The resonant structure of the trans-Neptunian space

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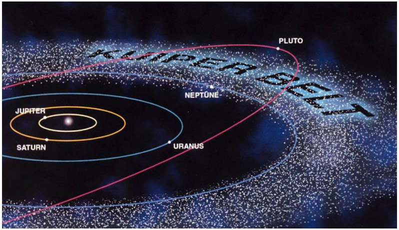
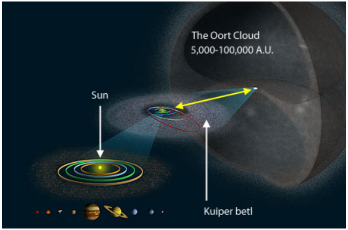
The trans-Neptunian space

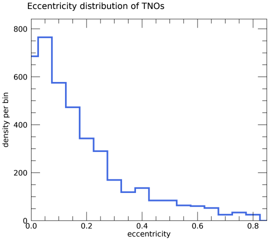
Credit: NASA

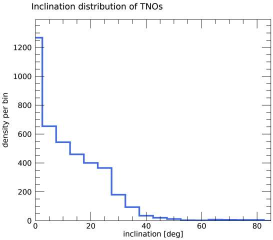


Credit: Wikimedia



Data: ~ 4200 trans-Neptunian objects (TNOs) between 30.1 and 2000 AU:



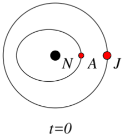


Mean-motion resonances

Definition:

- mean-motion commensurability: periodic repeat of a particular orbital configuration (the ratio of the mean motions n, n' are the ratio of small integers):

$$\frac{n}{n'} \approx \frac{p+q}{p}$$



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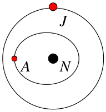

t=0,25 T_J

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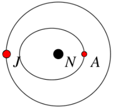

t=0,5 T_J

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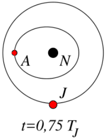

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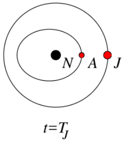

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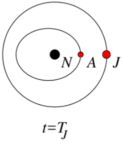

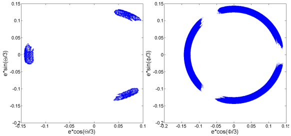

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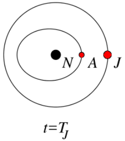
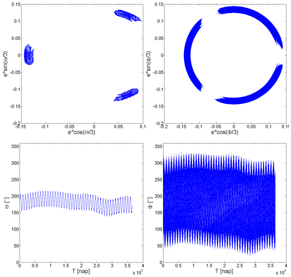


- mean-motion resonance (MMR): commensurability + libration of the critical argument θ around a mean value:



$$\theta = (p+q)\lambda - p\lambda' - q\widetilde{\omega}'$$

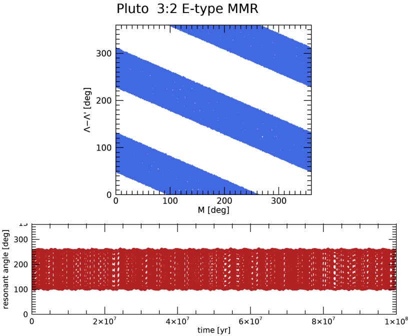
λ, λ' : mean longitudes

$\widetilde{\omega}'$: longitude of the perihelion

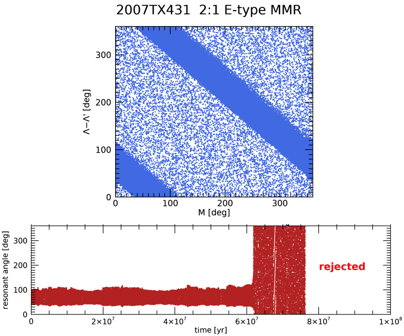
Mean-motion resonances

Identification of MMRs: FAIR method: (Forgács-Dajka et al., 2018)

- long-term MMR:



- short-term MMR:



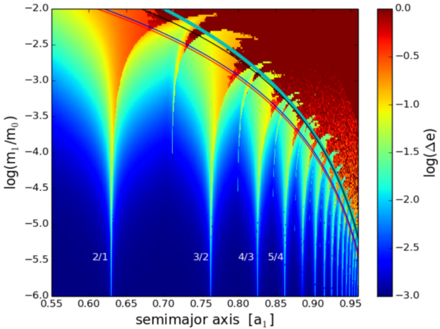
Chaotic diffusion and its measures

Chaotic diffusion:

- drift of a phase point in the phase space (this might lead to orbital instabilities)
- **slow/Arnold diffusion:**
  small perturbations, chaos is confined to thin layers around single resonances

  fast diffusion:
  large perturbations, resonance overlap, extended chaotic domains

Credit: Correa-Otto & Beaugé (2015)



An efficient indicator of chaos:

- maximal variation of the eccentricity e throughout the total integration time span T_{tot}:

$$\Delta e := \max_{t \leq T_{\mathrm{tot}}}(e) - \min_{t \leq T_{\mathrm{tot}}}(e)$$

Chaotic diffusion and its measures

Quantification of the diffusion coefficient:

- diffusion coefficient from the time derivative of the variance of some phase space variable X:

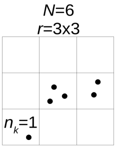
$$\mathrm{Var}(X) = \langle (X - \langle X \rangle)^2 \rangle = 2D_{\mathrm{Var}} t$$

$$D_{\mathrm{Var}} = \frac{1}{2} \frac{\mathrm{dVar}(X)}{\mathrm{d}t}$$

- diffusion coefficient from the time derivative of the Shannon entropy:

$$S(X) = -\sum_{k=1}^{r} \frac{n_k}{N} \ln\left(\frac{n_k}{N}\right)$$

$$D_S = \frac{(X_{\max} - X_{\min})^2}{r} r_0 \frac{\mathrm{d}S(X)}{\mathrm{d}t}$$



Characteristic times of stability:

$$\tau_{\mathrm{Var}} = \frac{1}{\langle D_{\mathrm{Var}} \rangle}$$

$$\tau_S = \frac{1}{\langle D_S \rangle}$$

Computations

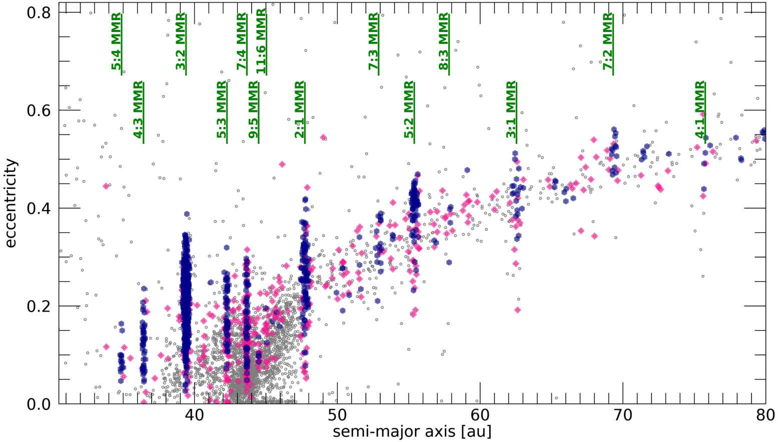
Direct long-term integrations:

- barycentric coordinate system
- Sun + 4 giants + small body (4200 TNOs, $6 \cdot 10^5$ test particles)
- integrator: Runge–Kutta–Nyström 7(6) (Dormand & Prince, 1978) (tolerance: 10^{-14})
- total integration time span:
  - TNOs: 100 Myr
  - test particles (for the dynamical maps): 200 kyr
- sampling time step: 100 yr

For the computation of the Shannon entropy of a given TNO:

- selected variables: the Delaunay variables L (energy) and G (angular momentum)
- total grid size: $L_0 \pm 0.005$, $H_0 \pm 0.005$
- number of cells of the grid: $r = 500 \times 500$

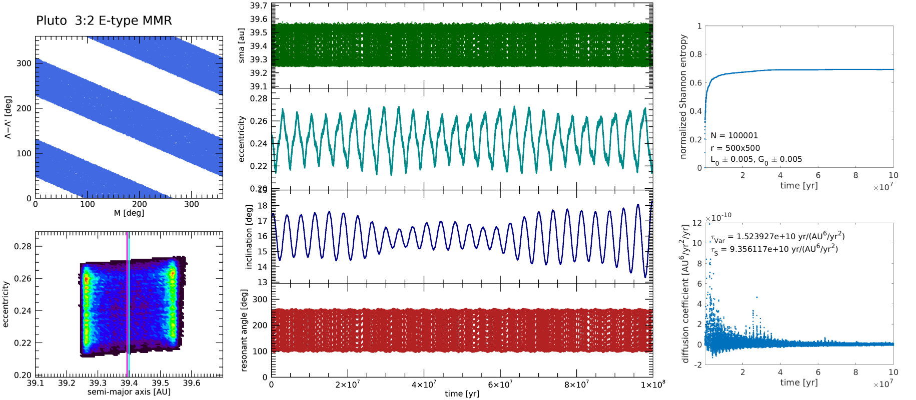
Resonant distribution of TNOs



●: TNO in long-term MMR, ◆: TNO in short-term MMR

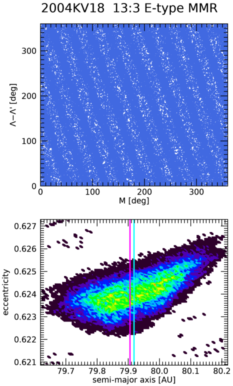
Long-term evolution of TNOs (examples)

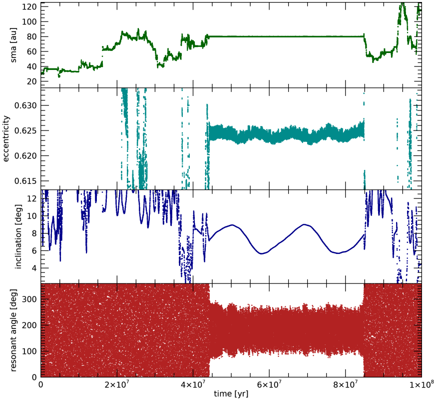
Long-term MMR $(3:2)$: $\tau_{\mathrm{Var}} \simeq 1.5 \cdot 10^{10} \frac{\mathrm{yr}}{\mathrm{AU}^6/\mathrm{yr}^2}$, $\tau_S \simeq 9.4 \cdot 10^{10} \frac{\mathrm{yr}}{\mathrm{AU}^6/\mathrm{yr}^2}$

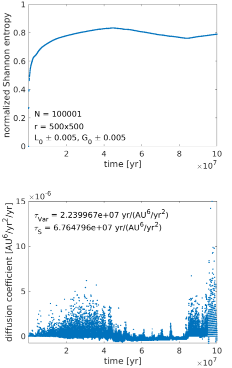


Long-term evolution of TNOs (examples)

Short-term MMR $(13:3)$: $\tau_{\mathrm{Var}} \simeq 2.2 \cdot 10^7 \,\frac{\mathrm{yr}}{\mathrm{AU}^6/\mathrm{yr}^2}, \qquad \tau_S \simeq 6.8 \cdot 10^7 \,\frac{\mathrm{yr}}{\mathrm{AU}^6/\mathrm{yr}^2}$

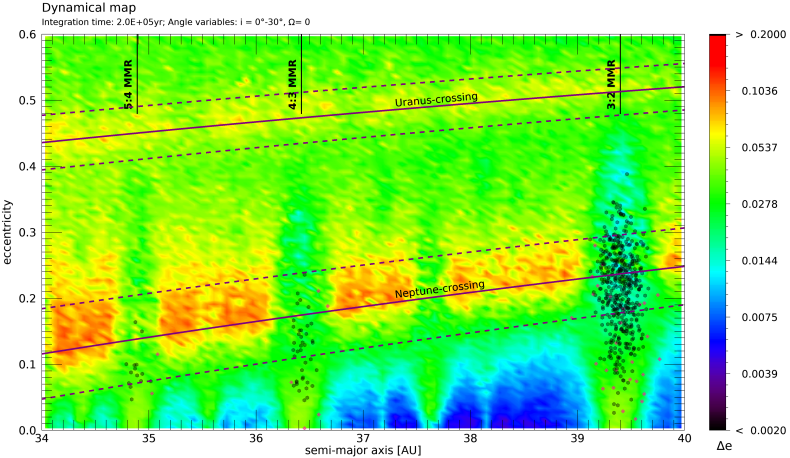






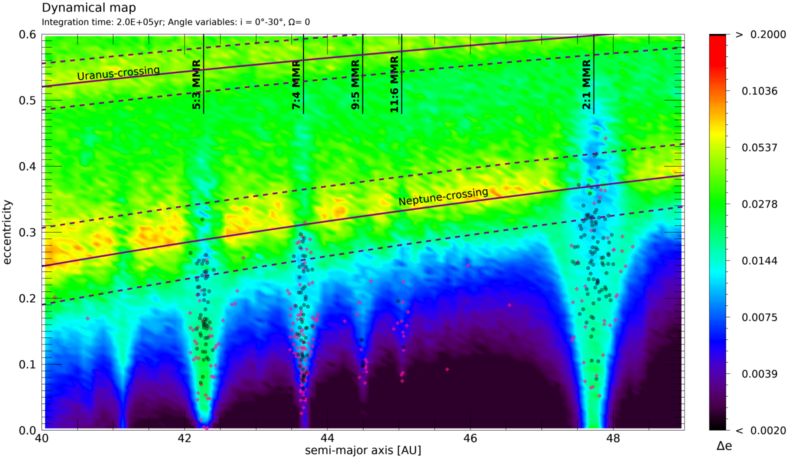
Dynamical maps

$34 - 40$ AU:



Dynamical maps

$40 - 49$ AU:



Summary

Importance of studying the trans-Neptunian region:

- helps to better understand the formation and evolution of planetary systems
- serves as a reservoir of dynamically unstable minor bodies
- feeds the population of near-Earth and of potentially hazardous asteroids

Significance of mean-motion resonances:

- periodic repeat of given configurations ⇒ large gravitational perturbations ⇒ big changes in the orbit of the minor body
- resonance overlap ⇒ strongly chaotic behaviour
- certain MMRs: offer safe harbours to minor bodies (see e.g.: Plutinos, Trojan asteroids, etc.)

Conclusions:

- semi-automatic identification of resonant TNOs in a remarkably large sample (~ 4200 bodies)
- distinction between the short- and long-term MMRs
- exploration of the dynamical structure of the $30-50$ AU region via dynamical maps of test particles
- study of chaotic processes via probabilistic chaos indicators (Shannon entropy)