SØ1

Analízis I (M.T.)

$\emptyset,\ \{\emptyset\},\ \{\emptyset,\{\emptyset\}\}$
$\ \ \ \ \shortparallel \ \ \ \ \ \shortparallel \ \ \ \ \ \ \ \ \ \shortparallel$
$\ \ \ \ 0 \ \ \ \ \ \ 1 \ \ \ \ \ \ \ \ \ \ 2$

~~$(a, b, c) = \{$~~

$(a,b) := \{a, \{a,b\}\}$

Analízis II. (M.T.)

Vektor-tér: $(V, \mathbb{K}, +, \cdot)$

$a = \lambda b$

$(a_1, a_2, a_3 \ldots)$
$(\lambda_1, \lambda_2, \lambda_3 \ldots)$ $\sum_{i=1}^{n} \lambda_i a_i = v$

n

$\in \mathbb{N}$

$\exists$ n elemű bázis $\Rightarrow \forall$ n elemű

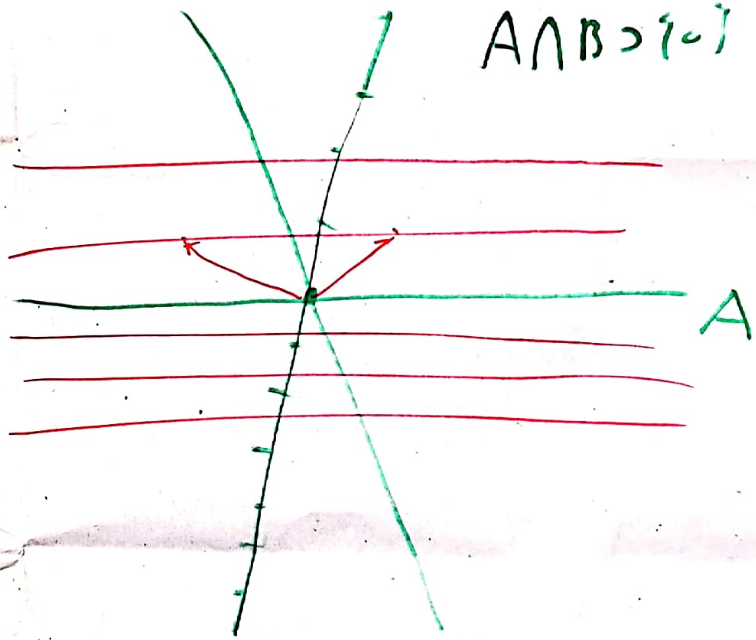
$\dim(V) := n$

$V \supset A$ $\quad B$ $\quad V/A$

$A \cap B \supset \{0\}$

$x \sim y \Leftrightarrow x - y \in A$

$V/\sim$



$V \times V$

S02

$f: V \to W$ $\quad Lin(V,W)$ $\quad Lin(V)$

$(Lin(V,W), +, \cdot, \circ)$
V, V

$Lin(V,W)$ $\quad Lin(W,V)$

$Lin(V,V)$
(V)

$(f+g)(v) := f(v) + g(v)$

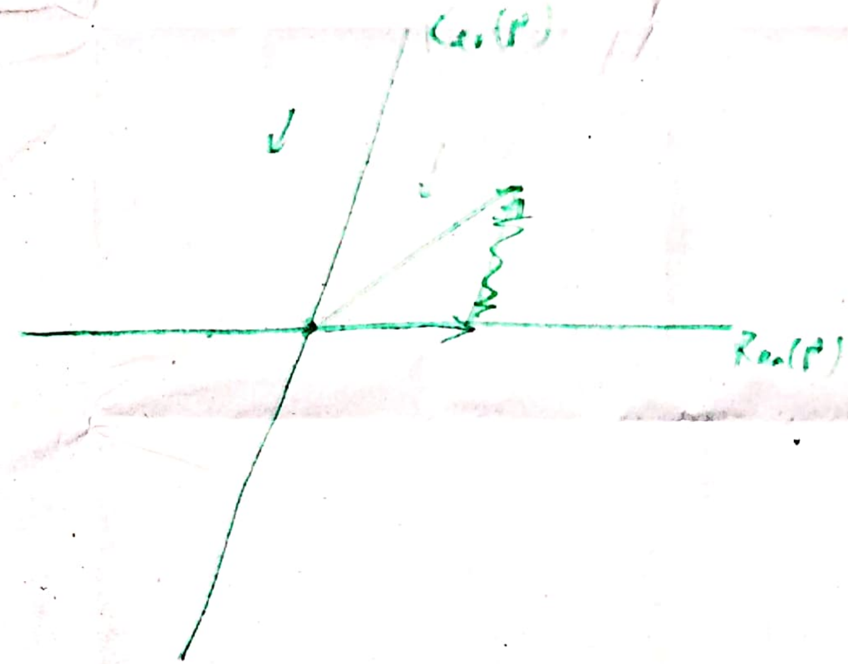
$(\lambda f)(v) := \lambda(f(v))$ $\quad f(\cancel{\lambda v})(v) =$

$(f \circ g)(v) = f(g(v))$

$PP = P$

$P^2 = P$

$Ran(P) \oplus Ker(P) = V$



$f(\lambda v) = \bar{\lambda} f(v)$

$Ran(f)$, $Ker(f)$
$\subset W$ $\quad \subset V$

$\boxed{5\text{Ø}3}$

$\mathrm{Lin}(V,\mathbb{K}) =: V^*$

$\overline{\mathrm{Lin}}(V,\mathbb{K}) =: \overline{V}^*$

$p(v)$

$=: (p|v)$

$\mathrm{Lin}(\overline{V}^*,\mathbb{K}) =: \overline{V}^{**}$

$\updownarrow \; \overline{V}$

$\mathrm{Lin}(V^*,\mathbb{K}) =: V^{**} \equiv V$

$\updownarrow$

V

$\cancel{\mathrm{Lin}(V^*,\mathbb{K}) =: V^{**}}$

$\mathrm{Lin}(W^*,V^*)$

$\mathrm{Lin}(V,W)$

$A: V \longrightarrow W \qquad A^*: W^* \longrightarrow V^*$

$(p|Av) \doteq (A^*p|v)$

$(p\circ A)(v) \doteq (A^*(p))(v)$

$\mathrm{Lin}(V,V) \qquad \mathrm{Lin}(V^*,V^*)$

5Ø4

EØZ

tenzorszorzat:

U, V

(Z, r) $\quad r: U \times V \to Z$ $\quad \exists!\, i: Z \to W$ $\quad \varphi = i \circ r$.

$\varphi: U \times V \to W$

(Z', r') $\quad Z \to Z'$

~~$\{f: U^* \times V^* \to \mathbb{K}$ bilin$\} =: U \otimes V$ $\quad x \quad y$~~

~~$(u, v) \mapsto$~~

$x \otimes y : V^* \to U$ lin

$x \otimes y(p) = x(y|p)$

$V \ (e_1 \dots e_n)$

$\downarrow$

$V^* \ (p_1 \dots p_n)$

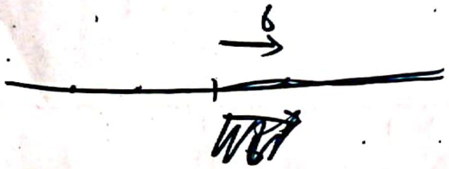
$(p_i | e_j) = \delta_{ij}$

$p_i(e_j)$

$(p | e) = 1$

1 dim vektorterek:

$L = L^- \cup \{0\} \cup L^+$ $\qquad L_0^+ = \{0\} \cup L^+$



$L \otimes L$

$L^n := \overset{n}{\otimes} L$

$L^{-n} := \overset{n}{\otimes} L^*$

$\dim(V \otimes W) = \dim(V) \dim(W)$

$1 \qquad 1 \qquad 1.$

$L \otimes T^*$

S05

$b: V \times V \to \mathbb{K}$

L^2

$\mathrm{Lin}(V, V^* \otimes_{\mathbb{K}} L^2)$

(V, b, L^2)

$\forall v: \exists v': b(v, v') \neq 0$

$b(e_i, e_j) = 0 \quad \forall i \neq j$

$b(e_i, e_i) \quad L^{2+}_0$

L^-_0

$$\begin{pmatrix} + & & & \\ & + & & \\ & & - & \\ & & & - \end{pmatrix}$$

(p, q)

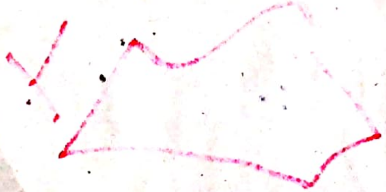
$(A, V, -)$

$-: A \times A \to V$

$\forall x, y, z: \ (x-y) + (y-z) + (z-x) = 0$

$\forall x \in A: \ (\cdot - x): A \to V$

$x + a$



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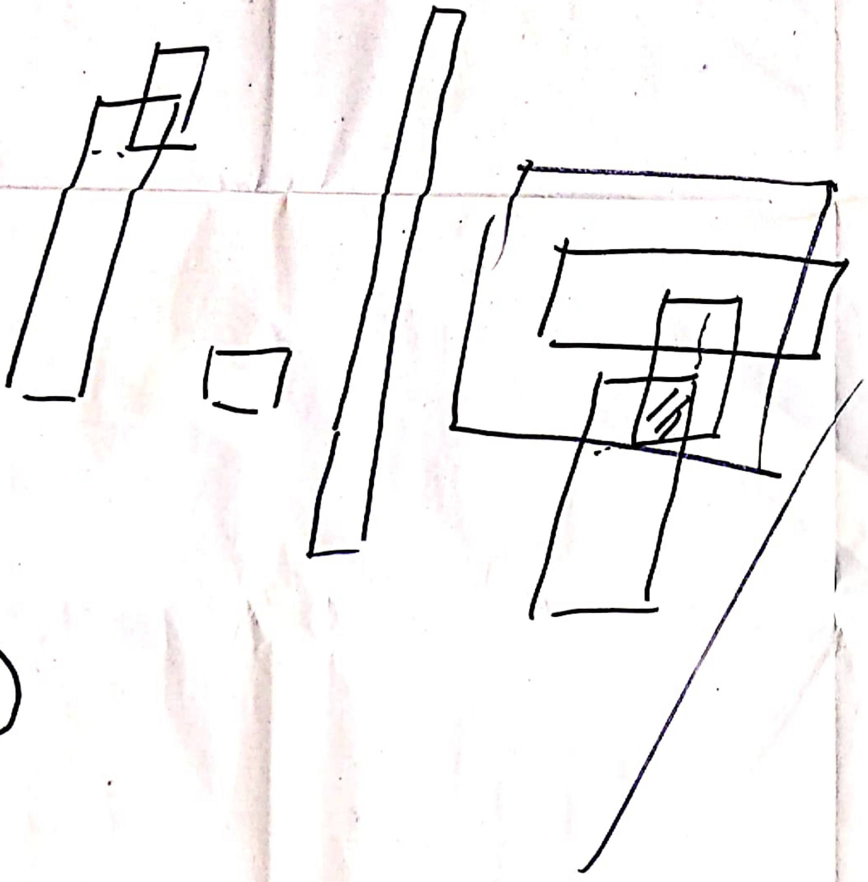
I	II	III	IV	V	VII	VIII	IX
HALM.	VEKT.	FOLYT.	DIFF.	INT.	DIF.EGY	FUNK.AN.	DISZTR.

$$\rho := \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V}$$

$$\delta(x) := \begin{cases} 0 & x \neq 0 \\ \infty & x = 0 \end{cases}$$

$$\int \delta(x)\,dx = 1$$



ρ

$X \quad \mathcal{A} \subset \mathcal{P}(X)$

σ - algebra

S07

$E_n \in \mathcal{R}$ $(n \in \mathbb{N})$ $\qquad \left.\begin{array}{l} \bigcap_n E_n \\ \bigcup_n E_n \end{array}\right\} \in \mathcal{R}$ $\qquad E \setminus F \in \mathcal{R}$

$X \in \mathcal{R}$

$$\mu : \mathcal{R} \to \mathbb{R}_0^+ \cup \{\infty\} \qquad \mu\Big(\dot{\bigcup_n} E_n\Big) = \sum_n \mu(E_n)$$

$a \in X$ $\qquad \delta_a(E) := \begin{cases} 0 & \text{ha } a \notin E \\ 1 & \text{ha } a \in E \end{cases}$

$$E \subset F \Rightarrow \mu(E) \leq \mu(F)$$

5Ø8

$\mathbb{R}$ $\mathbb{R}^2$ $\mathbb{R}^3$ Lebesgue-mérték

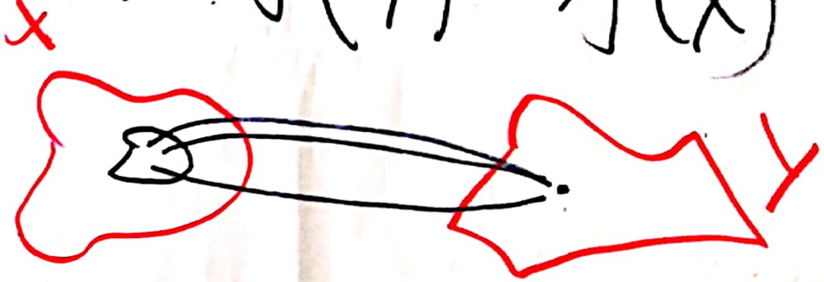
λ Borel-féle $\mathcal{B}(\mathbb{R})$ ...

$$\lim_{\substack{n\to\infty \\ \cancel{\lambda(E)\to 0}?}} \frac{m(E_n)}{\lambda(E_n)} =: \rho(x) \qquad \frac{\delta_Q(E_n)}{\lambda(E_n)} \qquad E_Q$$

$\bigcap_n E_n = \{x\}$

$$T : X \to Y \qquad T^{-1} : P(Y) \to P(X)$$

$$T^{-1}(H) := \{x \in X \mid T(x) \in H\}$$



$$\overset{-1}{T}\Big(\bigcap_{i\in I} H_i\Big) = \bigcap_{i\in I} \overset{-1}{T}(H_i)$$

$\bigcup$

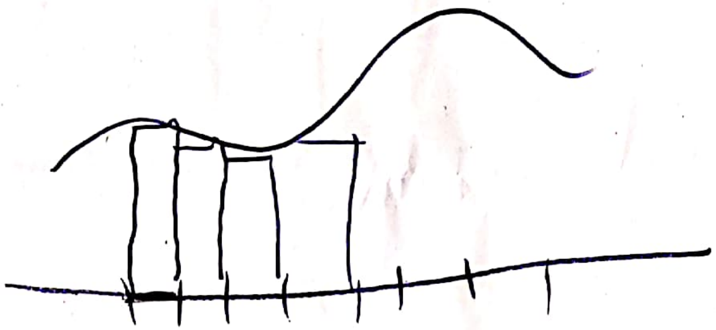
$\mathcal{A} \subset \mathcal{P}(X)$ $\mathcal{B} \subset \mathcal{P}(Y)$

$T: X \to Y$ ho: $\overset{-1}{T}: \mathcal{B} \to \mathcal{A}$

$\mu \circ \overset{-1}{T}$

$\mathcal{A}$-$\mathcal{B}$-mérhető

$f: X \to \mathbb{R}$ Borel-fv: $\mathcal{A}$-$\mathcal{B}(\mathbb{R})$-mérhető



$\boxed{f = g \ \mu\text{-m.m.}}$

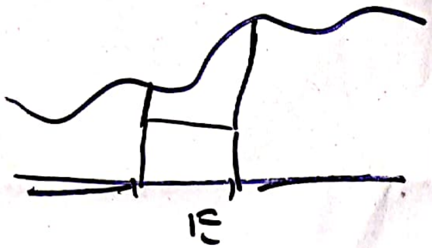
$$\int_X f\,d\mu = \int f(x)\,d\mu(x)$$

$d\lambda(x) = dx$

$$\boxed{\int f\, d\delta_a = f(a)}$$

$$\chi_E(x) = \begin{cases} 0 & x \notin E \\ 1 & x \in E \end{cases}$$

$$:= \int \chi_E\, d\mu = \mu(E)$$



f μ-int. E-n ha $f\chi_E$ μ-int.

$g > 0$ $\quad (g\mu)(E) := \begin{cases} \infty \\ \int g\chi_E\, d\mu \end{cases}$

$$\boxed{\int f\, d(g\mu) = \int f g\, d\mu}$$

$$\lim_n \frac{(g\mu)(E_n)}{\mu(E_n)} = g(x)$$

$\{x\} = \bigcap_n E_n$ $\quad$ μ-mm x-re

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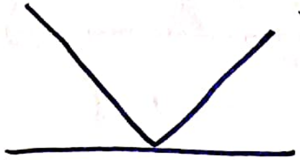
$\mu(E)=0 \Rightarrow (g\mu)(E)=0$ $\qquad \nu$

$\left(\text{ma } \mu(E)=0 \text{ allora } \nu(E)=0\right) \Rightarrow \exists\, g$

$\nu = g\mu$



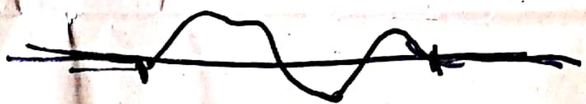
$\lim_n \int f_n \, d\mu \overset{?}{=} \int \lim_n f_n \, d\mu$

$((f\lambda_{\mathbb{R}})' | \varphi) := -\int f \varphi'$

$\emptyset \lambda_M$

$f\lambda_{\mathbb{R}}$

$\int f \varphi' = \cancel{\int (f\varphi)'} - \int f' \varphi$



$\int f' \varphi := -\int f \varphi' \quad \forall \varphi$

$\chi_{[0,\infty[}$

$$\left((\chi_{[0,\infty[}\lambda_{\mathbb{R}})' \,\middle|\, \varphi \right) = -\int \chi_{[0,\infty[}\varphi' \, d\lambda_{\mathbb{R}} =$$

$$\varphi\lambda_M \qquad -\int_0^\infty \varphi'(x)\,dx = \varphi(0)$$

$$\int \varphi' \, d\sigma$$