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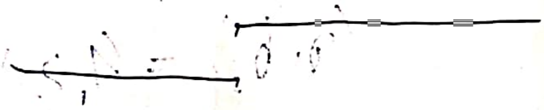
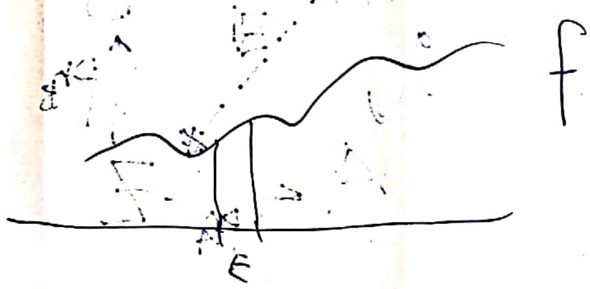
$$(\mu|\varphi) := \int \varphi \, d\mu$$

$$(\mu'|\varphi) := -\int \varphi' \, d\mu = -(\mu|\varphi')$$

$$(f\lambda_{\mathbb{R}}| \qquad (f\lambda_{\mathbb{R}})' = f'\lambda_{\mathbb{R}}$$

$\rho\lambda$ ha f diffható





5.14

① $A \quad \mathcal{A} \quad a$

② $A\times B = \{(a,b) \mid a\in A,\ b\in B\}$

$(a,b) := \{a, \{a,b\}\}$

$d\setminus c$

$\{c,d\} \longmapsto \begin{cases} (c, d) & \text{ha } c\in d \\ (d, c) & \text{ha } d\in c \end{cases}$

$pr_A : A\times B \longrightarrow A$

$pr_B : A\times B \longrightarrow B$

$A\times B = \{ f : \{1,2\} \longrightarrow A\cup B \mid f(1)\in A,\ f(2)\in B\}$

$pr_A : A\times B \longrightarrow A \quad f\longmapsto f(1)$

$pr_B : A\times B \longrightarrow B \quad f\longmapsto f(2)$

$$\begin{tikzcd} Q \arrow[r, "f"] \arrow[d, "g"'] \arrow[dr, dashed, "\exists!"] & B \\ A & Z \arrow[l, "pr_A"] \arrow[u, "pr_B"'] \end{tikzcd}$$

$(a,b) = (1,2)$

③ V, W

vegés dim

$$\begin{tikzcd} V\times W \arrow[r, "i"] \arrow[dr, "b"', "\text{bilineáris}"] & V\otimes W \arrow[d, dashed, "\exists!\, \tilde{b}\ \text{lin.}"] \\ & X \end{tikzcd}$$

$V \quad v_1, \dots, v_n$

$W \quad w_1, \dots, w_m$

$i(v_k, w_\ell) \quad k,\ell$

$\shortparallel$

$v_k\otimes w_\ell$

$\mathrm{Lin}(V^*, W)$

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$$V \ni v^k \vec{e}_{(k)} = \vec{v}$$

$$\begin{pmatrix} v^1 \\ v^2 \\ \vdots \\ v^n \end{pmatrix}$$

$$V^* \ni \underset{\to}{\varphi} \qquad \underset{\to}{\varphi}(\vec{v}) = \alpha \in R$$

$$(\varphi_1, \varphi_2 \dots \varphi_n)$$

$$\underset{\to}{\varepsilon}^{(k)}(\vec{e}_{(l)}) = \delta^k_l$$

$$\underset{\to}{\varphi} = \varphi_l \underset{\to}{\varepsilon}^{(l)}$$

$$\underset{\to}{\varphi}(\vec{v}) = (\varphi_l \underset{\to}{\varepsilon}^{(l)})(v^k \vec{e}_{(k)}) = \varphi_l v^k \underbrace{(\underset{\to}{\varepsilon}^{(l)} \vec{e}_{(k)})}_{\delta^l_k} = \varphi_k v^k$$

$$V \times V \to \mathbb{R}$$

$$b(\vec{v}, \vec{u}) = \beta$$

$$b(\vec{e}_{(k)}, \vec{e}_{(l)}) = g_{kl}$$

$$b(\vec{v}, \vec{u}) = b(v^k \vec{e}_{(k)}, u^l \vec{e}_{(l)}) = v^k u^l \underbrace{b(\vec{e}_{(k)}, \vec{e}_{(l)})}_{g_{kl}}$$

$$= g_{kl} v^k u^l$$

$$W_k = g_{kl} u^l$$

$$V \to V^*$$

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5/6)

$\mathcal{H} \ni |v\rangle$ $\qquad |v\rangle + |u\rangle \qquad |v+w\rangle$

$\mathbb{K} = \mathbb{C}$ $\qquad \alpha|v\rangle \qquad \alpha \in \mathbb{C}$

ket

$\mathcal{H}^* \ni \langle u|$ $\qquad f_u(\vec{v}) \to \langle u|v\rangle \qquad \langle\ \rangle$

bra

$|v\rangle \to \langle v|$ $\qquad |u\rangle \quad |v\rangle$

$\langle u|$

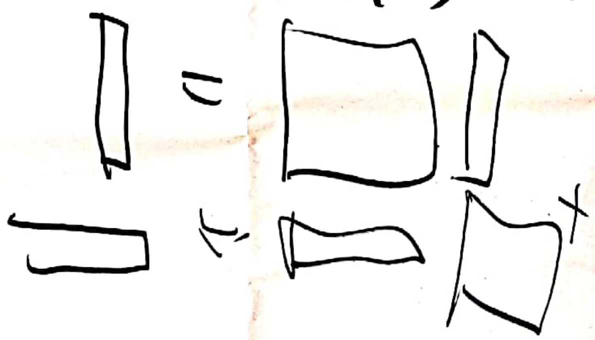
$|v\rangle \sim |w\rangle = \lambda|v\rangle \qquad \lambda \neq 0$

$\mathcal{H}/\sim \qquad \langle v|v\rangle = 1$

$|v(t)\rangle \qquad t=0 \qquad |v_0\rangle = |u_0\rangle + |w_0\rangle$

$\downarrow t \qquad |v(t)\rangle = |u(t)\rangle + |w(t)\rangle$

$|v(t)\rangle = \hat{G}(t)|v_0\rangle \quad \to \langle v(t)| = \langle v_0| G^+(t)$



$\langle v(t)|v(t)\rangle = \langle v_0|\hat{G}^+(t)\,G(t)|v_0\rangle$

$= \langle v_0|v_0\rangle = 1$

$\hat{G}^+(t)\,\hat{G}(t) = 1$

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$t_0 = 0 \longrightarrow t_1 \xrightarrow{t_2} t_1 + t_2$

$|v_0\rangle \to |v(t_1)\rangle = \hat{G}(t_1)|v_0\rangle$

$|v(t_1+t_2)\rangle = \hat{G}(t_2)|v(t_1)\rangle$

$= \hat{G}(t_2)\hat{G}(t_1)|v_0\rangle$

$\hat{G}(t_2+t_1)|v_0\rangle$ $\quad \to \hat{G}(0) = \hat{1}$ $\quad G(t)^{-1} = G(-t)$

$\hat{G}(t_2+t_1) = \hat{G}(t_2)\hat{G}(t_1)$

$\hat{G}(t) = e^{t\hat{B}} = \sum_{n=0}^{\infty} \frac{t^n}{n!} B^n \qquad t \in \mathbb{R}$

$G^+(t) = e^{t\hat{B}^+}$

$\hat{G}(t)^{-1} = e^{-t\hat{B}}$ $\qquad \hat{B}^+ = -\hat{B}$

$\hat{B} = i\hat{C} \qquad \hat{C}^+ = \hat{C}$

$G(t) = e^{-it\hat{C}} = e^{-\frac{i}{\hbar}t\hat{H}} \qquad \hat{H}^+ = \hat{H}$

$|v(t)\rangle = \hat{G}(t)|v_0\rangle = e^{-\frac{i}{\hbar}t\hat{H}}|v_0\rangle$

$\frac{d}{dt}|v(t)\rangle = -\frac{i}{\hbar}\hat{H}\underbrace{e^{-\frac{i}{\hbar}t\hat{H}}|v_0\rangle}_{|v(t)\rangle}$

$\boxed{i\hbar\frac{d}{dt}|v(t)\rangle = \hat{H}|v(t)\rangle}$

$\hat{H}|\varphi_n\rangle = E_n|\varphi_n\rangle$

$\hat{H} = \sum E_n|\varphi_n\rangle\langle\varphi_n|$

$\hat{G}(t) = \sum e^{-\frac{i}{\hbar}E_n t}|\varphi_n\rangle\langle\varphi_n|$

S18 | $H(q,p)$

$$\downarrow \hat{H} = H(\hat{q},\hat{p})$$

$$F(q,p) \rightarrow \hat{F} = F(\hat{q},\hat{p})$$

$$q_k^{\dagger} = q_k$$
$$\hat{p}_k^{\dagger} = \hat{p}_k$$
$$\hat{F}^{\dagger} = \hat{F}$$

$$\hat{F}|f_k\rangle = f_k|f_k\rangle$$

$$\mathcal{H} \ni |\psi\rangle = \sum_k c_k |f_k\rangle$$

$$W(F = f_k) = |c_k|^2$$

$$\langle\psi|\psi\rangle = 1 = \sum |c_k|^2$$

$$\overline{F} = \sum W_k f_k = \langle\psi|\hat{F}|\psi\rangle$$

$$|\psi(t)\rangle = \hat{G}(t)|\psi_0\rangle \rightarrow$$

$$\langle\psi(t)| = \langle\psi_0|\hat{G}^{\dagger}(t)$$

$$\overline{F}(t) = \langle\psi_0| \underbrace{\hat{G}^{\dagger}(t)\hat{F}\hat{G}(t)} |\psi_0\rangle$$

S17 S18