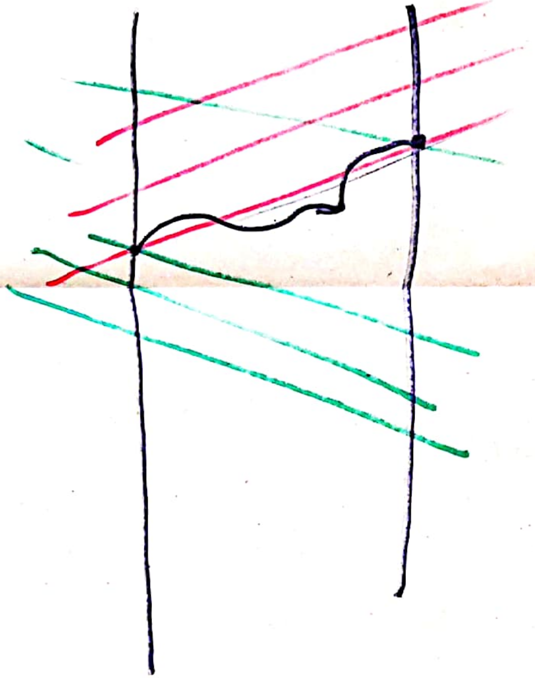
Z01

$S \times S^*$ u, m, k

$(\mathbb{R}^3 \times \mathbb{R}^3)$

$L^2(S)$

$(L^2(\mathbb{R}^3))$



Egyensúly . .

$$\partial_0(\hat{\varphi}\lambda_{T\times S}) = -i\hat{H}(\hat{\varphi}\lambda_{T\times S})$$

$$\frac{d\psi_t}{dt} = -iH_t\psi_t$$

$$\dot{\eta}_t\varphi = -iH_t\eta_t\varphi$$

$$|\psi_t\rangle\langle\psi_t|$$

$$\psi_t = \eta_t\varphi \qquad \boxed{i\frac{\dot{\eta}_t}{\eta_t}\varphi = H_t\varphi}$$

Z 02

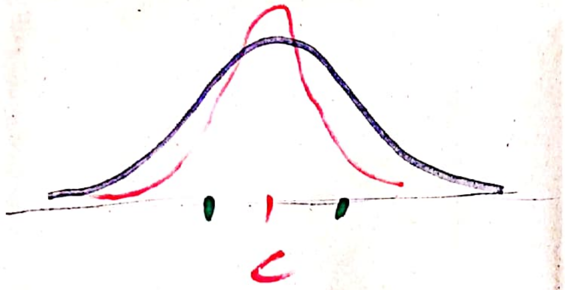
Szabad tömegpont $\sigma_p\left(\frac{\Delta}{2m}\right) = \{ E \in T^* \}$

$\boxed{\hat{\phi}_{c,a}}$ 26.6

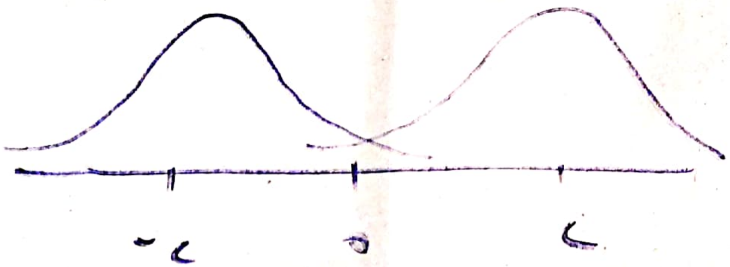
26.7

$\boxed{c}, a$

26.8 26.9



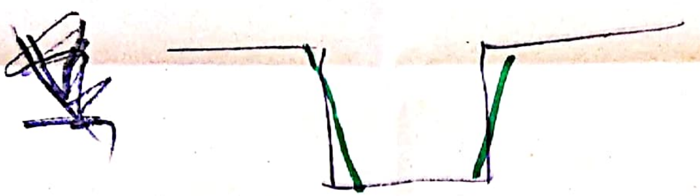
$$\frac{\hat{\phi}_{c,a} + \hat{\phi}_{-c,a}}{\|\hat{\phi}_{c,a} + \hat{\phi}_{-c,a}\|}$$



Z03

Gödör potenciál

$$-\frac{\Delta}{2m} + V$$



V_0

$0 < E < V_0$

$\varphi \in L^2(S)$

N nyílt halmaz

$\chi_N \wedge |\varphi\rangle\langle\varphi| =$

$\chi_N |\varphi\rangle\langle\varphi| \neq 0$

$|\varphi\rangle\langle\varphi| \neq 0$

Z04

$$V(q) = \frac{m^{*}}{2\hbar}\omega^2 |q|^2$$

$$-\frac{\Delta}{2m} + V$$

$$\varphi(q) = p(q)\, e^{-\frac{m\omega^2}{2}|q|^2}$$

$$\chi_N \quad |\varphi\rangle\langle\varphi|$$

$$V(q) = \frac{-\alpha}{|q|}$$

$$\varphi(q) = p(q)\, e^{-\frac{|q|}{\cdots}}$$

$$[\mathbb{L}^2, L_a] = 0$$

$$H + \lambda V_t$$

$$[H, L] = 0$$

Z05

$\langle \varphi_m | V_t \varphi_n \rangle$

Ált. sajátérték

λ

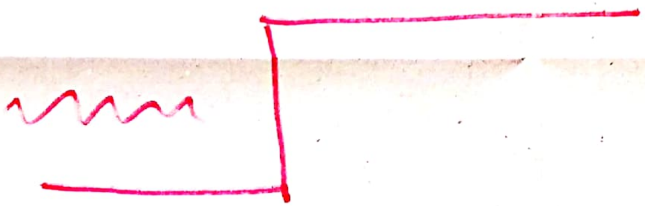
$A - \lambda I$

$\exists \varphi_n \quad \|\varphi_n\| = 1 \quad \forall n$

$\lim_n (A - \lambda I)\varphi_n = 0$

$\left(-\frac{\Delta}{2m}\right)\varphi = E\varphi$

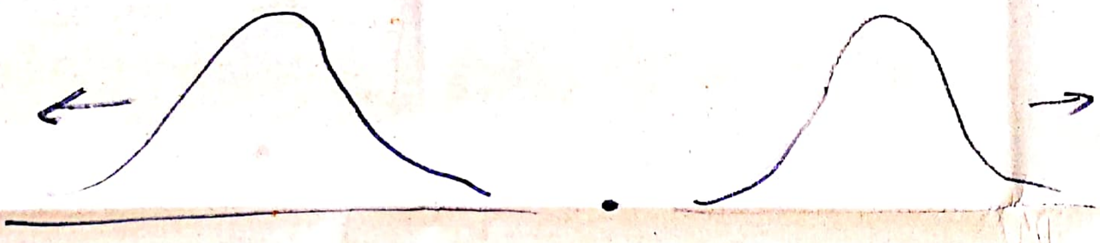
$\boxed{\varphi(y) = e^{ik \cdot y}} \quad |k| = \sqrt{\frac{2E}{m}}$



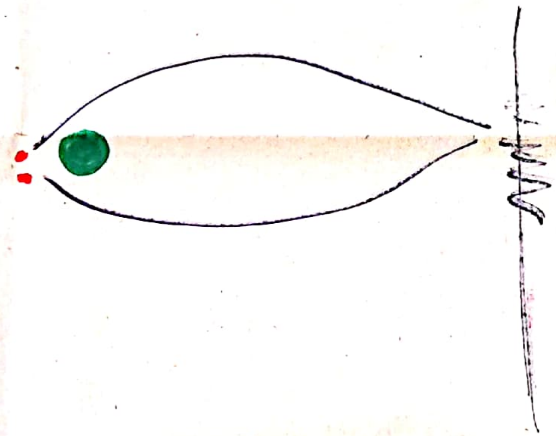
$\|\varphi_n\| = 1$

$\varphi \; \varphi_n$

Z06



Bohm-Aharonov



SZIMMETRIÁK

MIÉRT:

- PROBLÉMA MEGOLD.
- FOLY. ÁLLANDÓK (MEGM. MENNY.)
- ZÁRT RENDSZER DEF.
- ELVI ÉSZREV.
- MEGOLDÁSOK OSZTÁLY., RENDSZ., JELL., HASZNA

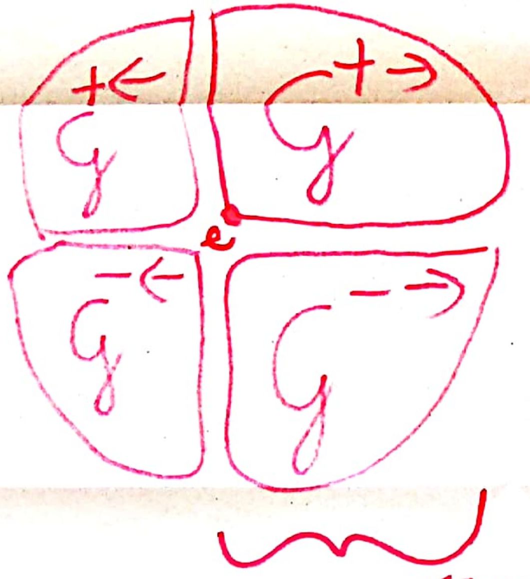
Z 07

TÉRIDŐ-SZIMM. $(M, \vec{T}, \mathbb{L}, \tau, h)$

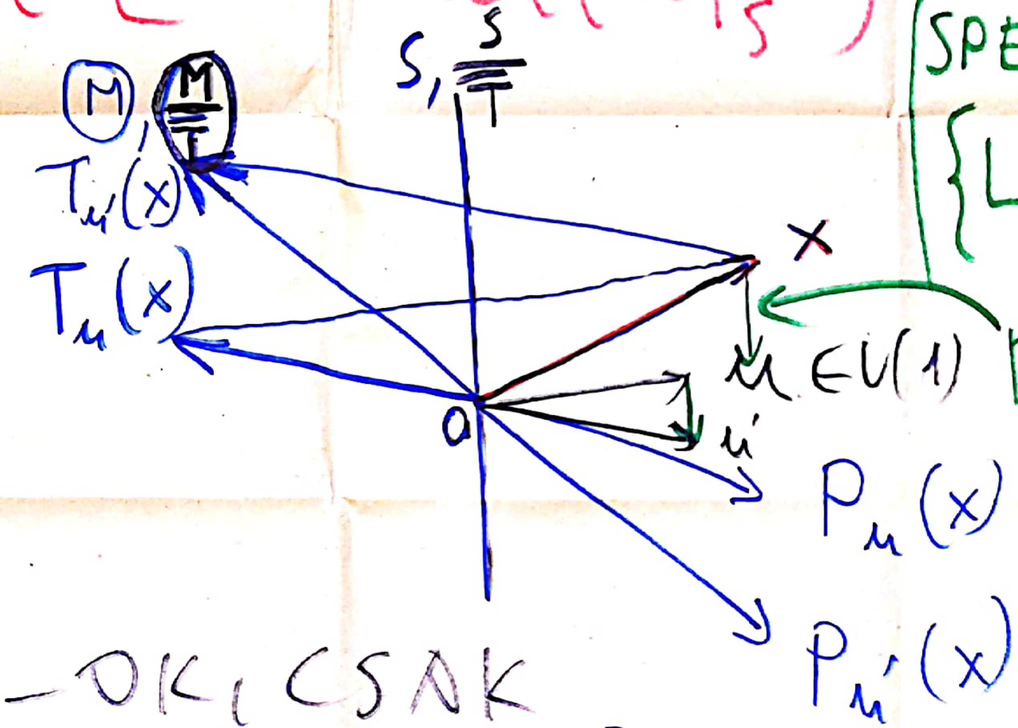
$M \to M$: GALILEI-CSOP.:

$$\mathcal{G} := \{ L \in \mathrm{Lin}(M,M) \mid \tau \circ L = \pm\tau,\ L|_S \in O(S) \}$$

$$\mathrm{sign}\, L := \det(L|_S)$$



MX.-TARTÓK



SPEC. GAL.-TR.:

$$\{ L \in \mathcal{G}^{+\to} \mid L|_S = id_S \}$$

$$B_{u'u} = id_M + (u'-u) \otimes \tau$$

WIGNER-FÉLE

✓ KISCS.

NINCSENEK FORG.-OK, CSAK

$$SO_u(S) := \{ L \in \mathcal{G}^{+\to} \mid Lu = u \}$$

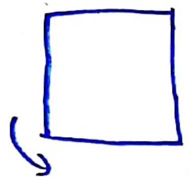
u-TÉRSZ. FORG.-OK

ZD [redacted] G CSOPORT, HA ADOTT EGY
8

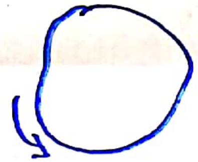
$G\times G \to G$, $(g,h)\mapsto gh$ ÚGY:

- $f(gh)=(fg)h$ $\quad(\forall f,g,h\in G)$
- $\exists e\in G$ ÚGY: $eg=ge=g$ $\quad(\forall g\in G)$
- $\forall g\in G$-HEZ $\exists g^{-1}\in G$: $gg^{-1}=g^{-1}g=e$

VÉGES/DISZKRÉT CSOPORT: G VÉGES.



(FOLYTONOS)/LIE-CSOP.: G DIFFH. SOK. IS, gh, g^{-1} DIFFH.



PL. SO(2): $\mathbb{R}^2$ FORG.-AI

$$\begin{pmatrix}x\\y\end{pmatrix}' = \underbrace{\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}}_{g(\alpha)}\begin{pmatrix}x\\y\end{pmatrix}$$

$\alpha\in[-\pi,\pi]$

$e: \alpha=0: \begin{pmatrix}1&0\\0&1\end{pmatrix}$

(INF. GEN.)

$$\frac{dg}{d\alpha}=\begin{pmatrix}-\sin\alpha & \cos\alpha\\ -\cos\alpha & -\sin\alpha\end{pmatrix},\quad \left.\frac{dg}{d\alpha}\right|_{\alpha=0}=\begin{pmatrix}0&1\\-1&0\end{pmatrix}=:\underset{\sim}{A}$$

$$\underset{\sim}{A}^2=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}=-\begin{pmatrix}1&\\&1\end{pmatrix}$$

$$e^{\alpha\underset{\sim}{A}}=\begin{pmatrix}1&\\&1\end{pmatrix}+\alpha\underset{\sim}{A}+\frac{\alpha^2}{2!}\underset{\sim}{A}^2+\frac{\alpha^3}{3!}\underset{\sim}{A}^3+\dots=$$

$$=\begin{pmatrix}1&\\&1\end{pmatrix}+\alpha\underset{\sim}{A}-\frac{\alpha^2}{2!}\begin{pmatrix}1&\\&1\end{pmatrix}+\frac{\alpha^3}{3!}(-\underset{\sim}{A})+\dots=$$

$$=\left[1-\frac{\alpha^2}{2!}+\frac{\alpha^4}{4!}-\dots\right]\begin{pmatrix}1&\\&1\end{pmatrix}+\left[\alpha-\frac{\alpha^3}{3!}+\dots\right]\underset{\sim}{A}=$$

$$=\cos\alpha\begin{pmatrix}1&\\&1\end{pmatrix}+\sin\alpha\begin{pmatrix}&1\\-1&\end{pmatrix}=g(\alpha)$$

$$\underset{\sim}{A}^+=\left[\begin{pmatrix}0&1\\-1&0\end{pmatrix}^T\right]^*=\begin{pmatrix}0&-1\\1&0\end{pmatrix}^*=\begin{pmatrix}0&-1\\1&0\end{pmatrix}=-\underset{\sim}{A}$$

$$\left(\frac{1}{i}\underset{\sim}{A}\right)^+=\left(\frac{1}{i}\underset{\sim}{A}^T\right)^*=\frac{1}{-i}(-\underset{\sim}{A})=\frac{1}{i}\underset{\sim}{A}$$: ÖNADJ.

$$e^{i\alpha\left(\frac{1}{i}\underset{\sim}{A}\right)}$$

Z09

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}' = \begin{pmatrix} \text{3 EULER-SZÖG} \\ \text{A SOKASÁG} \\ \text{PARAMÉTEREZÉSE} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

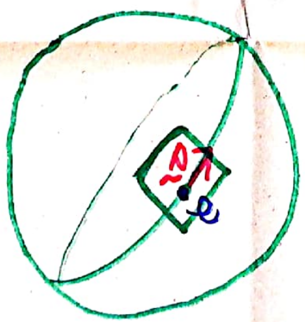
1-PARAMÉTERES RÉSZCSOP.-JA:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}' = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

1-PAR. RÉSZCSOP.:

1-DIM. RÉSZSOK.,

RÉSZCSOP.



$$\begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix}' = \begin{pmatrix} \cos\alpha & \sin\alpha & & \\ -\sin\alpha & \cos\alpha & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix}$$

A $\begin{pmatrix} 0 \\ 0 \\ z \\ u \end{pmatrix}$ ALAKÚAK INV.-AK

INVARIÁNS ALTÉR

Z10

SZÉTHAS. GAL.-CS.:

$$\xi_u : M \to \bar{T} \times S, \quad x \mapsto (\tau(x), \pi_u(x))$$

$$\Rightarrow \xi_u L \xi_u^{-1} : \bar{T} \times S \to \bar{T} \times S$$

$$: \left\{ \begin{pmatrix} \pm 1 & 0 \\ v & \pm R \end{pmatrix} \middle| \, v \in \frac{S}{\bar{T}},\ R \in SO(S) \right\}$$

u-FÜGGŐ AZ EREDMÉNY

$$M \ni x \xrightarrow{u} (t,q) \in \bar{T} \times S$$
$$\downarrow u'$$
$$(t', q')$$

$$\xi_u : x \mapsto (\tau x, \pi_u x)$$
$$\xi_u^{-1} : (t,q) \mapsto q + ut$$

SPEC. GAL.-TR.:

$$\begin{pmatrix} 1 & 0 \\ v & id_S \end{pmatrix} \begin{pmatrix} t \\ q \end{pmatrix} = \begin{pmatrix} t \\ q + vt \end{pmatrix}$$

AKTÍV / PASSZÍV TR.

$$\xi_{u'}(q+ut) = (t, \pi_{u'}(q+ut)) =$$
$$= (t, (q+ut) - \tau(q+ut)u') =$$
$$= (t, q + (u-u')t) = (t, q - v_{u'u} t)$$

11

NOETHER-CSOP.: $(\underset{\bullet}{M}, T, \mathbb{L}, \tau, h)$ $\quad \mathcal{G}: \tau, h$

$\underset{\bullet}{M} \to \underset{\bullet}{M}$:

$\mathcal{N} := \{ \underset{\bullet}{L} : \underset{\bullet}{M} \to \underset{\bullet}{M}$ AFFIN (BIJ.) $\mid L \in \mathcal{G} \}$

- TÉRIDŐ ELTOLÁSOK $\quad a \in M$ $\quad \underset{\bullet}{x} \mapsto \underset{\bullet}{x} + a$ $\quad \underset{\bullet}{M}$
- TÉRSZERŰ ELTOLÁSOK $\quad a \in S$
- u - IDŐELTOLÁSOK $\quad \underset{\bullet}{x} + tu$
- $\underset{\bullet}{o}$ KÖZÉPPONTÚ GALILEI-CS. $\quad L \in \mathcal{G}$ $\quad \underset{\bullet}{x} \mapsto L(\underset{\bullet}{x} - \underset{\bullet}{o})$ $\quad \underset{\bullet}{o} +$

SZÉTHASÍTÁS UTÁN $\quad T \times S \to T \times S$

„IDŐELTOLÁSOK RÉSZCS.-JA", „GALILEI-CS. MINT RÉSZCS."

ZH) KV. MECH.,
FEJL. TÉREN

$\underset{\cdot}{L}$ NOETHER-TR.

$$\phi\lambda_{\underset{\cdot}{M}} \mapsto \begin{cases} (\phi\lambda_{\underset{\cdot}{M}})\circ \underset{\cdot}{L}^{-1}, & \text{HA } \underset{\cdot}{L} \rightarrow \\ (\phi^{*}\lambda_{\underset{\cdot}{M}})\circ \underset{\cdot}{L}^{-1}, & \text{HA } \underset{\cdot}{L} \leftarrow \end{cases}$$

INV. $\underset{\cdot}{L}$-EKRE $\left[(\phi\lambda_{\underset{\cdot}{M}})\circ \underset{\cdot}{L}^{-1} = (\phi\circ \underset{\cdot}{L}^{-1})\lambda_{\underset{\cdot}{M}}\right]$

HA $\underset{\cdot}{L} \rightarrow$: $iD_{Lu}(\phi\circ \underset{\cdot}{L}^{-1}) = (iD_u\phi)\circ \underset{\cdot}{L}^{-1}$ $(\mathcal{F}^{\cdots}_{Lu})$

$\underset{\cdot}{L} \leftarrow$: $iD_{-Lu}(\phi\circ \underset{\cdot}{L}^{-1}) = (iD_u\phi)^{*}\circ \underset{\cdot}{L}^{-1}$ $(\mathcal{F}^{\cdots}_{-Lu})$

u, K $\quad u', K'$

Z13

MOSTANTÓL CSAK EGY σ-T VÁLASZTVA $\in M$

$\Rightarrow \left.\left(\phi \circ \zeta^{-1}\right)\right|_{t} = \left.\phi\right|_{\zeta[t]}$

$\Rightarrow \left.U_{\zeta}\right|_{\mathcal{F}_u^{m,K}}$ UNITÉR

$$h_{\zeta}(x) := \left(-\frac{m}{2}\left|L u - u\right|^2 \tau + m(Lu - u)\cdot \Pi_u\right)\left(x - \frac{L\sigma - \sigma}{2}\right)$$

$$\phi \mapsto U_{\zeta}\phi = e^{i h_{\zeta}}\, \phi \circ \zeta^{-1}$$

ζ (m,K) <u>SZIMMETRIÁJA</u>, HA $\exists\, g_{\zeta} : M \to \mathbb{R}$

$\forall\, \phi \in \mathcal{F}_u^{m,K}$-RE $U_{\zeta}\phi \in \mathcal{F}_u^{m, K + Dg_{\zeta}}$

$F = D \wedge K = D \wedge (K + Dg_{\zeta})$

<u>SZIGORÚ</u>, HA $Dg_{\zeta} = 0$

ZH)

SZIMM.: (MEGENGETT) FOLY.-HOZ

(MEGENGEDETT) FOLY.-OT

RENDEL [AKTÍV]

$\boxed{\text{WEINBERG: PASSZÍV TR.}}$

(m, k) <u>ZÁRT RENDSZER</u>, HA $\forall$ (e)

$\Rightarrow$ $k=0$ (q_i EREJÉIG) SZIMM.-JA

A SZABAD TÖMEGPONT A ZÁRT R.

ABR. ~~[illegible]~~ EGY SZABAD m KOLY.-

TERÉN

7/5

$$U_{\underset{\cdot}{L}'} U_{\underset{\cdot}{L}} = \omega(\underset{\cdot}{L}', \underset{\cdot}{L}) \cdot U_{\underset{\cdot}{L}' \circ \underset{\cdot}{L}}$$

$|\omega(\underset{\cdot}{L}', \underset{\cdot}{L})| = 1$ UNITÉR KOCIKLUS

UNITÉR SUGÁRÁBR.

$$\left\{ e^{i\alpha} \phi \;\middle|\; \alpha \in [-\pi, \pi) \right\}$$

EGY $\underset{\sim}{H}$ N.-CS.-LIE-ALG.-ELEM ÁBRÁZOLÓDIK:

INF. GEN.:

$$(S_{\underset{\sim}{H}} \phi)(\underset{\cdot}{x}) = i \frac{d}{d\alpha} \left(U_{e^{\alpha \underset{\sim}{H}} = \underset{\cdot}{L}} \phi(\underset{\cdot}{x}) \right) \Big|_{\alpha=0} =$$

$$= -m (\underset{\sim}{H} u) \cdot \Pi_{u} (\underset{\cdot}{x} - \underset{\cdot}{o}) \phi(\underset{\cdot}{x}) - i D_{M} \phi(\underset{\cdot}{x}) \underset{\cdot}{H}(\underset{\cdot}{x})$$

LD. WIGNER TÉTELE: ALTÉRH.-K KÖZ. ORTO-G.-ROM.-OK EKV. UNITÉR / ANTIUNITÉR SUG.ÁBR.-SAL

16)

- u IRÁNYÚ IDŐELTOLÁS: $-i D_u$
- $e \in S$ IRÁNYÚ TÉRELT.: $-i \nabla_e$ $F_u^{m,k} \to$
- $\underset{\cdot}{o}$ KÖZÉPP., v IRÁNYÚ SP. GAL.-TR.: $x \mapsto -m v \cdot \Pi_u(x - \underset{\cdot}{o}) + Lv$ $+ m\tau(x - \underset{\cdot}{o})(-i \nabla_v)$
- $\underset{\cdot}{o}$ KP., a AXIÁLVEKTOR KÖRÜLI I_u-FORG.: $x \mapsto (m \Pi_u (x - \underset{\cdot}{o}) \times (-i \nabla)) \cdot a$

A SZABAD R.

FOLX. ÁLL.-I:
- u-IMPULZUS (VETÜLETEI)
- $(u, \underset{\cdot}{o})$-DEVIÁCIÓ (VET.-EI)
- $(u, \underset{\cdot}{o})$-IMP. MOM. (...)
- AZ u-ENERGIA NEGATÍVJA

[redacted] SH HONNAN HOVA?

matolcsi.tamas
@gmail.com

217)

U-SZTATIKUS K

$$T \to L^2(S),\quad t \mapsto \Psi_t = \boxed{e^{-itH}\,\Psi_0}$$

$e^{i\alpha S_H}$

$S_H = -H$

U → K : KOMMUTÁTOR KOCIKLUS

$H = \frac{-\Delta}{2m}$ $\quad -H = \frac{\Delta}{2m}$

⇒ $m \neq m'$: UN. SUG.-ÁBR.-OK INEKV.-EK; IRREDUCIBILISEK

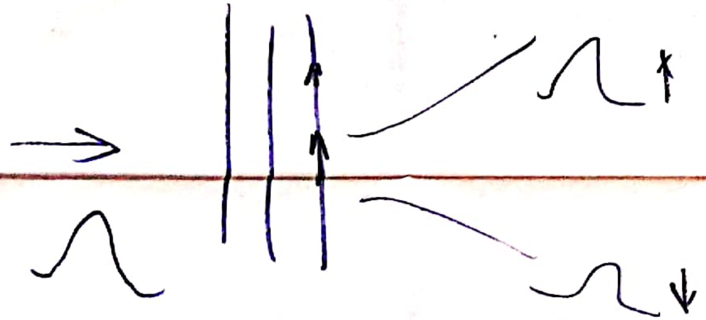
Z18

$\mathcal{D}(M)^*$ $\quad M \to \mathbb{C}$

$M \to Z$ két dim. $\quad Z(S)$

$\mathcal{D}(M,Z)^*$

$$\frac{(-i\nabla - A)^2}{2m} + V_u - B\cdot\mu \qquad \mu = \frac{eS}{2m}$$



m, S

$L^2(S)\otimes L^2(S)$

$$M \times_{\tau} M := \{(x,y)\in M\times M \mid \tau(x)=\tau(y)\}$$

219

$$\left(\mathcal{D}\left(M \underset{\tau}{\times} M,\ Z(S_1) \overset{\vee}{\underset{\wedge}{\otimes}} Z(S_2)\right)\right)^*$$

Különböző részecskék

$$(x_1, x_2) \mapsto V(\underbrace{|x_1 - x_2|}_{\in S})$$

$$Q_t(E) \otimes Q_t(E)$$

$$Q_t(E) \otimes I_t + I_t \otimes Q(E) -$$

$$x \mapsto \Pi_u(x - o)\big|_t$$

$$Q_{t,1}(E_{t,1}) \otimes I_{t,2} \qquad I_{t,1} \otimes Q_{t,2}(E_{t,2})$$

$$Q_{t,1}(E_{t,1}) \otimes Q_{t,2}(E_{t,2})$$

$\frac{1}{2}$

E_c

t

$$\alpha\, \phi_1 \otimes \phi_2 + \beta\, \psi_1 \otimes \psi_2$$