi(\boldsymbol{\theta}, \sigma | \mathcal{M}_j)$$

Then we can compute the marginal likelihood

$$P(\mathbf{y}_{\text{obs}} | \mathcal{M}) \propto \int p(\boldsymbol{\theta}, \sigma | \mathbf{y}_{\text{obs}}, \mathcal{M}_i) d\boldsymbol{\theta} d\sigma$$

So for any set of data, we have two tasks:

1. Specify the signal function $f(t)$
2. Specify the prior distribution $\pi(\boldsymbol{\theta} | \mathcal{M})$

Specify the signal function: Precession

- Spin-down rate:

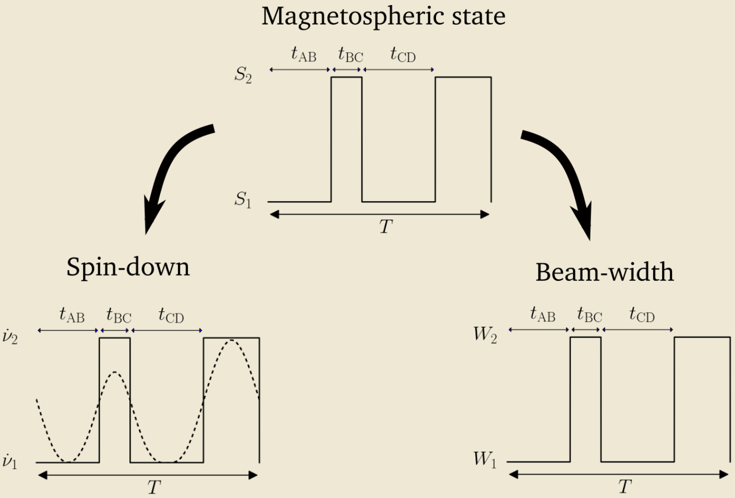
$$\Delta\dot{\nu}(t) \sim 2\theta \cot\chi \sin\psi - \frac{\theta^2}{2}\cos 2\psi$$

- Beam-width model

$$\Delta w(t) \sim 2\theta\xi \sin\psi - \frac{\theta^2}{2}\cos 2\psi$$

See for example: *Jones & Andersson (2001), Link & Epstein (2001), Akgun et al. (2006) Zanazzi & Lai (2015), Arzamasskiy et al. (2015)*

Specify the signal function: Switching



The prior distribution

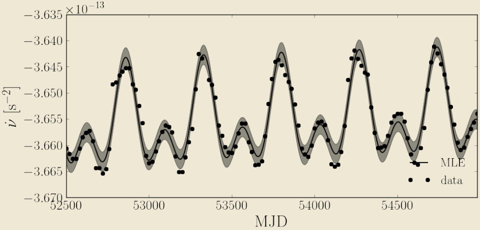
- For the switching model, no astrophysical priors exist for many of the parameters
- The odds-ratio can depend heavily on the prior volume

Solution

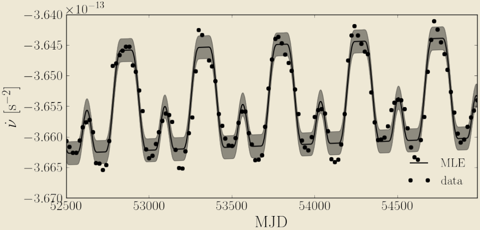
Use the spin-down data to generate prior distributions for the beam-width data: this allows a fair comparison between the methods without undue influence from the choice of priors.

Checking the fit: Spin-down data

Precession model:

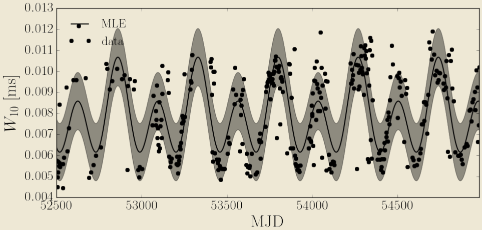


Switching model:

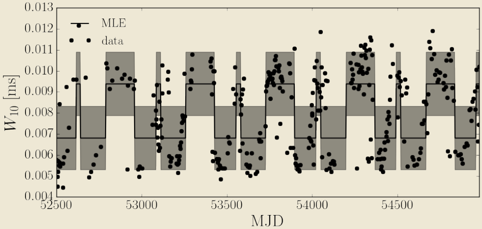


Checking the fit: Beam-width data

Precession model:



Switching model:



Results

- Currently we are finding the odds ratio favours the *precession* model
- This is not yet confirmed as we are in the process of examining the dependence on the prior distributions and the model assumptions
- Primarily we are interested in setting up the framework to evaluate models

Conclusions

- We can learn about neutron stars from the physical mechanisms producing timing noise: implications of precession for super-fluid vortices pinning to the crust
- Need a quantifiable framework to test models and argue their merits
- For B1828-11 a simple precession model is preferred by the data to a phenomenological switching model
- Models are extensible: we can test different types of beams or torques
- In the future, we intend to form a hybrid model where the precession biases the switching