C01

$|\langle\psi|\varphi\rangle|^2$

$|\psi\rangle\langle\psi|$ esemény

$|\varphi\rangle\langle\varphi|$ körülmény

$A\psi = a\psi$

$[A,B] \subseteq iC$ $\quad |X|$ körülmény

$[Q,P] = iI$ $\quad \Delta_\omega(A)\Delta_\omega(B) \geq \frac{1}{2}\eta_\omega(C)$

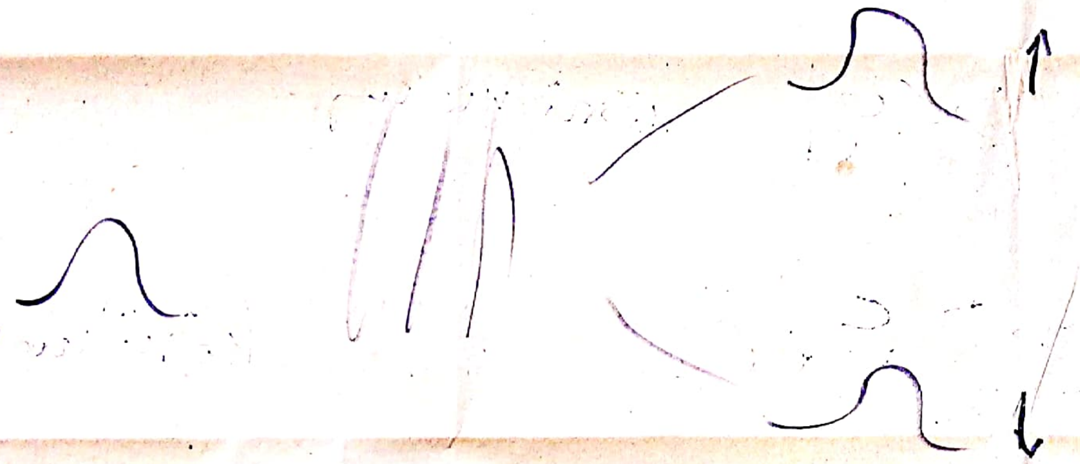
$\boxed{\Delta x \Delta p \geq \frac{1}{2}}$ $\quad \Delta x \sim \frac{1}{2\Delta p}$

$\begin{pmatrix} \square & 0 \\ & 0 \\ 0 \; 0 & 1 \end{pmatrix} \quad \begin{pmatrix} \square & 0 \\ & 0 \\ 0 \; 0 & 1 \end{pmatrix}$

$\Delta t \Delta E \geq \frac{1}{2}$

CØLJ

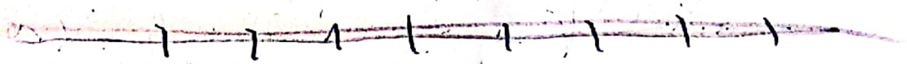
$$\frac{\alpha\varphi+\beta\psi}{\|\alpha\varphi+\beta\psi\|}$$



$$\lambda|\varphi\rangle\langle\varphi| + \mu|\psi\rangle\langle\psi|$$

$$|\alpha\varphi+\beta\psi\rangle\langle\alpha\varphi+\beta\psi|$$

a	$\overleftarrow{b}$	c	a	$\vec{b}$	c
+	+	+	−	−	−
+	+	−	+	+	+



Cs03 ## Fontos algebrai tétel:

X és X' vektor tér, $\dim X' = \dim X$.

Ha $i : Lin(X) \to Lin(X')$ algebra izomorfizmus $\Rightarrow$

$\exists\, A : X \to X'$ lin vagy konj-lin bijekció, hogy $L \mapsto i(L) = ALA^{-1}$.

Ha A és B $X \to X'$ lin vagy konj-lin bijekció, akkor $B(\cdot)B^{-1} = A(\cdot)A^{-1} \Rightarrow$

$\exists\, \alpha \in \mathbb{K} \setminus \{0\} : B = \alpha A$.

Szeszkvilineáris formák:

X véges dim $\mathbb{C}$-vektor tér. $\langle \cdot , \cdot \rangle : X \times X \to \mathbb{C}$, $(x,y) \mapsto \langle x, y \rangle$ ha elben konj. lin., ebben lin., és

$\langle x,y \rangle^* = \langle y, x \rangle$ $\forall x,y \in X$-re.

$\Rightarrow \langle x,x \rangle$ valós.

nemelfajuló: $\forall y \neq 0$: $\exists x : \langle y,x \rangle \neq 0$. poz(neg) definit: $x \neq 0 \Rightarrow \langle x,x \rangle > 0$ (<0).

$\Rightarrow$ $\exists\, j : X \to X^*$ konj. lin. bijekció, hogy $\langle x,y \rangle = (jx|y)$. $A : X \to X$ lin., akkor van $A^* : X \to X$ lin., hogy

$U : X \to X$ csillag(-)unitér, ha $\langle Ux, Uy \rangle = (-)\langle x,y \rangle$. $\langle A^*x, y \rangle = \langle x, Ay \rangle$ $\forall x,y \in X$. csillagadjungált

C(*)

$A \to A,\ A \mapsto A^*$

$(\alpha A)^* = \alpha^* A^*$

$(AB)^* = B^* A^*$

$(A^*)^* = A$

$(A, *)$

$S^* = S \quad \lambda, \mu \in Eig(S)$

$\lambda^* = \mu \quad \langle \cdot, \cdot \rangle$

$\{n_k \mid k = 1, \dots, N\}$ N-ben:

$\underset{\xi(n_k)}{I} \quad \dim(A) = 2^N$

$\xi(n_1) \cdots \xi(n_N) \quad A = Lin(X)$

$(N, \cdot) \quad A = Lin(X),\ \xi : N \to A$ lin.

$\xi(a)\xi(b) + \xi(b)\xi(a) = 2(a \cdot b) I \quad \forall a, b \in N$

$\xi(a)^2 = (a \cdot a) I \Rightarrow \left(\frac{\xi(a)}{\pm\sqrt{a \cdot a}} \right)^2 = I$

$\Rightarrow Eig\, \xi(a) = \{\pm\sqrt{a \cdot a}\}$

$Z_{a+} \oplus Z_{a-} = X$

$((N, \cdot), A, \xi) \quad \forall \xi' : N \to A'$ Clifford

$\Rightarrow \exists!\ h : A \to A' : \xi' = h \circ \xi$

Cliff-alg

$((N, \cdot), (A, *), \xi) \quad \xi(a)^* = \xi(a) \quad \forall a \in N$

$\boxed{C\Phi 5}$ $((N,\cdot), \mathrm{Lin}(X); \xi, *)$

$L : N \to N$ $\quad \xi \circ L$

$\xi \circ L(a)\, \xi \circ L(b) + \xi \circ L(b)\, \xi \circ L(a) = 2\underbrace{(La \cdot Lb)}_{= a\cdot b} I$

$A_L : X \to X \qquad \xi(La) = A_L \xi(a) A_L^{-1}$

$a \cdot a \neq 0 \qquad \gamma(a)[Z_{a\pm}] = Z_{a\pm}, \quad \gamma_5[Z_{a\pm}] = Z_{a\mp} \quad \forall a \in N$

I	1
$\gamma(n_{a})_3$	4

$\gamma(n_0) \sim \gamma(n_3) \sim \gamma_5 \quad 1$

$\langle \cdot, \cdot \rangle : X \times X \to \mathbb{C}$

Z_{a+} poz. def. $\quad a \cdot a > 0$ időszerű

Z_{a-} neg. def.

$\gamma_5 \gamma(a) + \gamma(a)\gamma_5 = 0 \quad \forall a \in N$

$\gamma_5^* = -\gamma_5 \quad (\gamma_5)^2 = I$

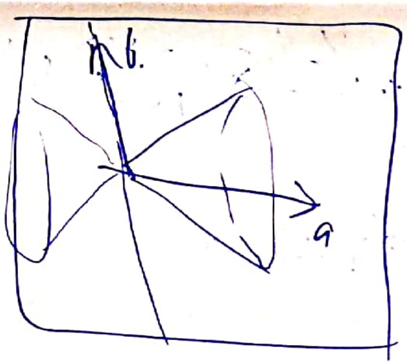
$$\begin{pmatrix} + & & & \\ & - & & \\ & & - & \\ & & & - \end{pmatrix}$$

$a \cdot a > 0 \quad \langle x, \gamma(a) y \rangle$

$\langle x, \gamma(a) x \rangle > 0$

[CØ6] $T_a : X \to Z_{a+} \oplus Z_{a+}$ $\quad x \mapsto \left(\frac{I+\gamma(a)}{2} x, \frac{I-\gamma(a)}{2} \gamma_5 x \right)$ $\quad a \cdot a = 1$ $\quad N = \frac{M}{T}$

$\gamma(a) = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$, $\quad \gamma(b) = \begin{pmatrix} 0 & -\sigma_a(b) \\ \sigma_a(b) & 0 \end{pmatrix}$ $\quad b \cdot a = 0$, $\quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$



$\bar{\Psi} = \Psi^+ \gamma^a$
$\gamma(a)$

$\gamma(\Psi)$

Feles spinű relativisztikus részecskék

① A Dirac$_{1/2}$-egyenlet

$(M, \mathbb{T}, g)$ rel. téridő $\quad \gamma : (M, g) \longrightarrow \mathrm{Lin}\, X$ $\quad$ kx. vektortér $\quad \dim_{\mathbb{C}} X = 4$

~~$(X, \langle , \rangle)$~~ $\langle , \rangle : X \times X \to \mathbb{C}$ nemelf. szeszkvilin. forma

γ kiterjesztése $\quad$ pl. ~~$k \in M^*, x \in X, \gamma(k \otimes x) \quad \gamma(x)$~~

$k \in M^* \equiv \frac{M}{T \otimes T}$, $x \in X$ $\quad \gamma(k \otimes x) = \gamma(k) x \in X$

$\phi : M \longrightarrow X$ $\quad x \in M$ $\quad D\phi[x] \in \mathrm{Lin}(M, X) = M^* \otimes X \Rightarrow D\phi : M \longrightarrow M^* \otimes X$

$\phi(x+v) - \phi(x) = D\phi[x] v + \mathrm{ordo}(v)$ $\quad$ **Def** $\quad \gamma(D\phi)[x] := \gamma(D\phi[x])$ $\quad \gamma\phi$

$D\phi_x v \quad D_v \phi$ $\quad \gamma(D)\phi := \gamma(D\phi) \sim \gamma^k \partial_k \phi$

$\boxed{C\emptyset 7}$ $L(I'X)$
$E_{t,\cdot} < L^2(t,X)$

téridőn

$X_{m,b} = \ldots$

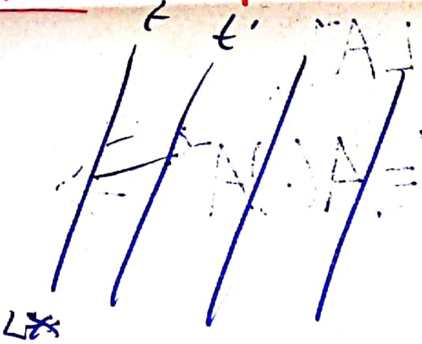
tömeghéj on

$V(m) = \{k \in M^* \mid k\cdot k = m^2, k \text{ jövő}\}$

$L^2(Di_{\frac{1}{2}}(m)) = \{\phi : V(m) \to X \mid \gamma(k)\phi(k) = m\phi(k) \ \forall k \in V\}$

absz. $\phi\, \mathcal{F}^{m,1/2,0} := \{\varphi \in T^{(3/2)} \otimes \mathcal{D}(M, X)^* \mid (\gamma(-iD-k)-m)\phi\lambda_u = 0$

$\langle\phi|\phi\rangle_u \in L^1$ és $\forall$ Re u-tól $\}$



széthas. $\hat{\mathcal{F}}^{m,1/2,0} = \{\hat\phi \in T^{(3/2)} \otimes \mathcal{D}(T_0 \times S_u, X)^* \longleftarrow L^2(\hat{D}i_{\frac{1}{2}}(m)) := \{\hat\phi : S_u^* \to X \mid (m\beta + \alpha\cdot p)\hat\phi(p) = p_0 \hat\phi(p)$

$\partial_0 \hat\phi = -i(m\beta + \alpha(-i\nabla_u))\hat\phi$

és $\hat\phi(t,\cdot) \in E_{u,t}$

$$\hat\phi(t,q) := \frac{1}{(2\pi)^{3/2}} \int_{S_u^*} e^{-i\sqrt{|p|^2+m^2}\,t + i p\cdot q}\, \hat\phi(p)\, dp$$

$Sp(m\beta + \alpha_u) = (-\infty, -m] \cup [+m, \infty)$

P_+

C08

Def. Fejlődési tér $\mathcal{E} = T^{(1/2)} \otimes D(M,X)^*$ $\quad \dim_{\mathbb{C}} X = 4$

Def. Dirac$_{1/2}$ egyenlet $\gamma(-iD-k)\phi\lambda_M = m\phi\lambda_M$

Def. $\phi\lambda_M$ mo. Dirac$_{1/2}$ $\quad J_\phi : M \longrightarrow M^*$ $\quad J_\phi(x) = \langle \phi(x), \gamma(\cdot)\phi(x)\rangle$

$$\Big[\underbrace{\{J_\phi(x), v\}}_{M\text{-beli szorzás}} = \underbrace{\langle \phi(x), \gamma(v)\phi(x)\rangle}_{X\text{-beli szorzás}}\Big]$$ ϕ részecskeárama

Áll. $\operatorname{div} J_\phi = 0$

$\phi\lambda_M$ mo. $\Rightarrow \langle \phi, \phi\rangle_u = \int_{S_u} \langle \phi, \gamma(u)\phi\rangle \, d\lambda$ ftlen u-tól

$$\langle \phi, \psi\rangle = \frac{1}{4}\left(\langle \phi+\psi, \phi+\psi\rangle - \langle \phi-\psi, \phi-\psi\rangle\right)$$

Def. Folyamattér $\mathcal{F}^{m,\frac{1}{2},0} = \{\phi\lambda_M \in \mathcal{E} \mid \text{mo}, \langle \phi, \phi\rangle \in L^1(S_u) \text{ és ftlen } S_u\text{-tól}\}$

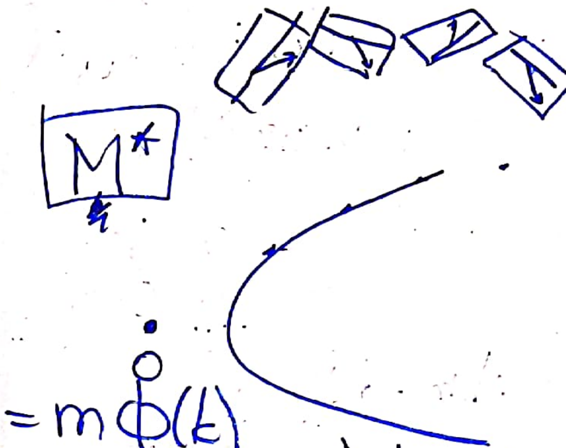
C09

$$\mathcal{F}^{m,1/2,0} \longrightarrow L^2(S_u, X) := \{ \varphi : S_u \to X \mid x \mapsto \langle \varphi(x), \varphi(x)\rangle_u \ \lambda_S\text{-int}\}$$

$$\varphi \longmapsto \lambda|_{S_u}$$

izometria, de vajon unitér is?

③ Abszolút impulzusokkal felírt folyamatok



$$V(m) := \{ k \in M^* \mid k \cdot k = m^2,\ k \text{ jövőbeni} \}$$

Dirac$_{1/2}$-egyenlet: $\phi : V(m) \to X$ $\quad \gamma(k)\phi(k) = m\mathring{\phi}(k)$ $\quad \forall k \in M^*$

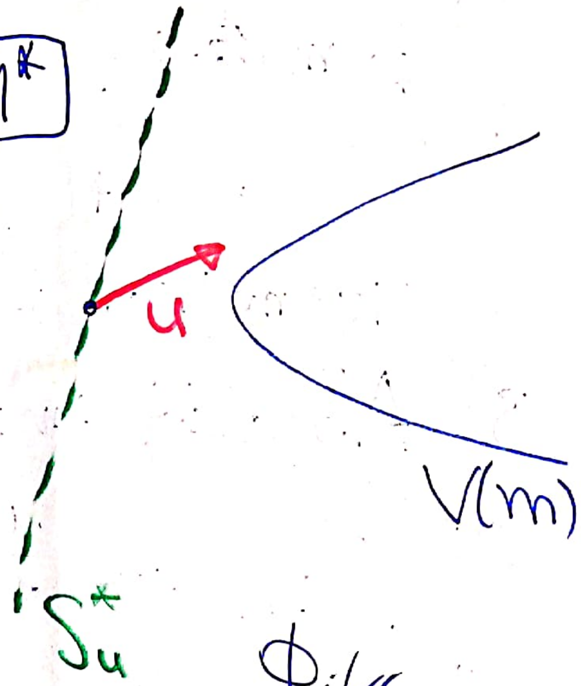
$$L^2(Di_{1/2}(m)) := \{ \phi : V(m) \to X \mid \text{mo. és } \phi \in L^2(V(m)) \}$$

$$\langle \phi | \chi \rangle_{L^2} := \int \langle \phi, \chi \rangle \, d\lambda_{V(m)}$$

Igaz-e az, hogy $L^2(Di_{1/2}(m)) \cong \mathcal{F}^{m,1/2,0}$?

C10 | Folyamatok rel. impulzusokkal

$\boxed{M^*}$



$$r_u : S_u^* \longrightarrow V(m), \quad p \longmapsto \underbrace{\sqrt{|p|^2+m^2}}_{p_0(p)}\, u + p$$

paraméterezi $V(m)$-et

$$\int f \, d\lambda_{V(m)} = \int_{S_u^*} (f \circ r_u) \frac{m\, d^3p}{\sqrt{|p|^2+m^2}}$$

$f : V(m) \to_{\mathbb{C}} V(m)$

$$\begin{tikzcd} & V(m) \arrow[dr, "\phi"] & \\ S_u^* \arrow[ur, "r_u"] \arrow[rr, "\hat\phi"'] & & X \end{tikzcd}$$

Dirac$_{1/2}$-egyenlet $\left(\sqrt{|p|^2+m^2}\, \underbrace{\gamma(u)}_{\beta} + \gamma(p)\right)\hat\phi(p) = m\,\hat\phi(p)$

$$(m\beta + \alpha \cdot p)\,\hat\phi = \sqrt{|p|^2+m^2}\,\hat\phi$$

$$\mathcal{F}\; L^2(\hat{D}(m)) \subset L^2(S_u^*, X)$$

$$\lambda_{V(m)} \circ (r_u^{-1})$$

$$Sp(m\beta + \alpha \cdot \cdot) = (-\infty, -m] \cup [m, \infty)$$