

LOBSTER: Chemical-Bonding Information from Plane Waves (and Orbitals)

Richard Dronskowski



H₂ within LCAO-MO Theory and Bloch States

Bands, DOS, COHP

Bonding Classics: Tellurium, Iron, Phase-Change Materials

Pseudopotentials & Plane Waves

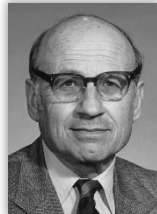
Bonding Information projected from Plane Waves: LOBSTER

Charges and Orbital Mixing (Pb₂Si₅N₈)

Bond Indices (István Mayer's Heritage)

Tellurides and Phase-Change Materials, one more time

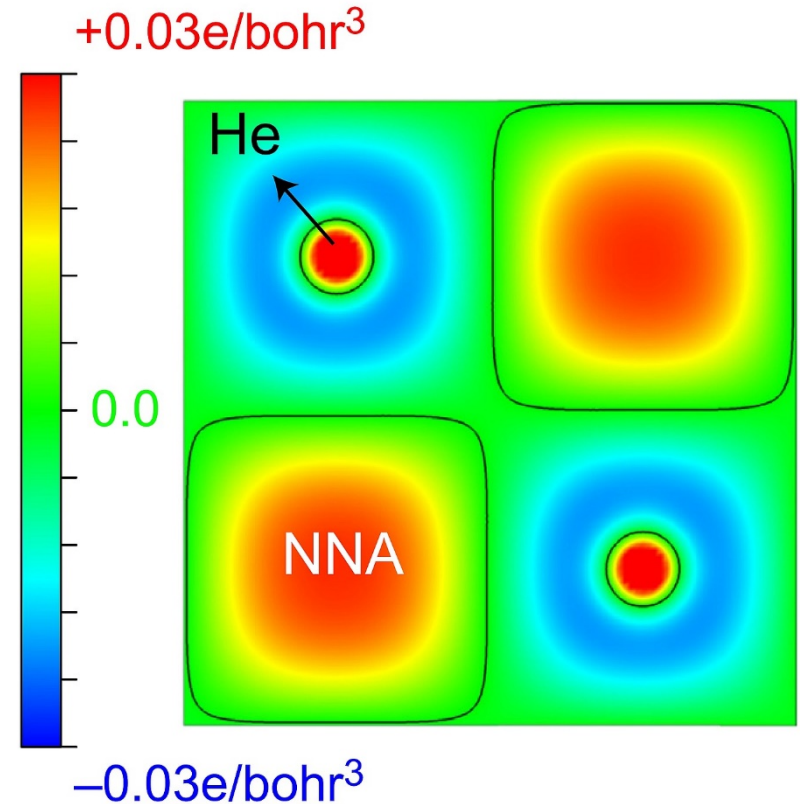
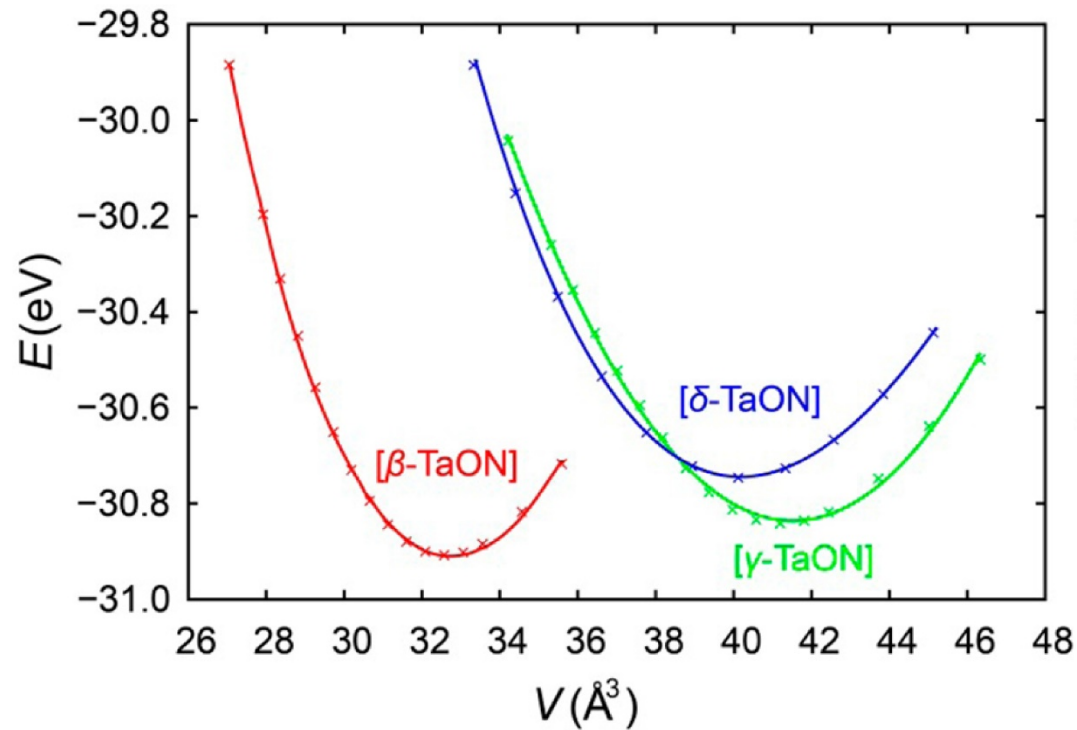
ab initio Computational Materials Science



$$v_{\text{eff}}(\mathbf{r}) = v(\mathbf{r}) + \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' + v_{\text{XC}}(\mathbf{r})$$



Total energy, $\rho(r)$, E vs. V , DOS... is *insufficient*



Total energy, $\rho(r)$, E vs. V , DOS... is *insufficient*



*To **understand** an observable means being able to predict, albeit qualitatively, the result that a perfectly reliable calculation would yield for that observable.*

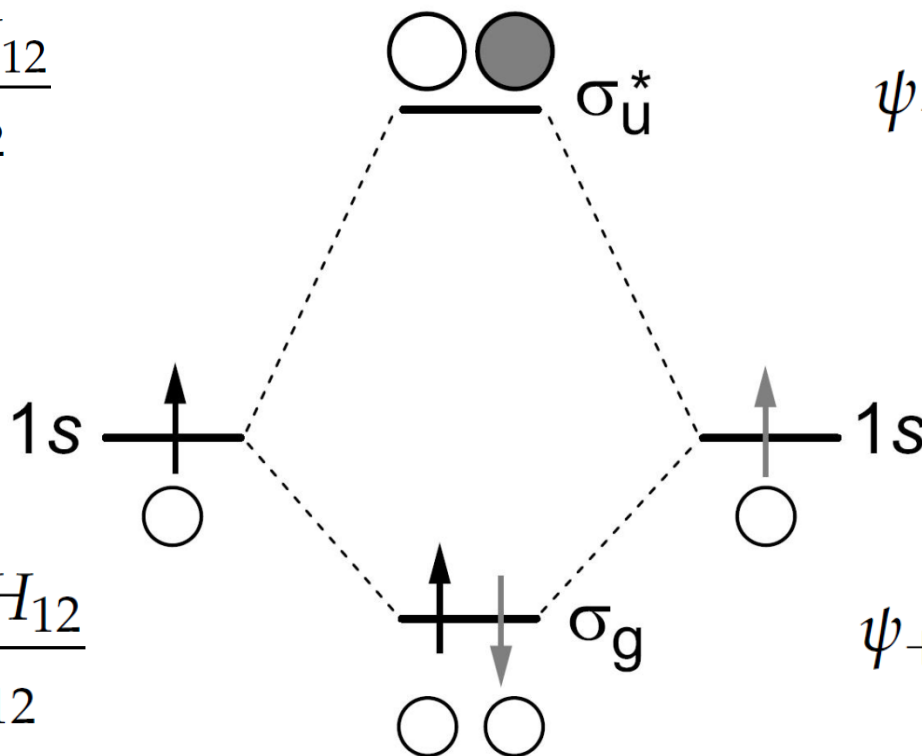
Roald Hoffmann,
Acc. Chem. Res.
1971, 4, 1

LCAO-MO theory of the hydrogen molecule, H₂

$$\psi = c_1\phi_1 + c_2\phi_2$$

$$E_- = \frac{H_{11} - H_{12}}{1 - S_{12}}$$

$$\psi_- = \frac{\phi_1 - \phi_2}{\sqrt{2(1 - S_{12})}}$$

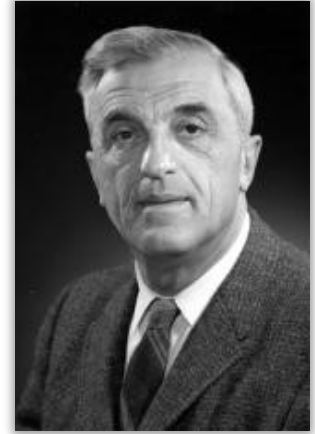


$$E_+ = \frac{H_{11} + H_{12}}{1 + S_{12}}$$

$$\psi_+ = \frac{\phi_1 + \phi_2}{\sqrt{2(1 + S_{12})}}$$

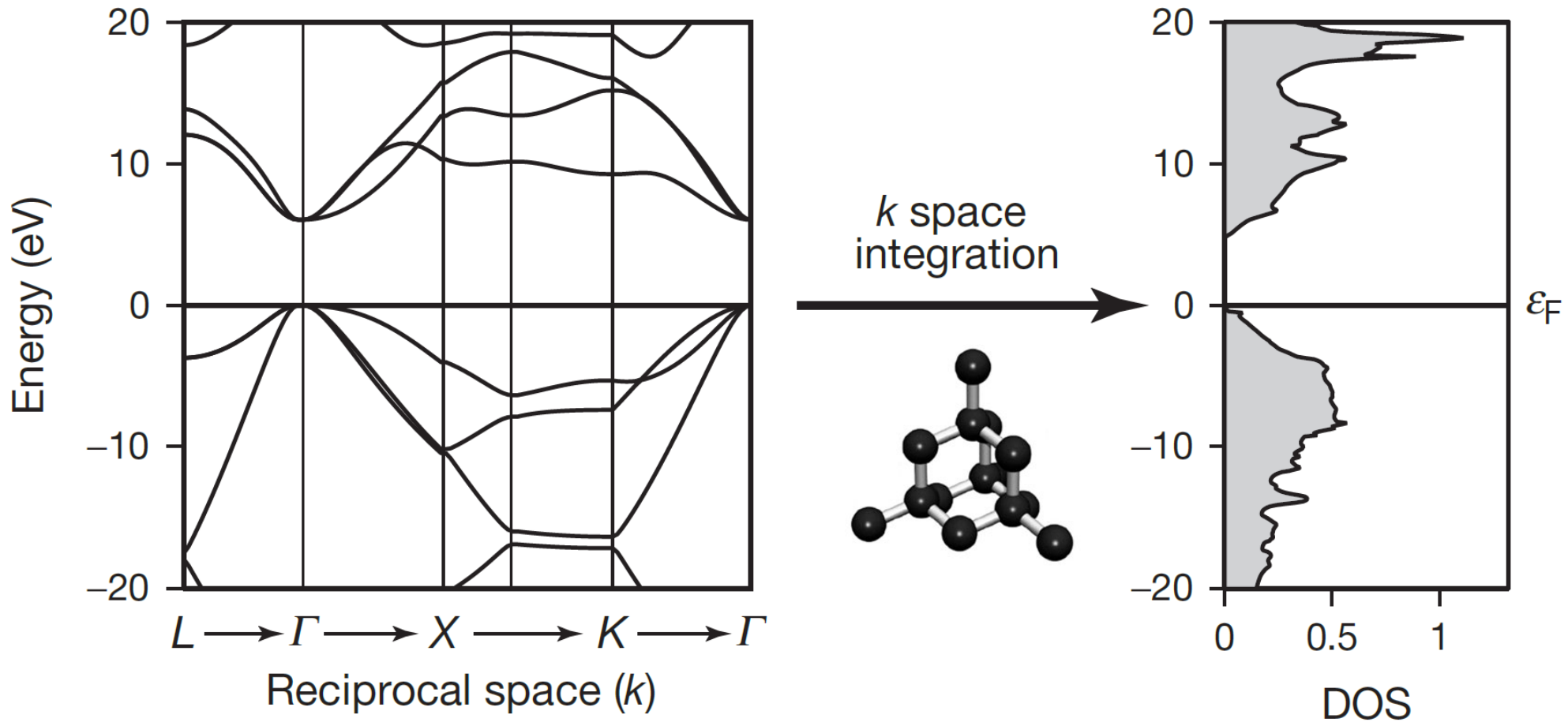
Felix Bloch's famous theorem

$$\psi(\mathbf{k}, \mathbf{r} + \mathbf{T}) = e^{i\mathbf{k}\mathbf{T}} \psi(\mathbf{k}, \mathbf{r})$$



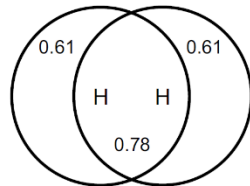
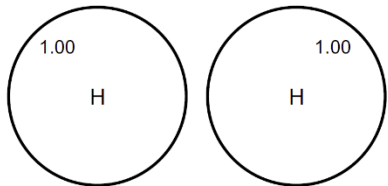
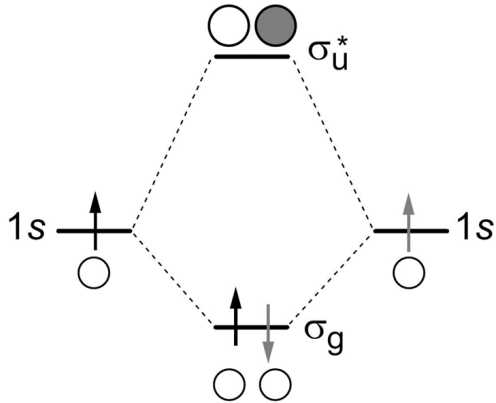
“When I started thinking about the problem, I realized that the main difficulty was to explain how the electrons could pass all the ions in the metal undisturbed [...] To my delight I found by a simple Fourier analysis that the wave differed from a plane wave of a free electron only by a periodic modulation.”

3dim diamond: bands and DOS (real system, DFT)



H₂: Population Analysis by Robert S. Mulliken

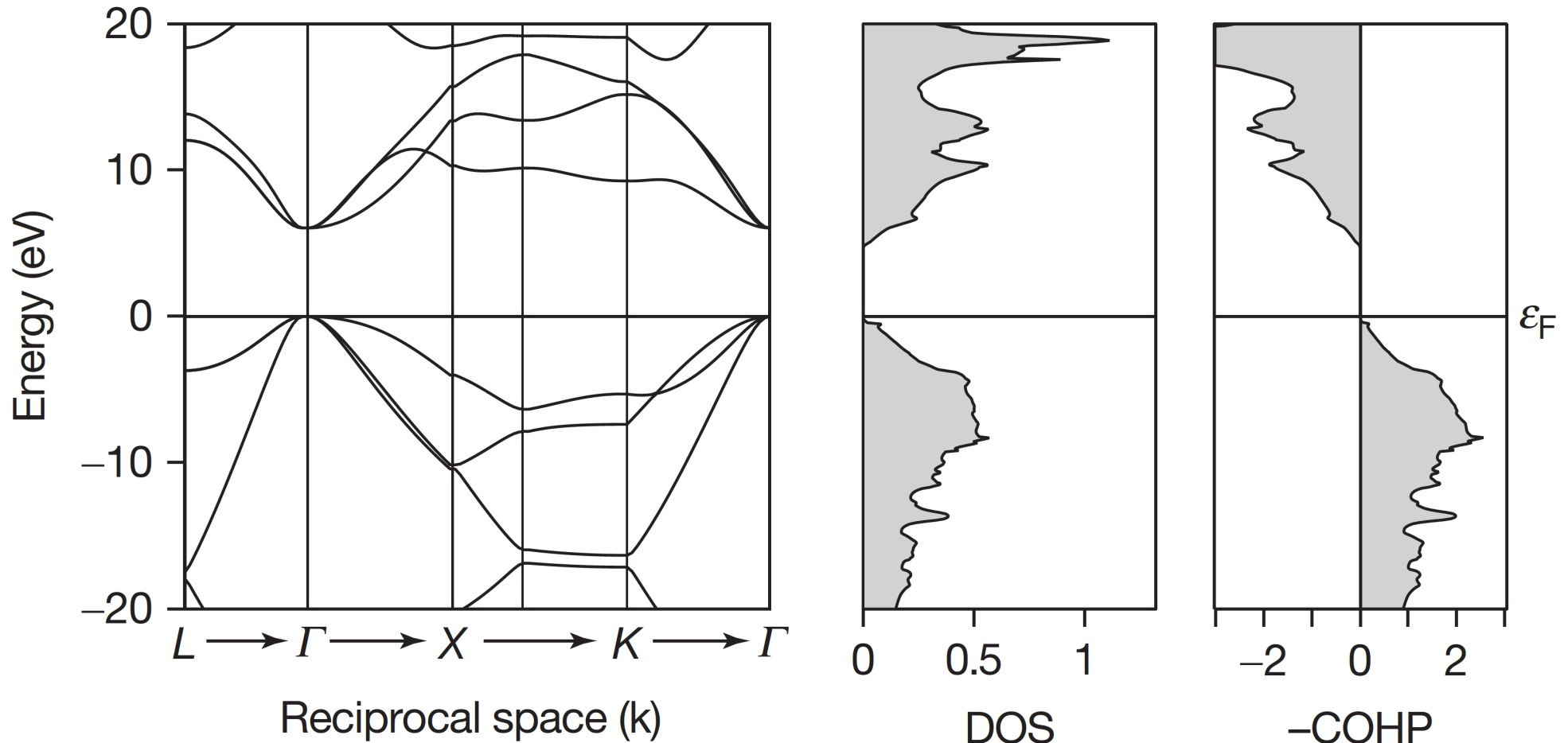
$$\int \psi^* \psi d\tau = \underbrace{\int \psi^2 d\tau}_{\equiv 1} = c_1^2 \underbrace{\int \phi_1^2 d\tau}_{\equiv 1} + c_2^2 \underbrace{\int \phi_2^2 d\tau}_{\equiv 1} + 2c_1c_2 \underbrace{\int \phi_1\phi_2 d\tau}_{S_{12}}$$



R. S. Mulliken,
J. Chem. Phys.
1955, 23, 1833

none of the aforementioned plane-wave electronic-structure codes can do that because *they are all lacking the atomic orbitals*

3dim diamond: band structure, DOS, COHP



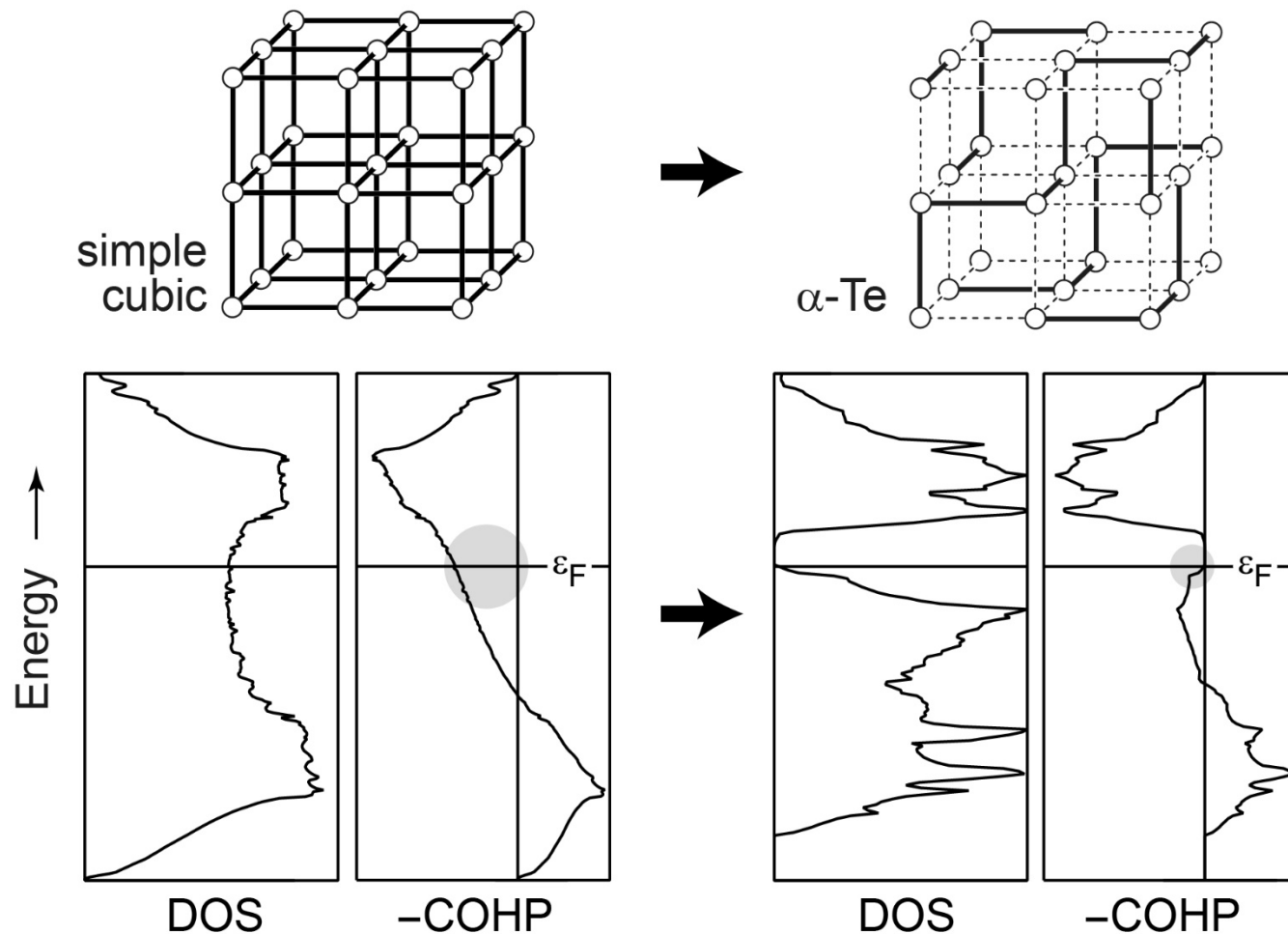
R. Dronskowski, P. E. Blöchl,
J. Phys. Chem. **1993**, 97, 8617

DFT: Crystal Orbital Hamilton Population, COHP

$$E = \underbrace{\int^{\varepsilon_F} \sum_A \sum_{\substack{\mu \\ \mu \in A}} P_{\mu\mu}(E) H_{\mu\mu}(E) dE}_{\text{net atomic energies}} + \underbrace{\int^{\varepsilon_F} 2 \sum_A \sum_{B>A} \sum_{\substack{\mu \\ \mu \in A}} \sum_{\substack{\nu \\ \nu \in B}} \text{Re}[P_{\mu\nu}(E) H_{\mu\nu}(E)] dE}_{\text{bonding energies}}$$

*partitioning the energy, not the electrons,
between the atoms and the bonds...*

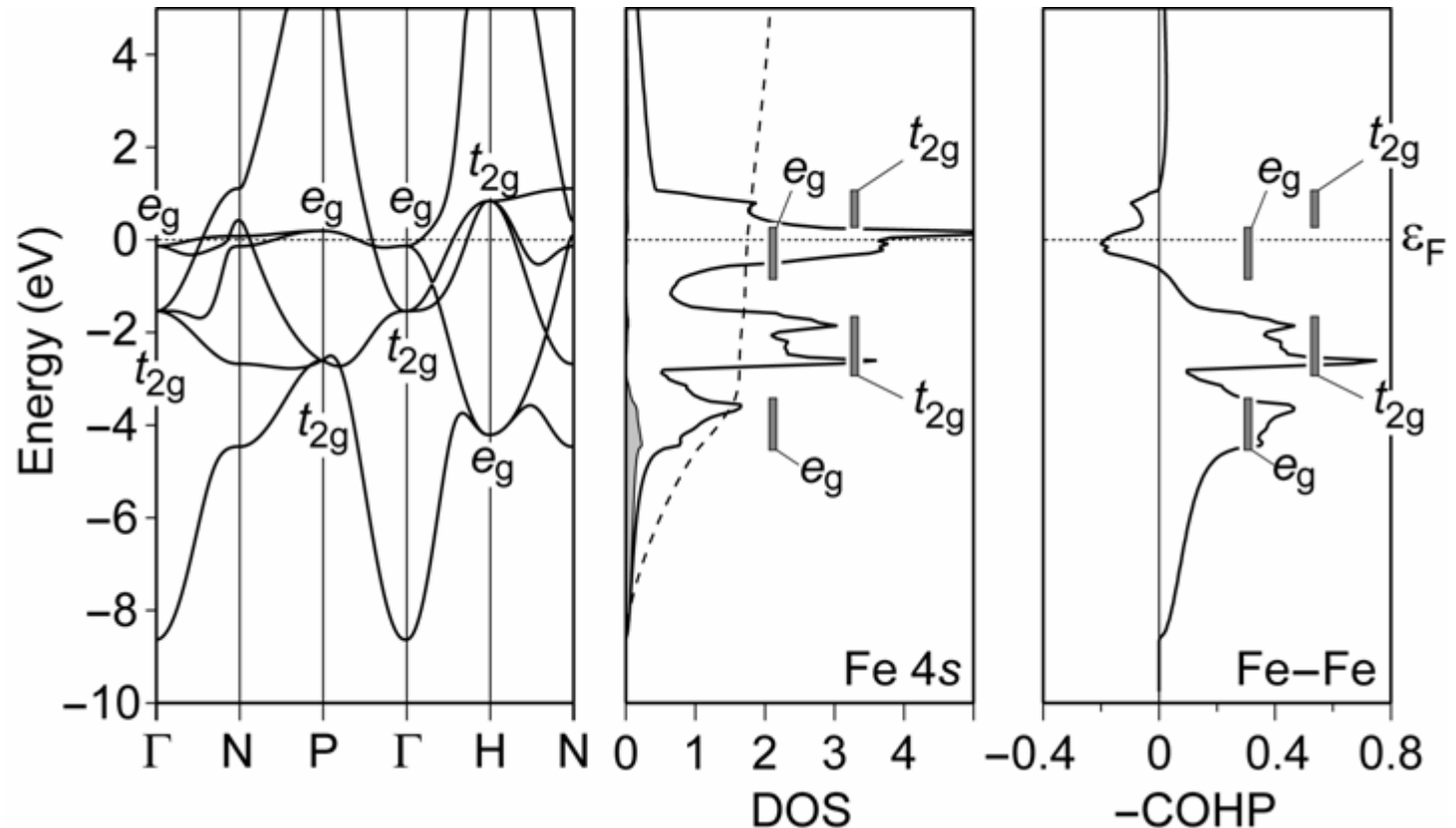
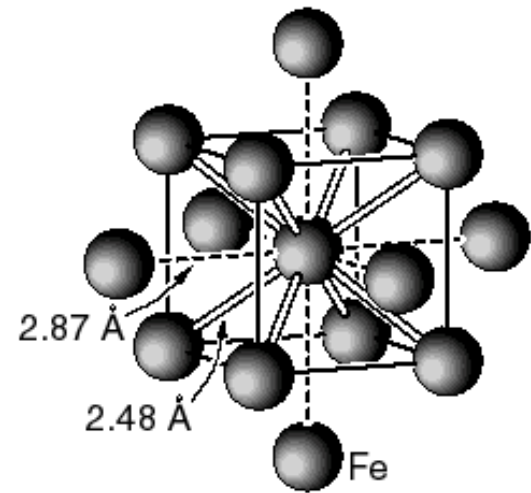
Example I: sc Tellurium is **Peierls-unstable**



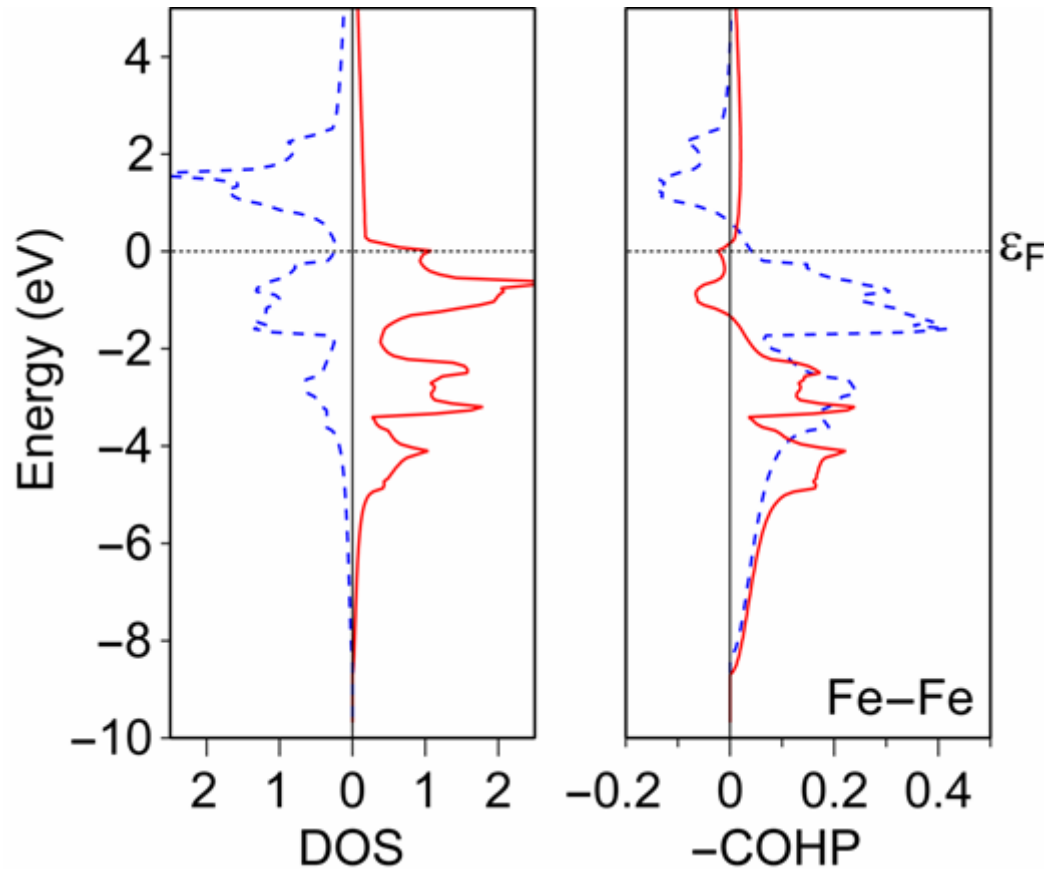
A. Decker, G. A. Landrum, R. Dronskowski,
Z. Anorg. Allg. Chem. **2002**, 628, 295

Example II: body-centered cubic Fe

and its corresponding non-realistic LDA band structure without spin-polarization, on purpose:



Spin Polarization: Chemical Bonding



magnetic moment: $2.27 \mu_B$
(exp.: $2.21 \mu_B$)

majority spin orbitals *contract*

minority spin orbitals *expand*

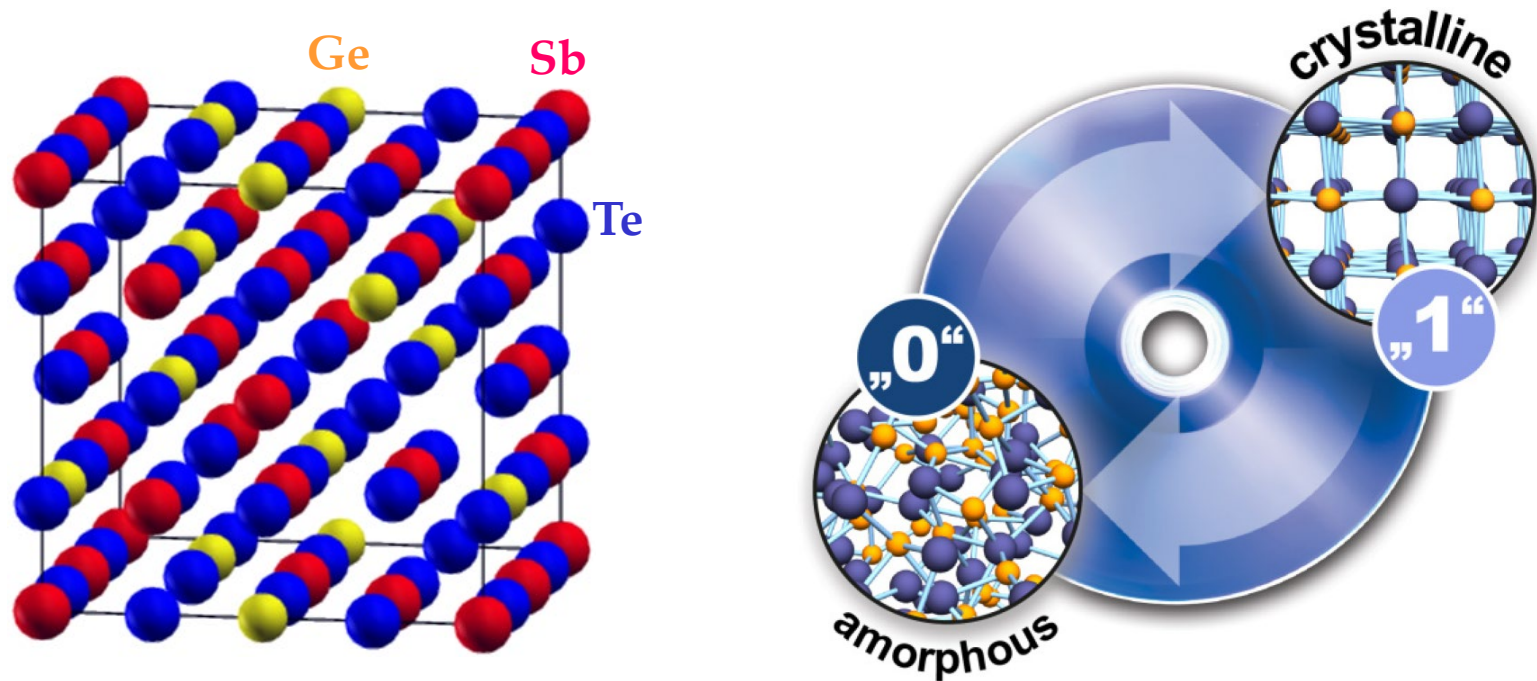
lowering of total energy
by about 0.43 eV

*strengthening of iron-iron
bonding by about 5%*

minority spins roughly twice
as strongly bonding as
majority spins

G. A. Landrum, R. Dronskowski,
Angew. Chem. Int. Ed. **2000**, 39, 1560

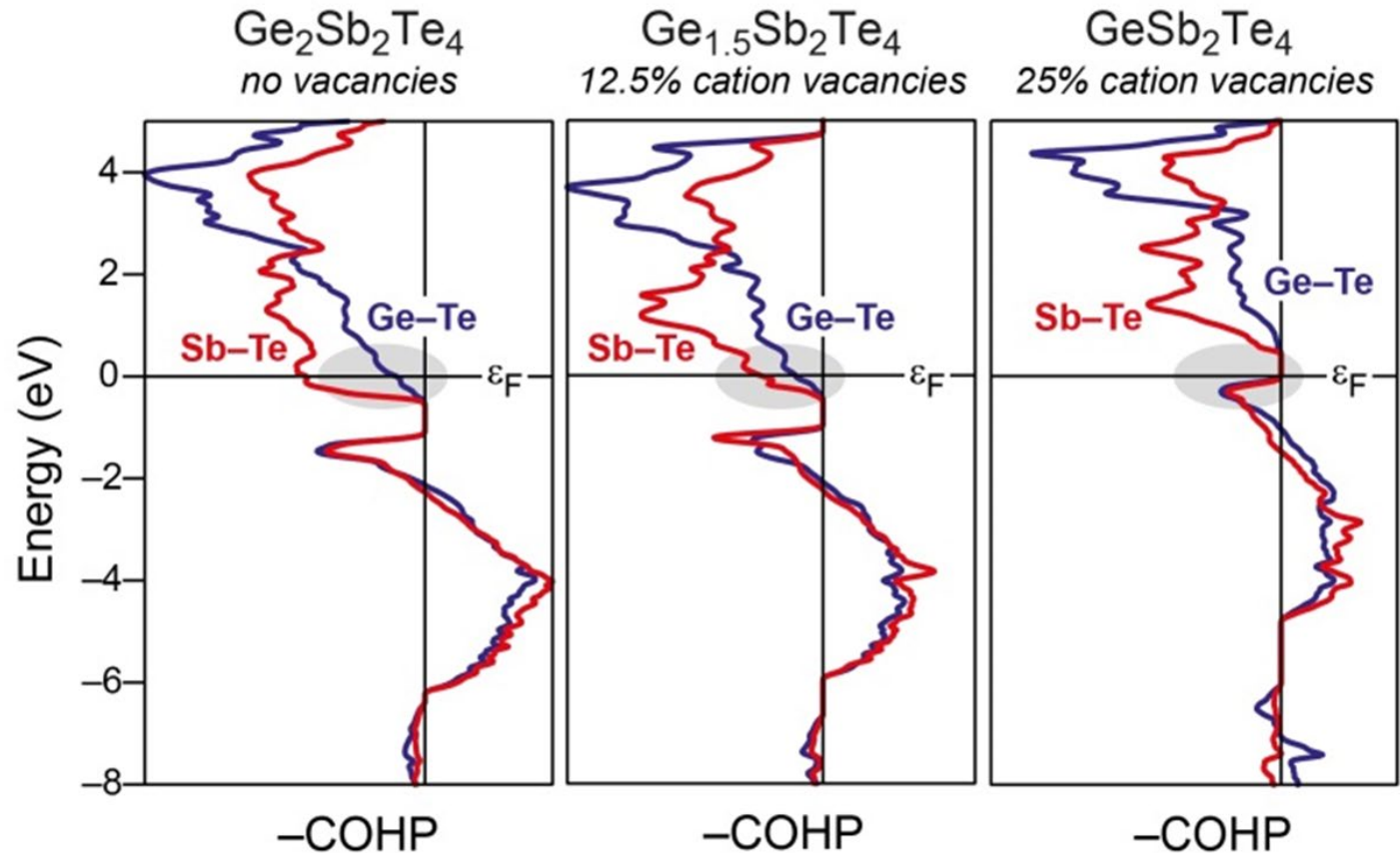
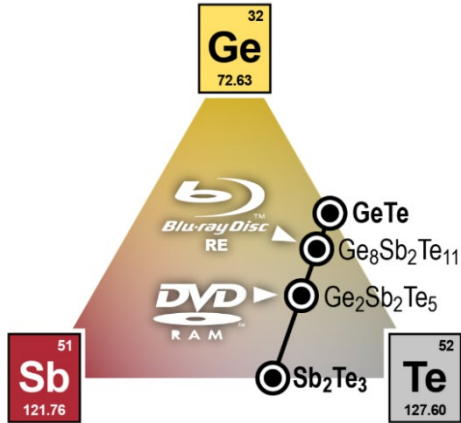
Example III: Ge–Sb–Te Phase-change Materials



e.g., $\text{Ge}_2\text{Sb}_2\text{Te}_4$ with lots of Ge vacancies ($\approx 20\%$) – **why?**

switching mechanism = $f(\text{vacancy nature})$

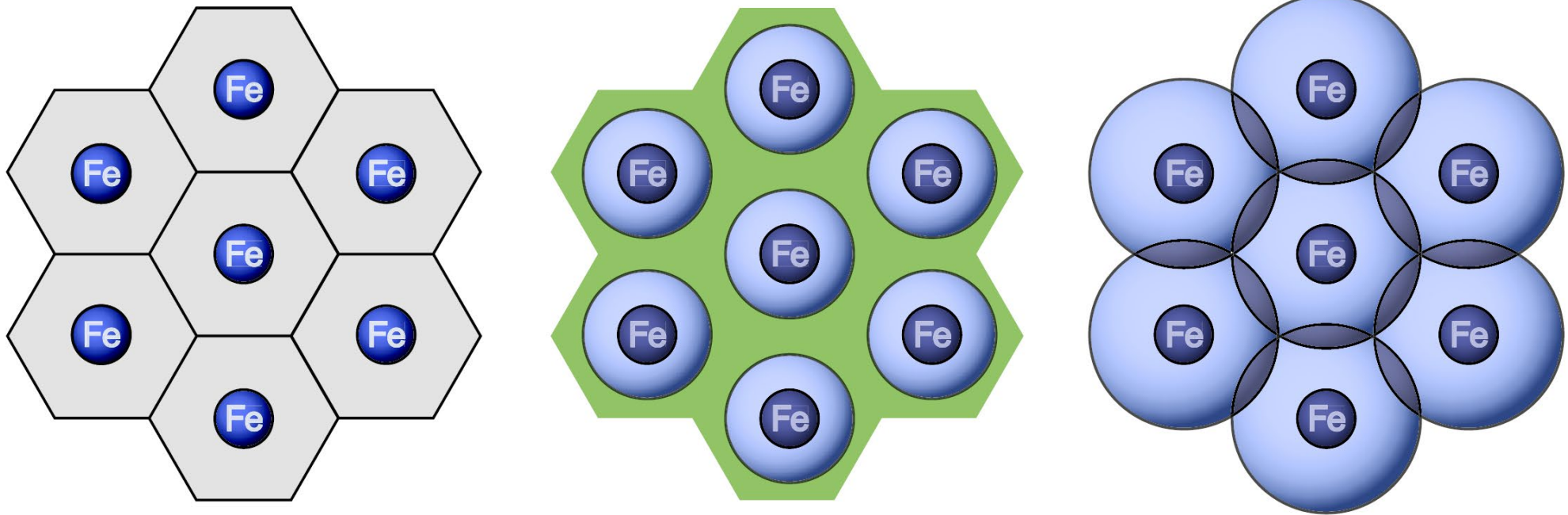
Phase-change Materials: first COHP study



antibonding Ge-Te and Sb-Te interactions in the highest bands;
germanium vacancies annihilate antibonding states

M. Wuttig, D. Lüsebrink, D. Wamwangi, W. Wełnic,
M. Gilleßen, R. Dronskowski, *Nature Mater.* **2007**, 6, 122

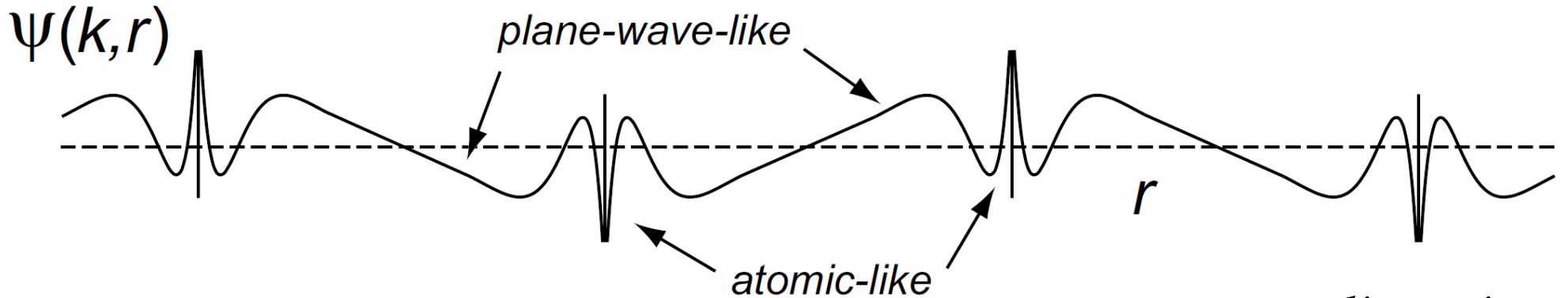
Wigner–Seitz, Muffin-Tin, **Atomic Spheres** → LMTO



*Tight-Binding Linear-Muffin-Tin Orbitals
using the Atomic-Spheres-Approximation (ASA)*

*probably the most influential (among the chemists)
periodic **orbital-based** electronic-structure method*

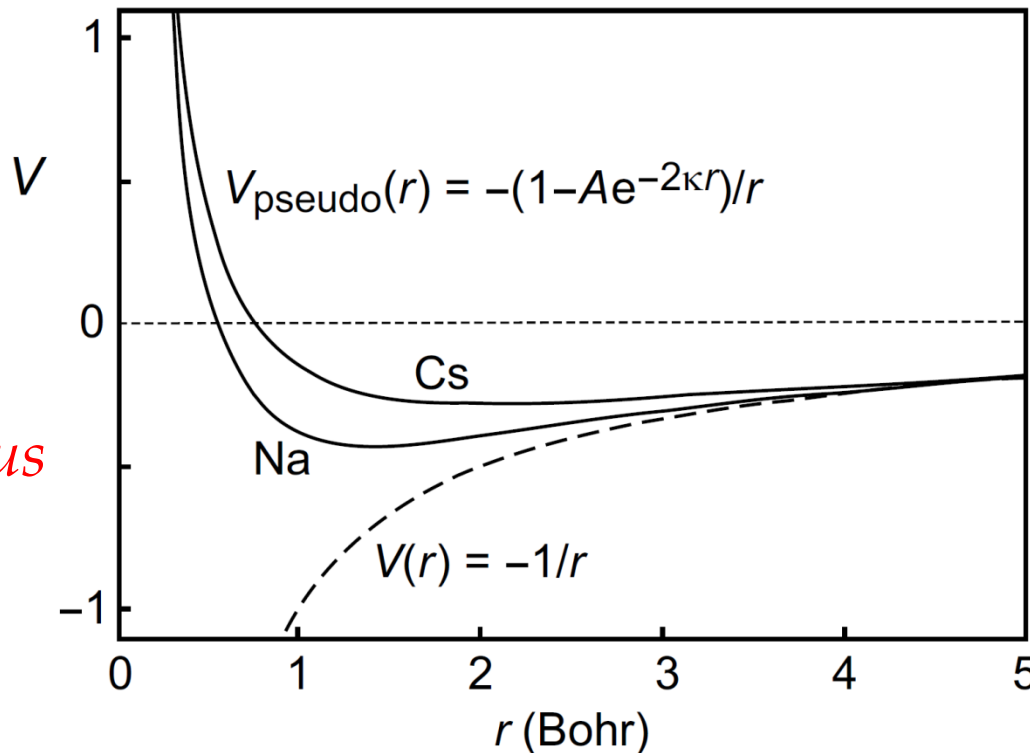
Hans Hellmann, *J. Chem. Phys.* 1935, 3, 61



*one-dimensional
Na-3s Bloch
function at X*

*the world's first
pseudopotentials:
Na and Cs*

*→ plane waves, thus
reliable forces!*



1933: Emigration to the Soviet Union

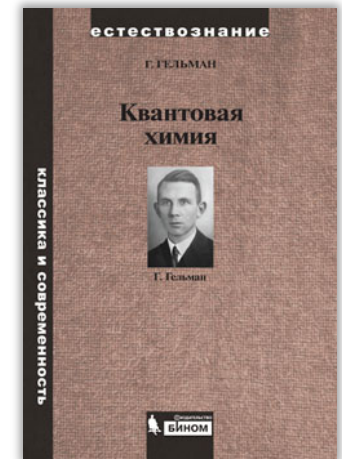
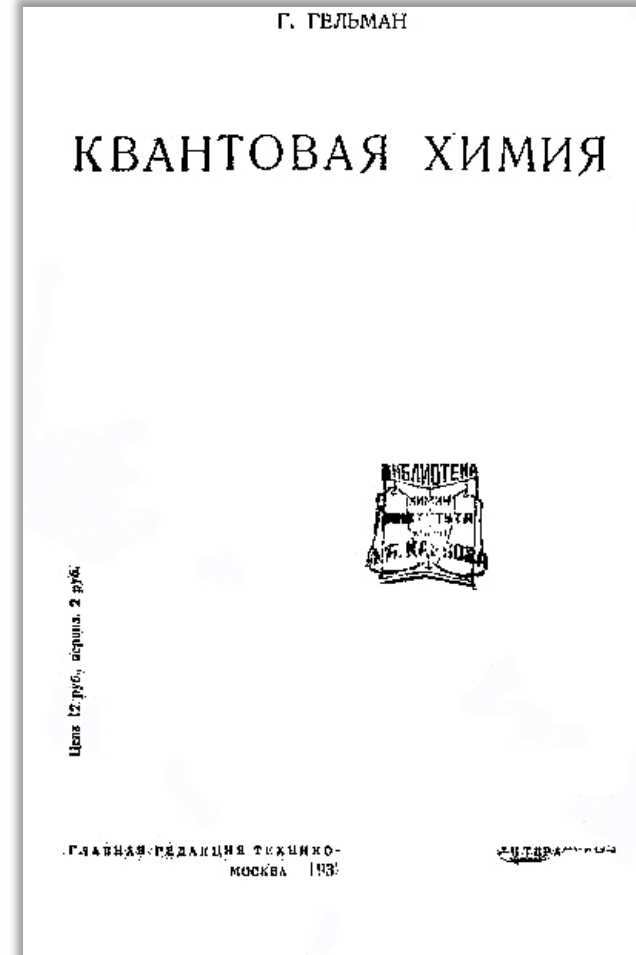
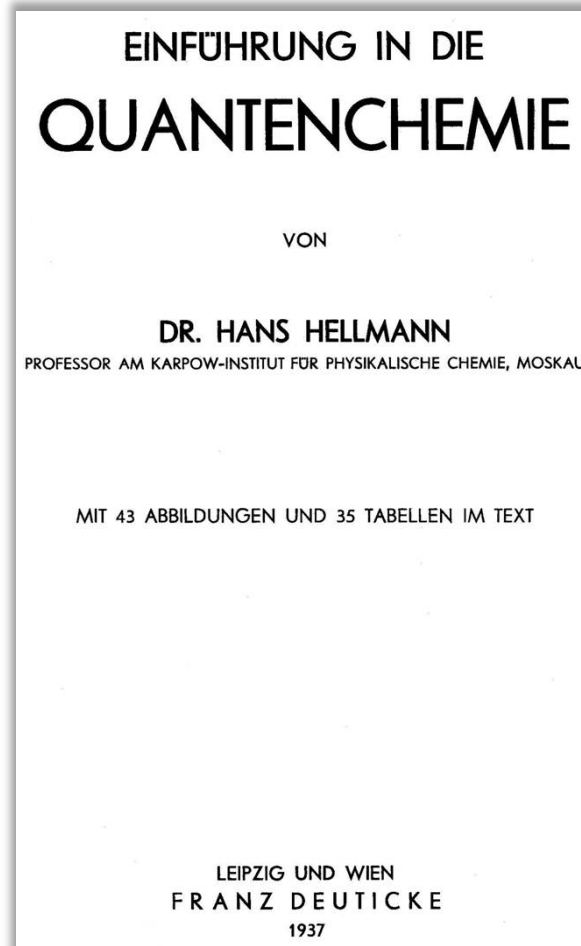


Greta

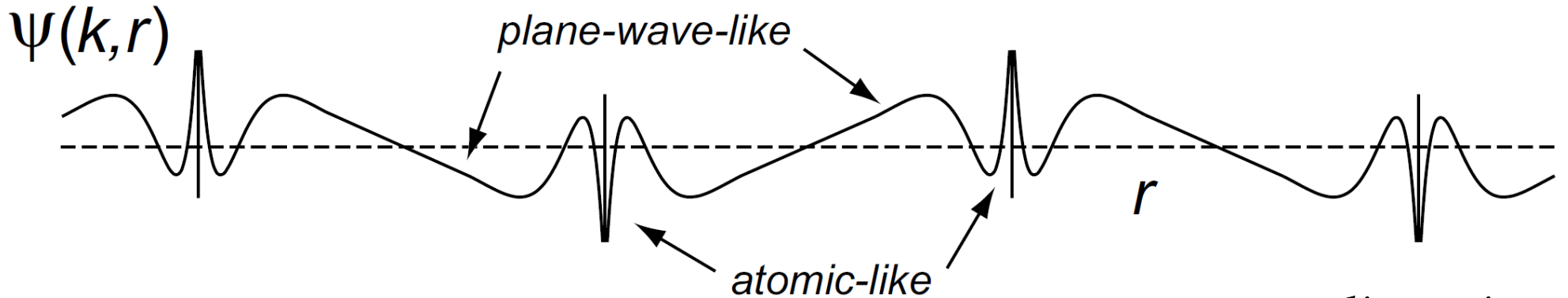
Hermine

Hans

The World's first Quantum-Chemistry Textbooks



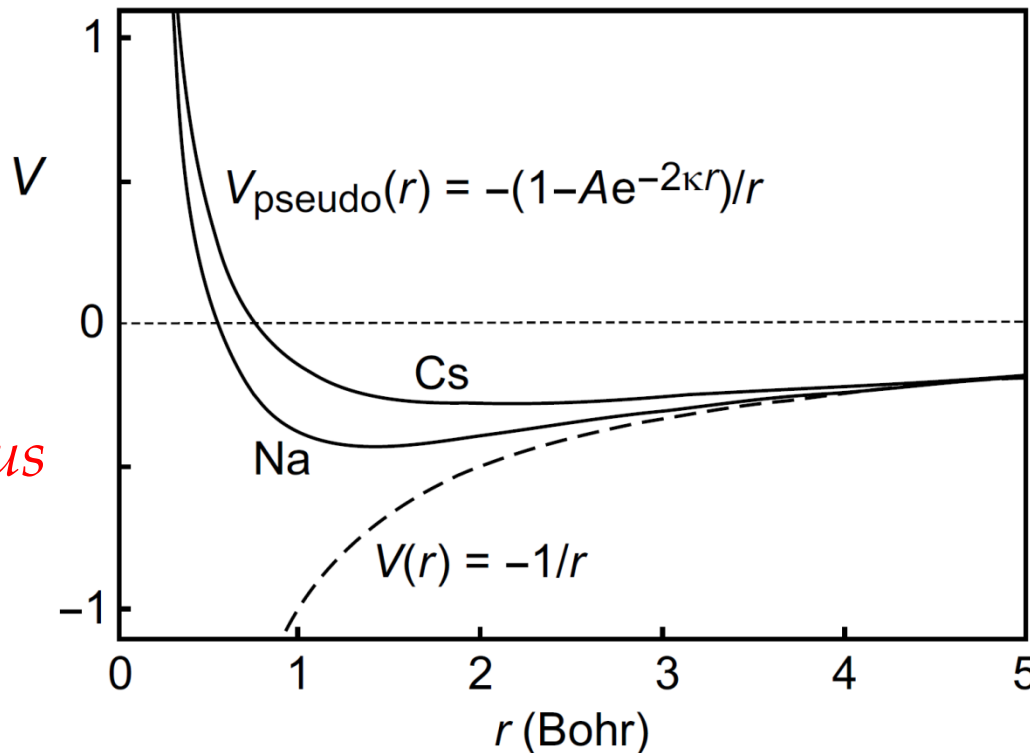
Hans Hellmann, *J. Chem. Phys.* 1935, 3, 61



*one-dimensional
Na-3s Bloch
function at X*

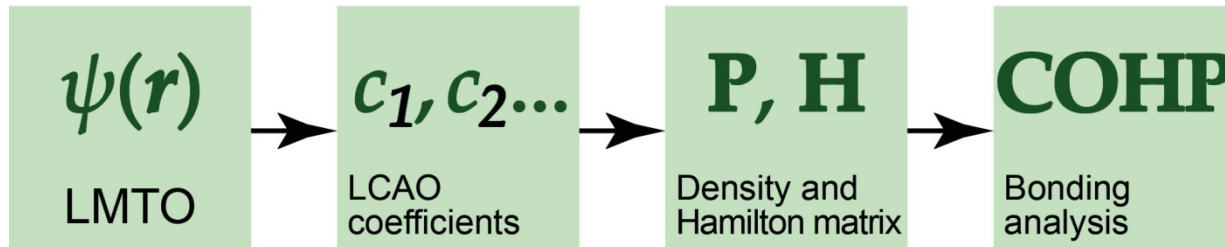
*the world's first
pseudopotentials:
Na and Cs*

*→ plane waves, thus
reliable forces!*

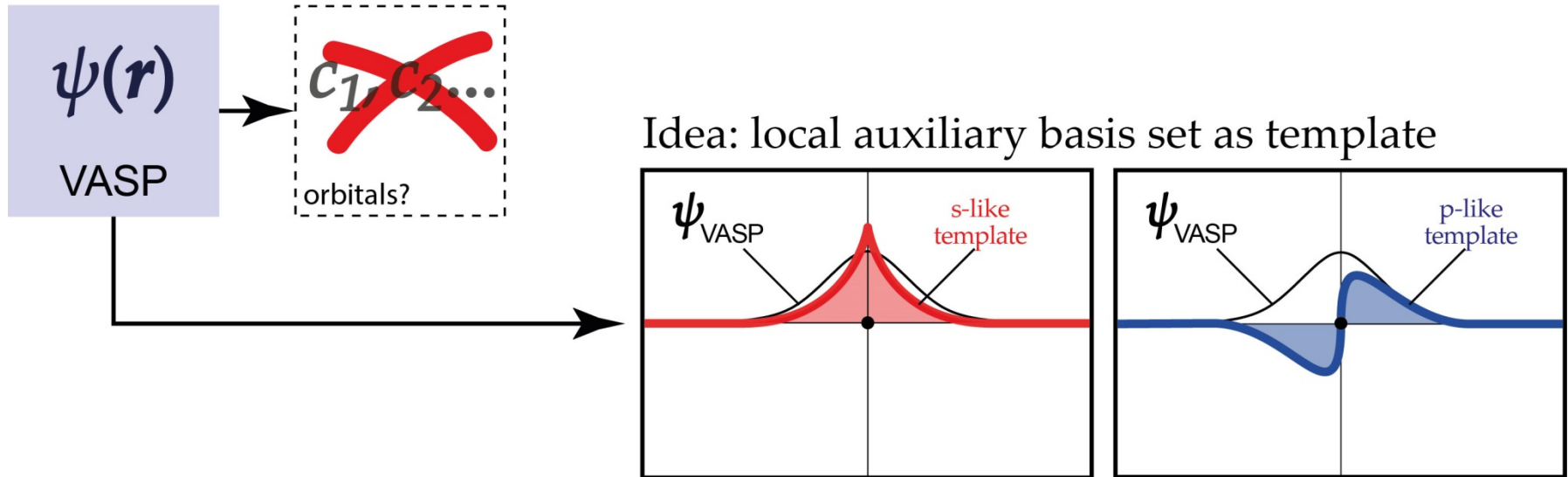


Retrieving the Chemistry

Traditionally: Tight-Binding LMTO-ASA (= densely packed atomic spheres)



Modern: countless program packages with plane waves (e.g., VASP)



LOBSTER transforms plane-wave DFT data

PAW Output

$$|\psi_i\rangle = [1 + \sum_{\mu} (|\phi_{\mu}\rangle - |\tilde{\phi}_{\mu}\rangle) \langle \tilde{p}_{\mu}|] |\tilde{\psi}_i\rangle; \varepsilon_i$$

VASP, QE, ABINIT

$$|\phi_{\mu}\rangle, |\tilde{\phi}_{\mu}\rangle, |\tilde{p}_{\mu}\rangle, |\tilde{\psi}_i\rangle, \varepsilon_i$$



LOBSTER

Local-Orbital Basis Suite Towards
Electronic-Structure Reconstruction

Slater-type orbitals

$$|\chi_{\mu}\rangle$$

(Bunge, Koga, pbeVaspFit2015)

Projection

$$T_{\mu j}(\mathbf{k}) = \langle \chi_{\mu}(\mathbf{k}) | \psi_i(\mathbf{k}) \rangle$$
$$= \sum_{\nu} S_{\mu\nu}(\mathbf{k}) C_{\mu j}(\mathbf{k})$$

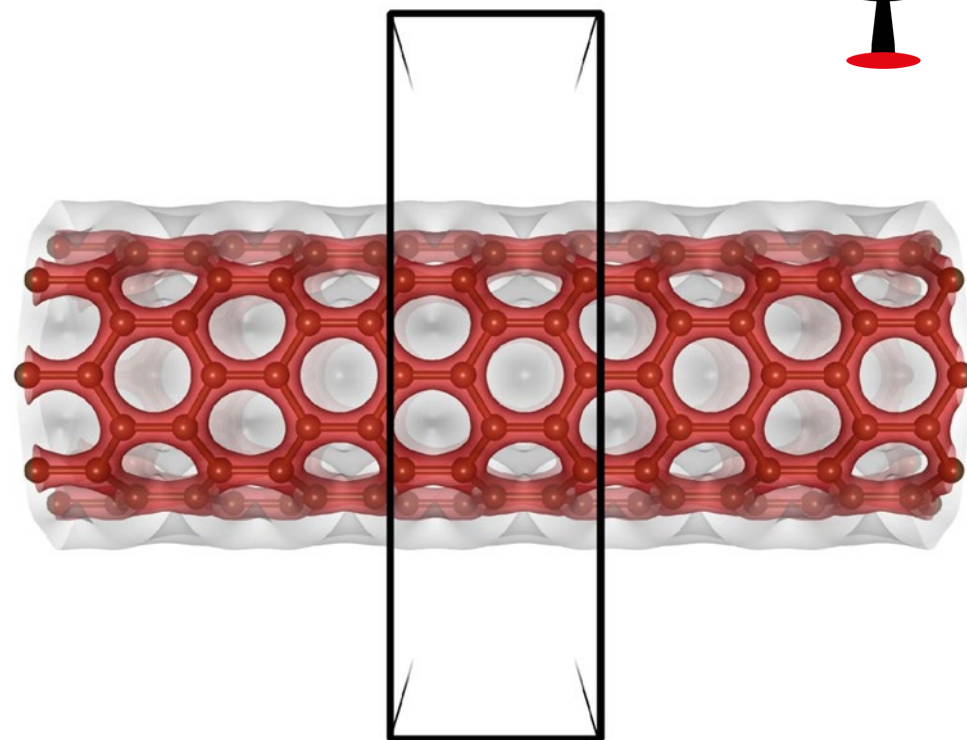
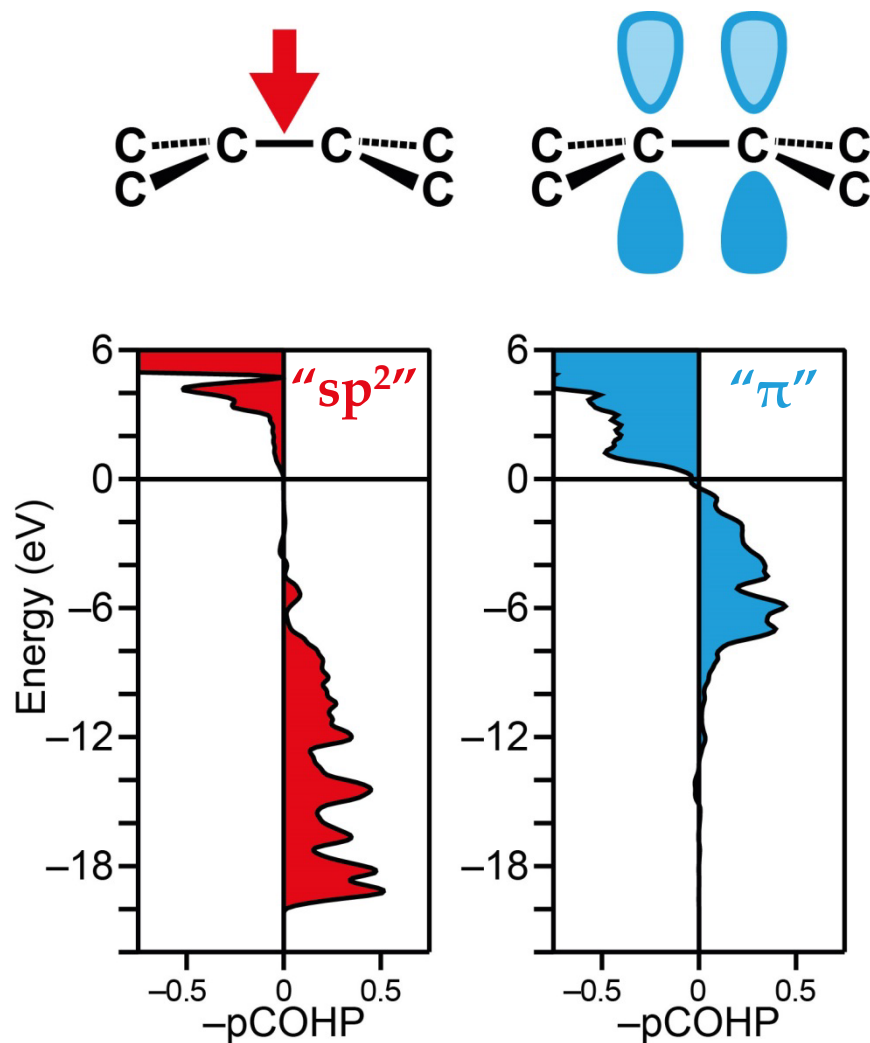
Chemical Bonding Analysis

$$|\psi_i\rangle = \sum_{\mu} C_{\mu j}(\mathbf{k}) |\chi_{\mu}\rangle$$

$C(\mathbf{k})$: pDOS, pCOOP, pCOHP, DOE, charge

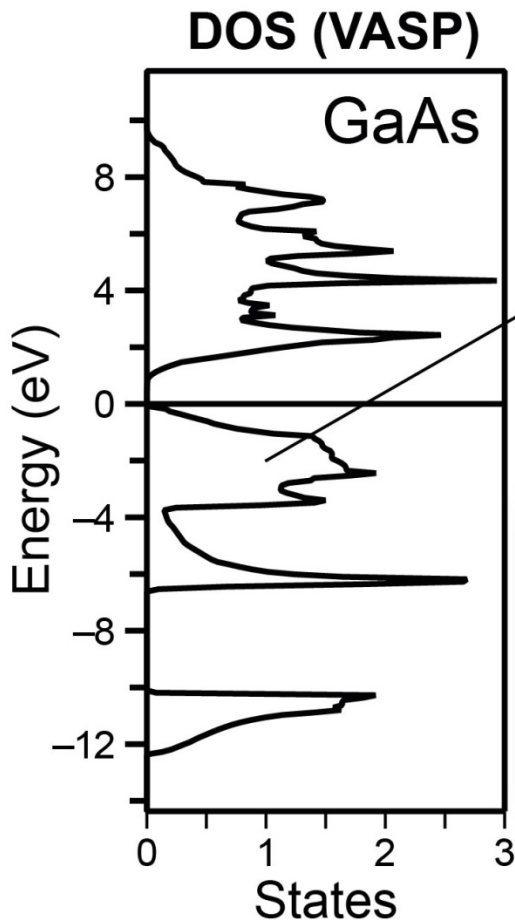
www.cohp.de

Chemical Bonding in the Carbon Nanotube

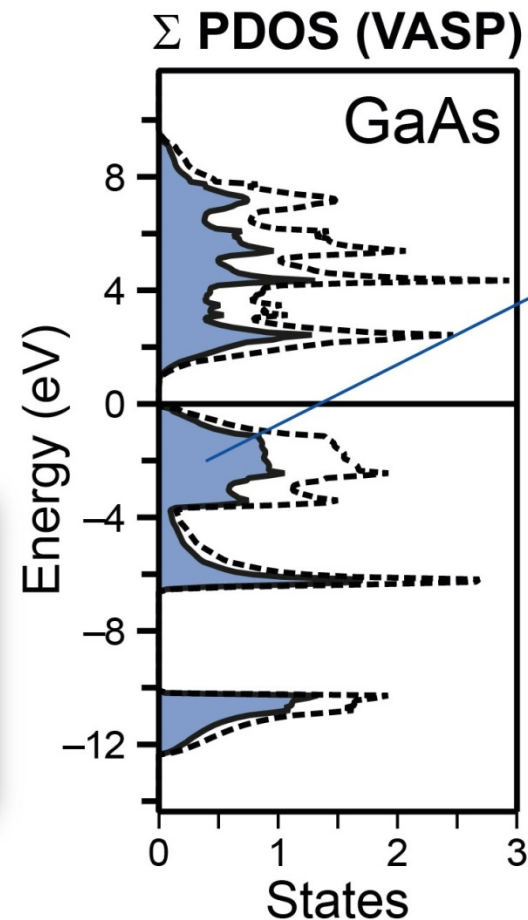
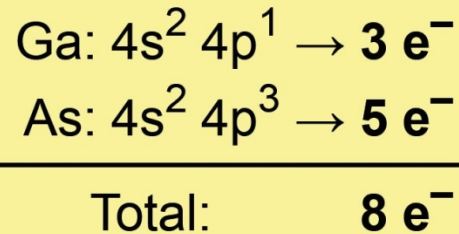


S. Maintz, V. L. Deringer,
A. L. Tchougréeff, R. Dronskowski,
J. Comput. Chem. **2016**, *37*, 1030

GaAs: **fast & incorrect** local DOS projections



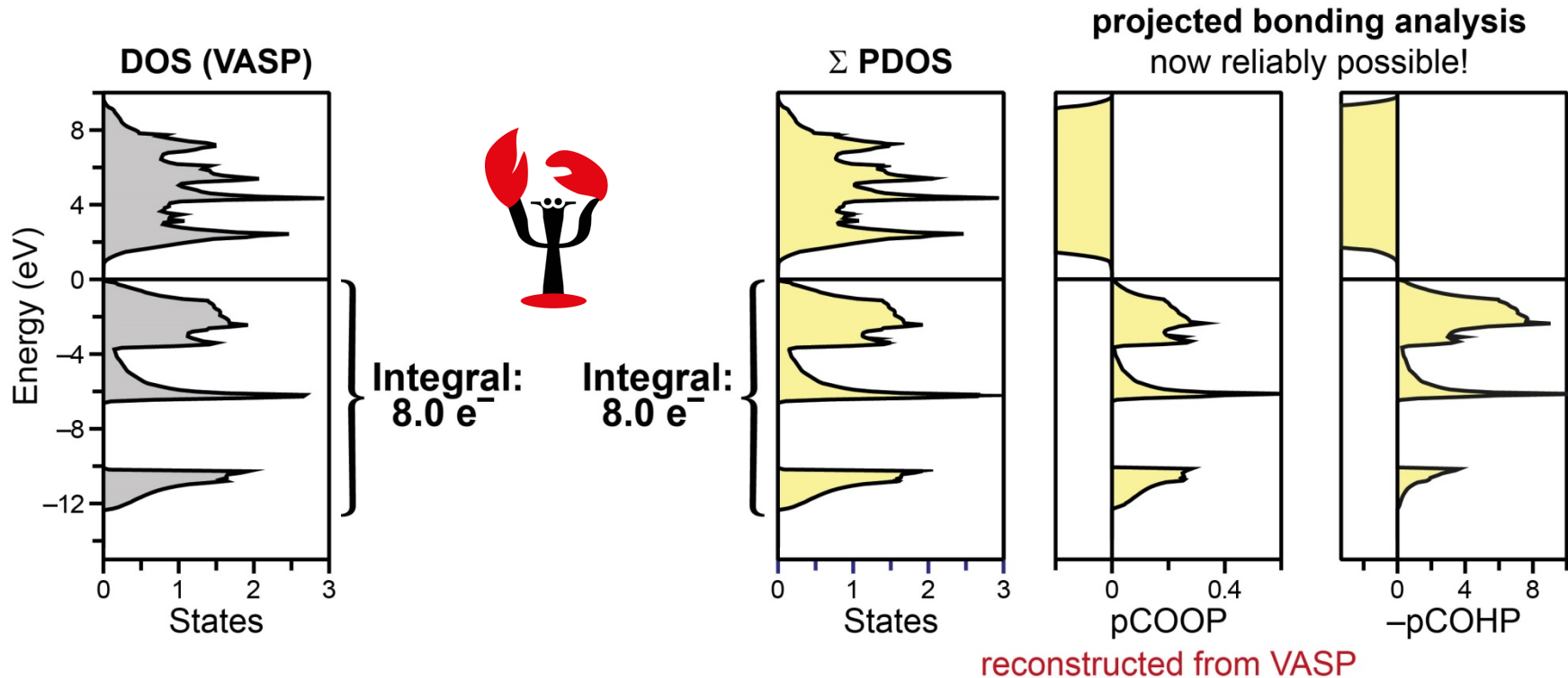
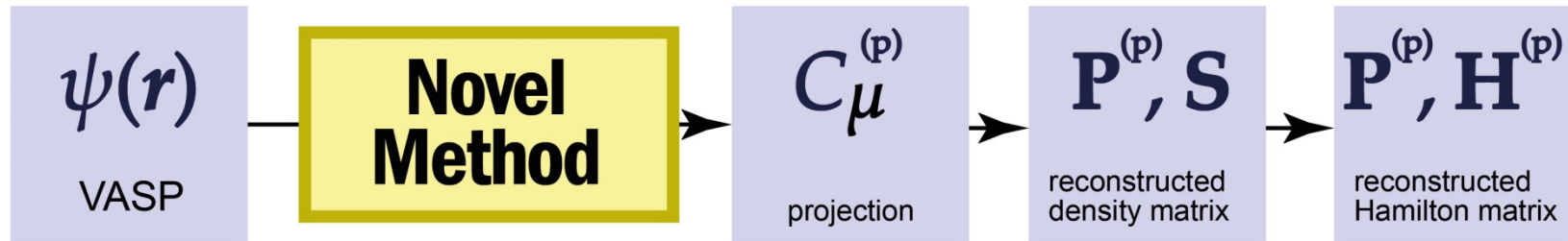
Integral:
8.0 electrons



Integral:
4.8 electrons
40% lost!

*within VASP, the local DOS almost **never** add up to the total DOS due to the chosen algorithm*

Correct Local Projection for GaAs using **LOBSTER**



How to calculate **atomic charges**



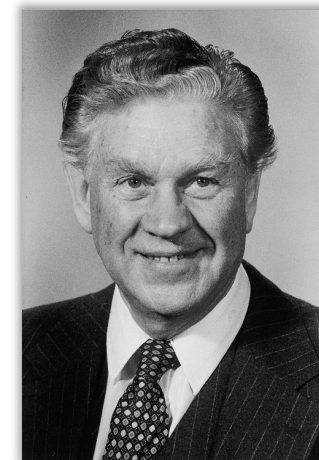
Bader

from the density

$$\rho(\mathbf{r}) = \sum_{i=1}^N f_i |\psi_i(\mathbf{x})|^2$$



Mulliken

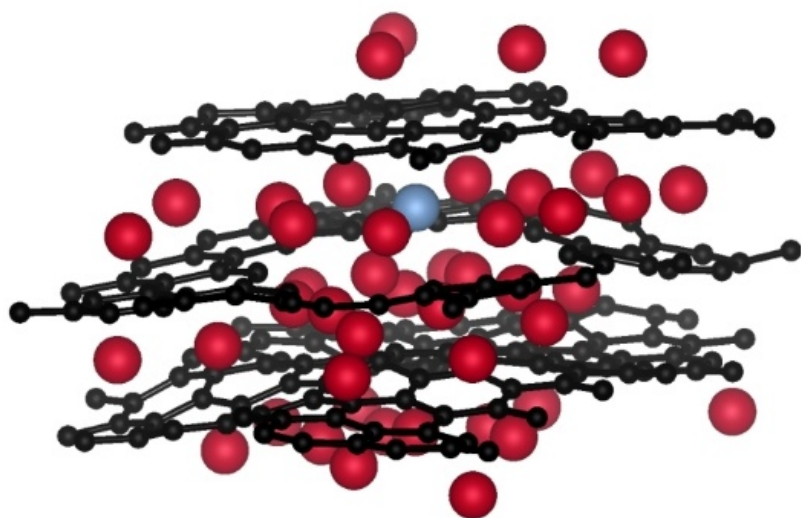


Löwdin

from the wave function (orbitals)

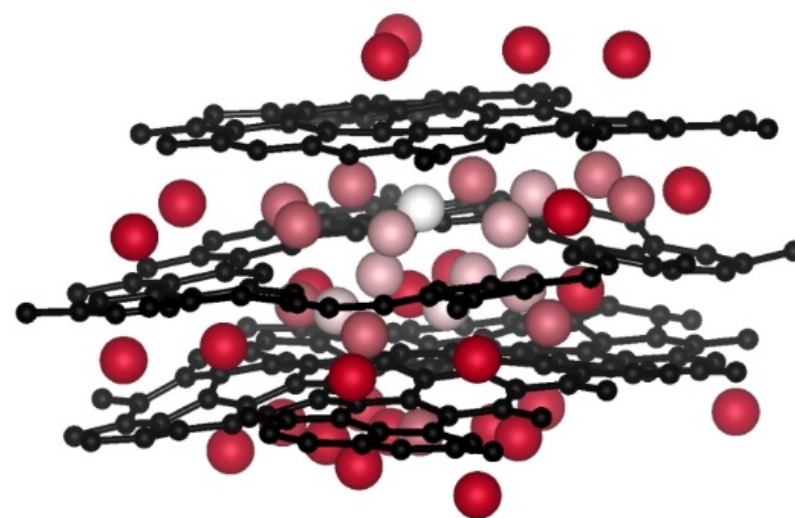
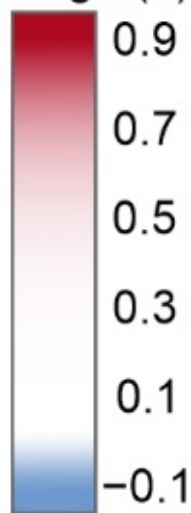
$$\psi_i(\mathbf{x})$$

Amorphous $\text{Li}_{48}\text{C}_{216}$ – Bader vs Löwdin



Bader charge

charge (e)

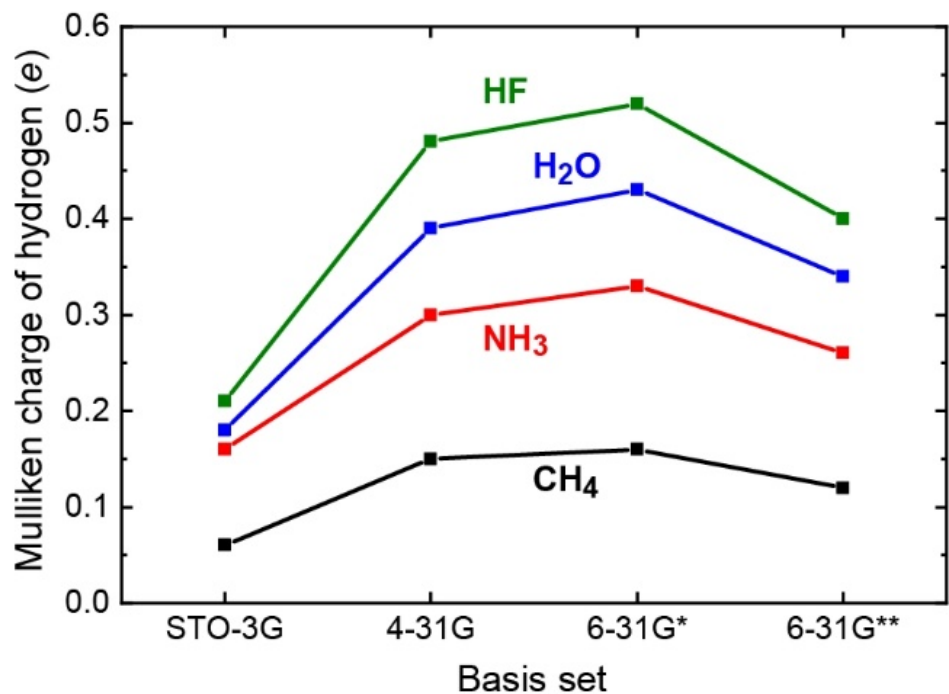


Löwdin charge

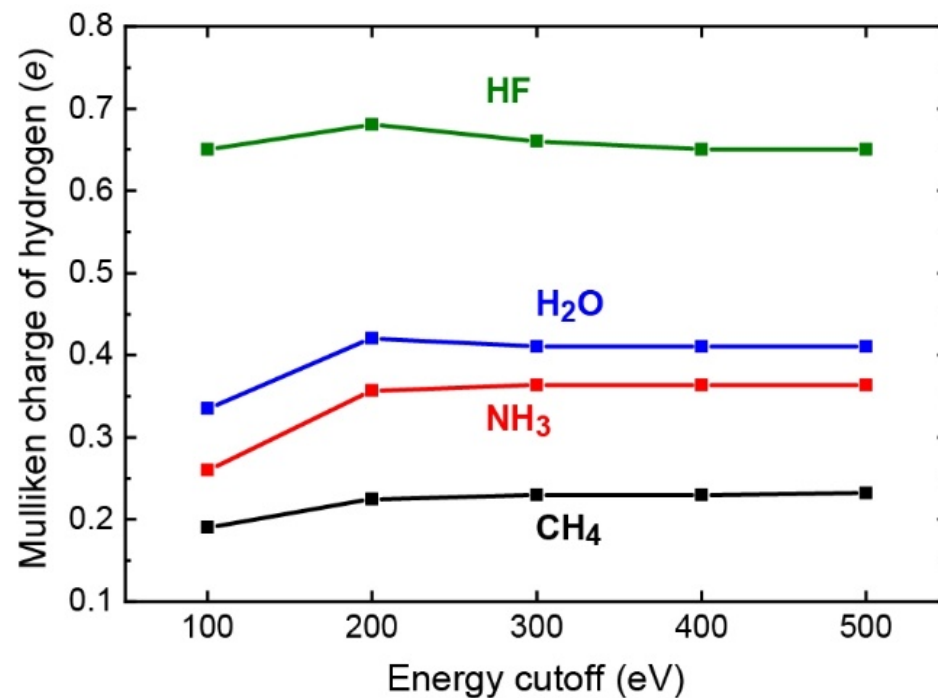
J.-X. Huang, G. Csányi, J.-B. Zhao,
J. Cheng, V. L. Deringer,
J. Mater. Chem. A **2019**, *7*, 19070

R. Nelson, C. Ertural, J. George,
V. L. Deringer, G. Hautier, R. Dronskowski,
J. Comput. Chem. **2020**, *41*, 1931

No basis-set dependency from plane waves



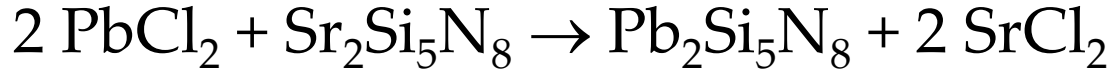
local basis sets (HF)



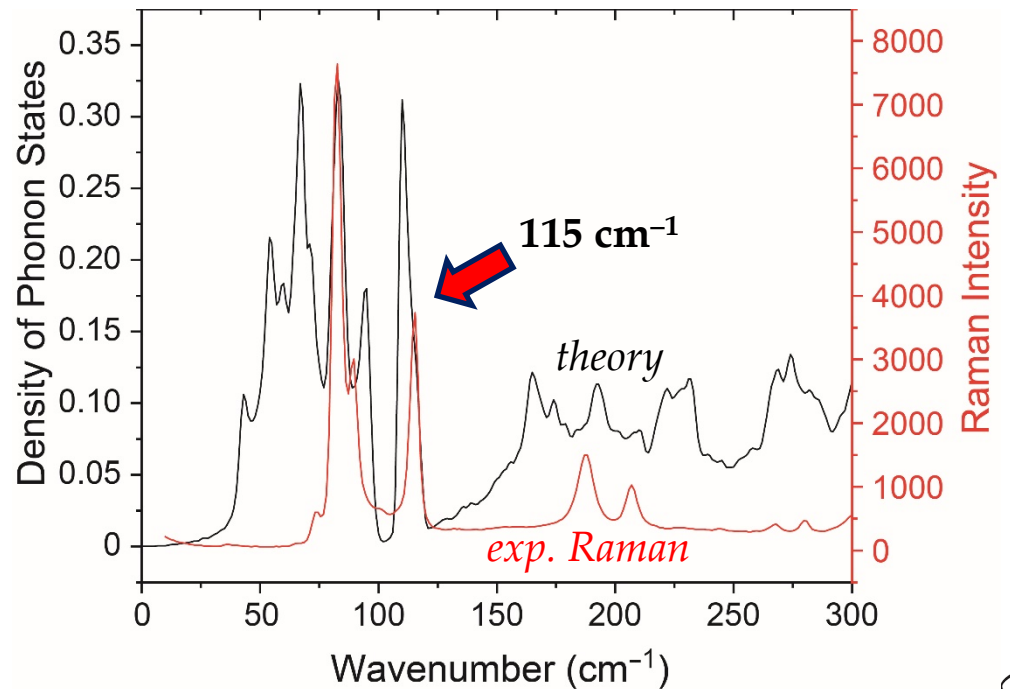
plane waves (LDA)

C. Ertural, S. Steinberg, R. Dronskowski,
RSC Advances 2019, 9, 29821

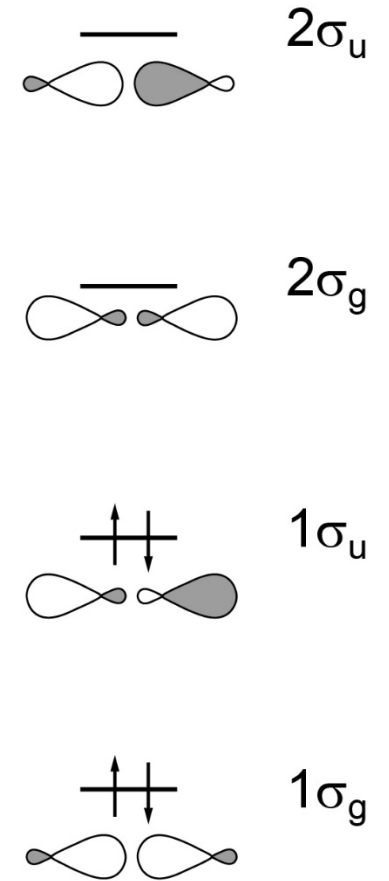
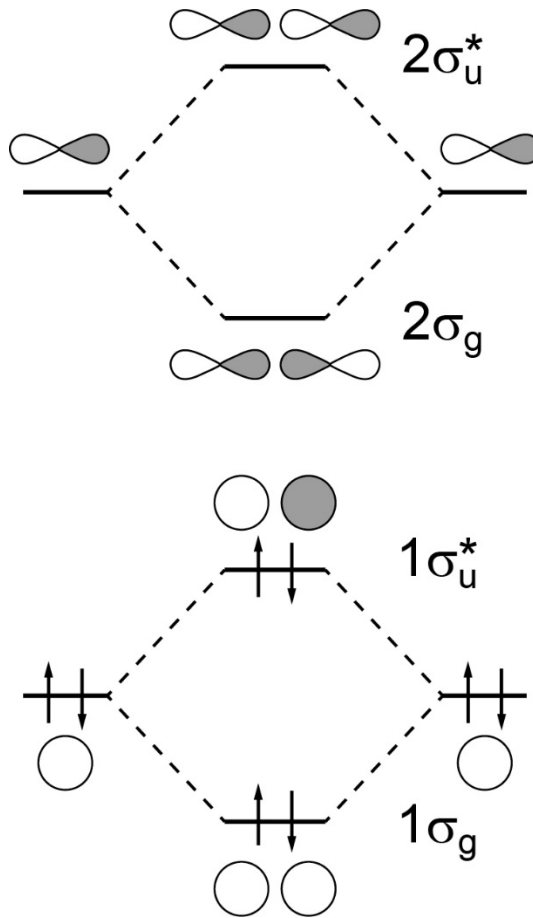
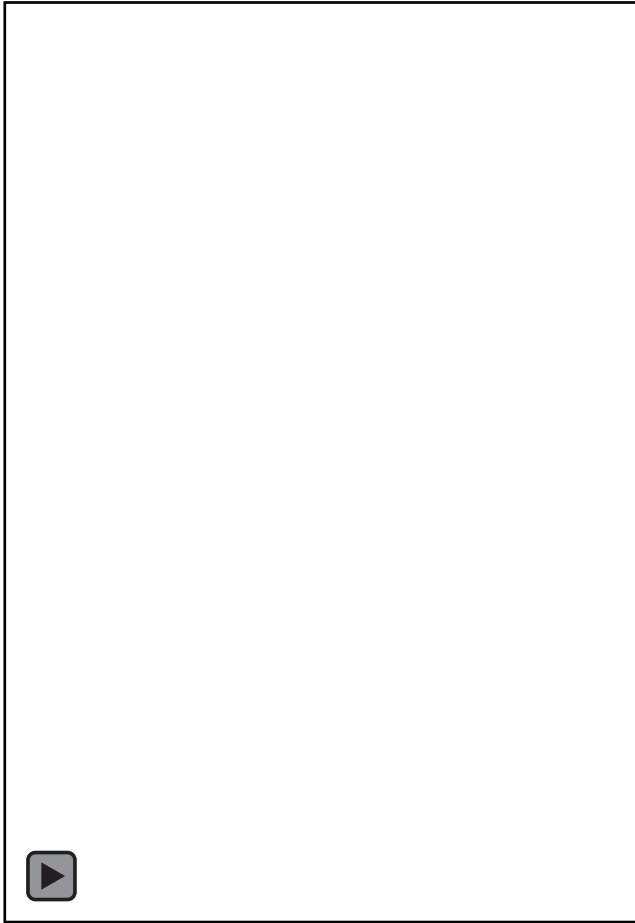
Pb₂Si₅N₈ from Schnick's group



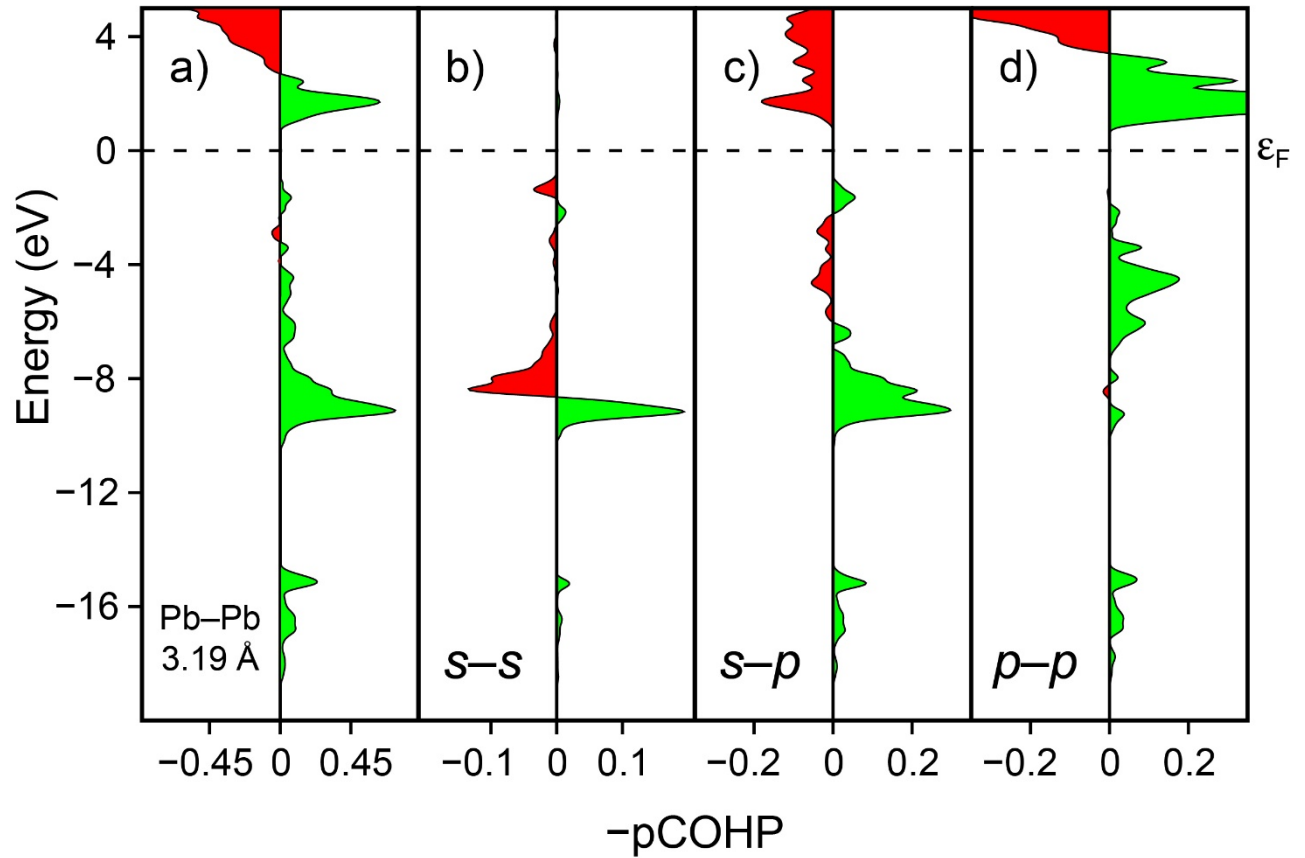
orthorhombic $Pmn2_1$,
 $a = 5.774(1) \text{ \AA}$,
 $b = 6.837(1) \text{ \AA}$,
 $c = 9.350(1) \text{ \AA}$, $Z = 2$



Attractive $\text{Pb}^{2+}-\text{Pb}^{2+}$ ($6s^2-6s^2$) interactions

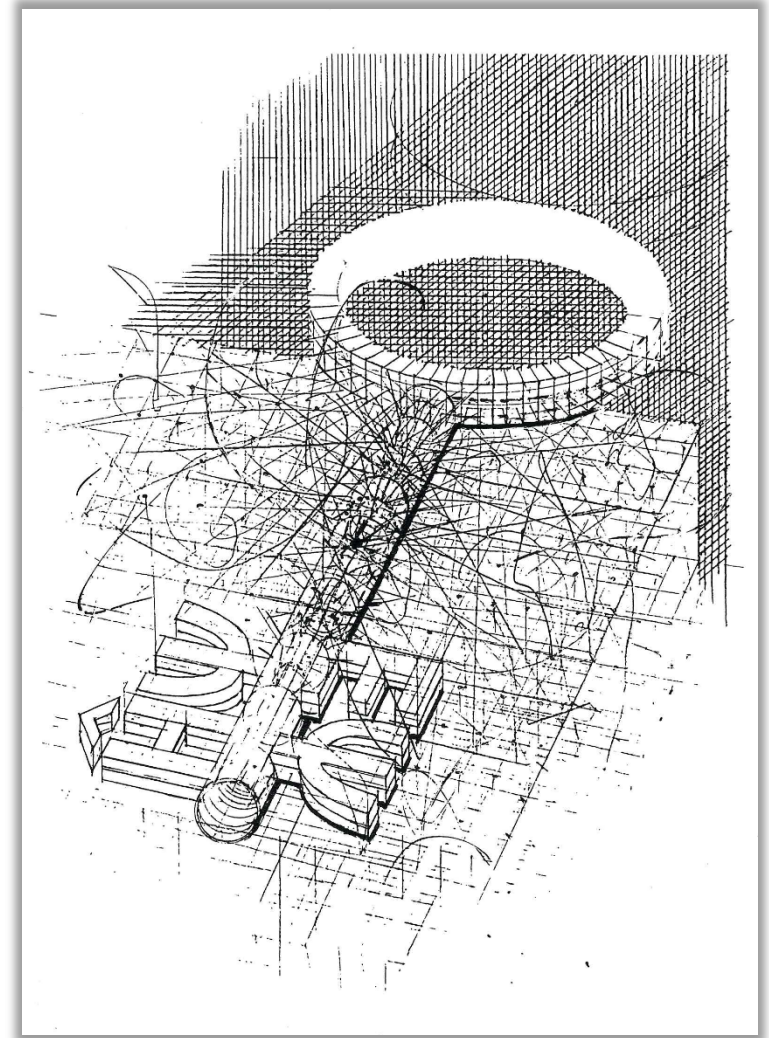
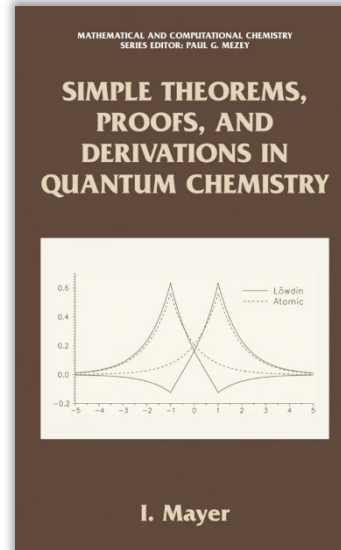


$\text{Pb}_2\text{Si}_5\text{N}_8$ with attractive $\text{Pb}^{2+}-\text{Pb}^{2+}$ interactions



P. Bielec, R. Nelson, R. P. Stoffel,
L. Eisenburger, D. Günther, A.-K. Henß,
O. Oeckler, R. Dronskowski, W. Schnick,
Angew. Chem. Int. Ed. **2019**, *58*, 1432

Hungarian Quantum Chemist **István Mayer**



$$BI_{\text{Mayer}} = \text{Tr}(P_{\mu\nu} \cdot P_{\nu\mu})$$

I. Mayer, *Chem. Phys. Lett.* **1983**, 97, 270

Crystal-Orbital Bond Index (COBI)

$$\text{COBI}_{\mu\nu}(E) = P_{\mu\nu} \sum_{j,\mathbf{k}} w_{\mathbf{k}} \text{Re}(c_{\mu,j\mathbf{k}}^* c_{\nu,j\mathbf{k}}) \cdot \delta(\varepsilon_j(\mathbf{k}) - E)$$

between 2 atoms or orbitals

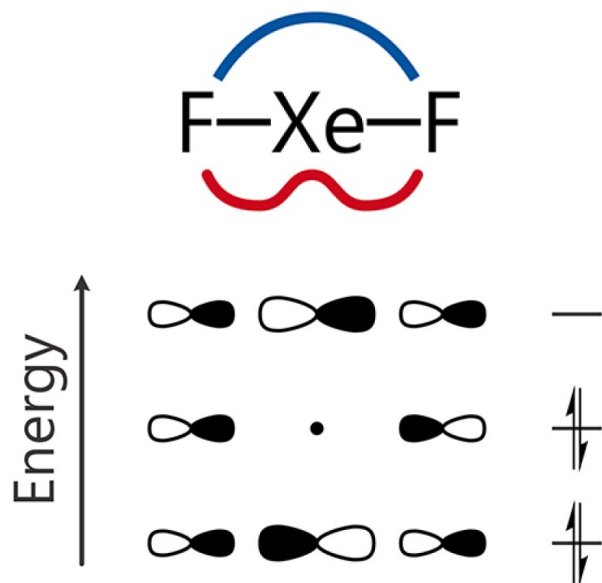
$$\text{COBI}^{(z)}(E) = \frac{1}{2^{z-1}} \sum_p P_{\mu\dots\chi} \sum_j w_{\mathbf{k}} \text{Re}(c_{\chi,j\mathbf{k}}^* c_{\mu,j\mathbf{k}}) \cdot \delta(\varepsilon_j(\mathbf{k}) - E)$$

between z atoms or orbitals

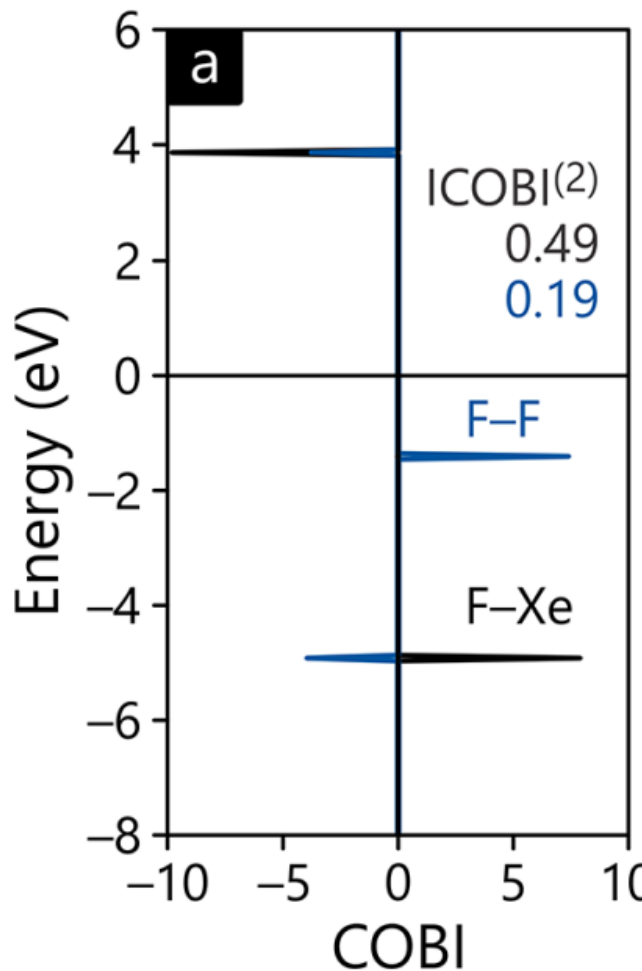


P. C. Müller, C. Ertural, J. Hempelmann,
R. Dronskowski, *J. Phys. Chem. C* **2021**, 125, 7959

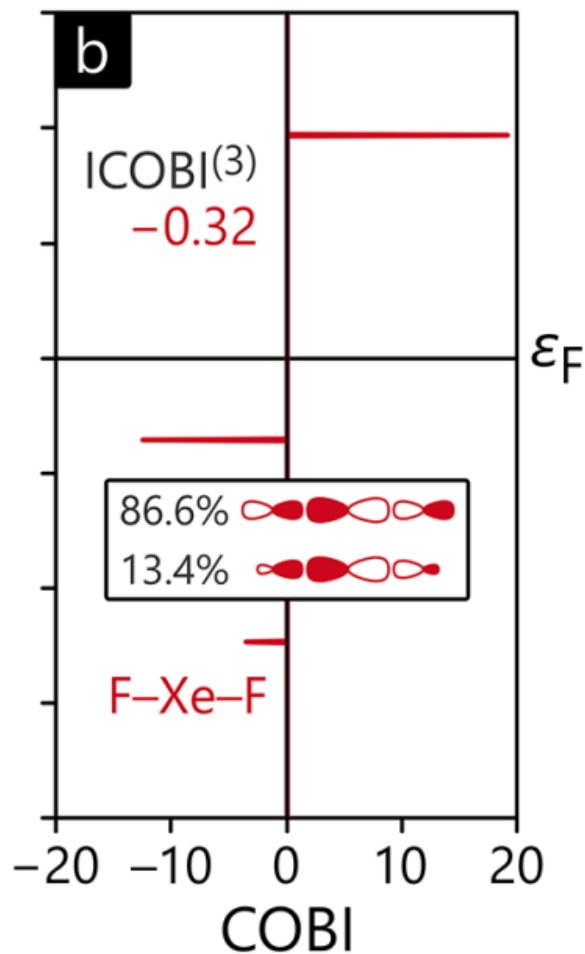
Xenon difluoride, a stable **ten**-electron species



COBI(2)



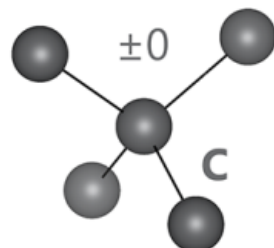
COBI(3)



Covalency and ionicity in solids



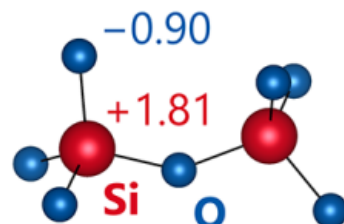
Diamond



ICOBI = 0.95

Covalent

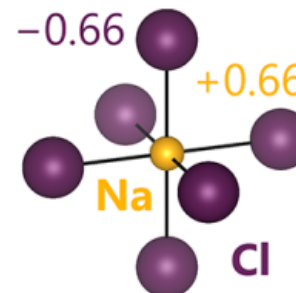
α -Quartz



ICOBI = 0.76

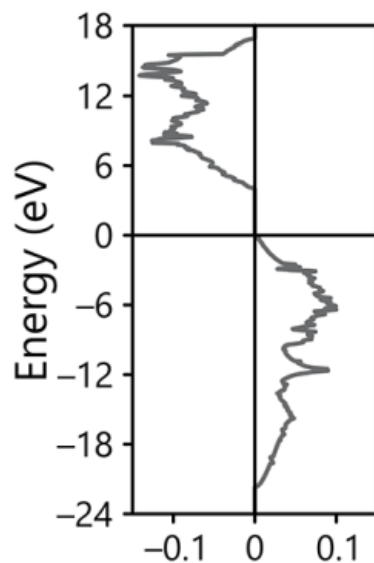
Covalent and ionic

Rock salt

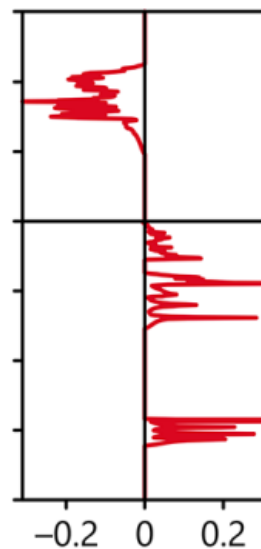


ICOBI = 0.09

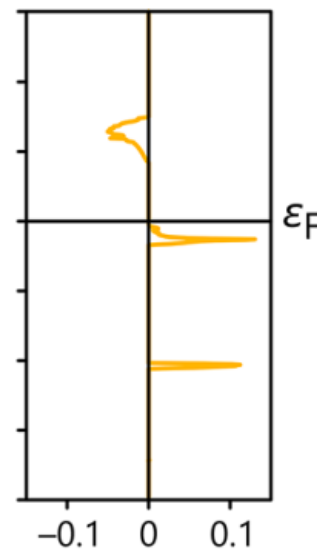
Ionic



COBI

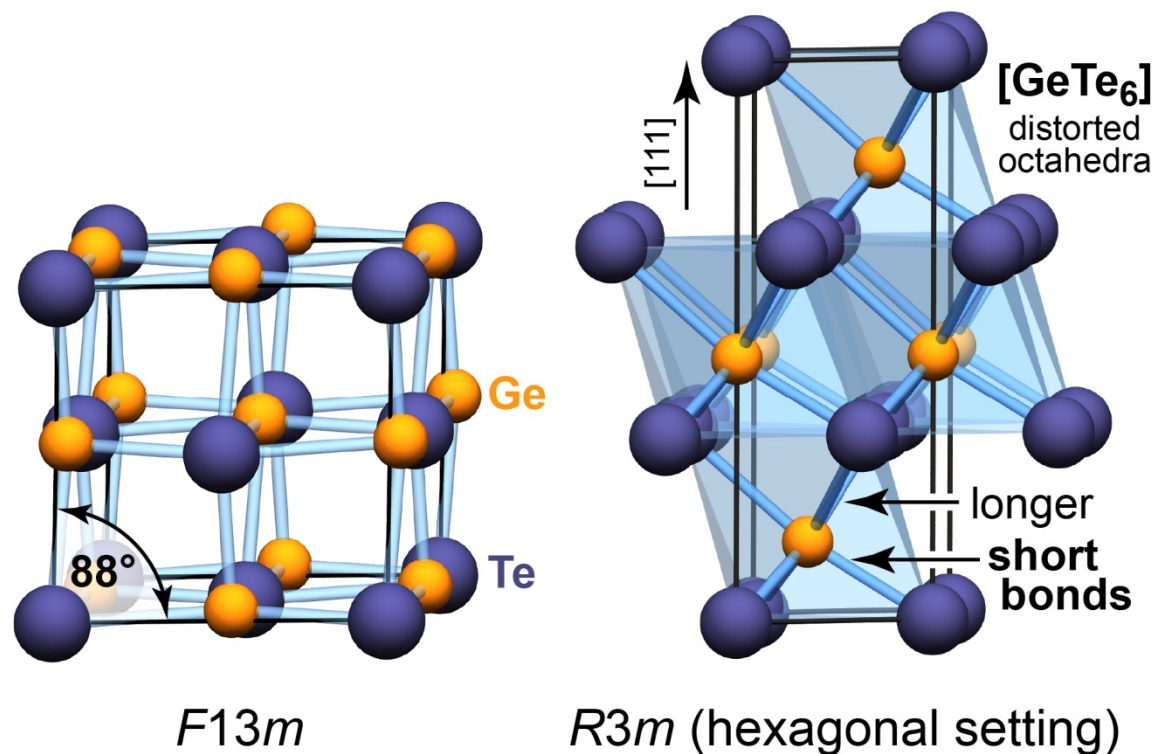


COBI



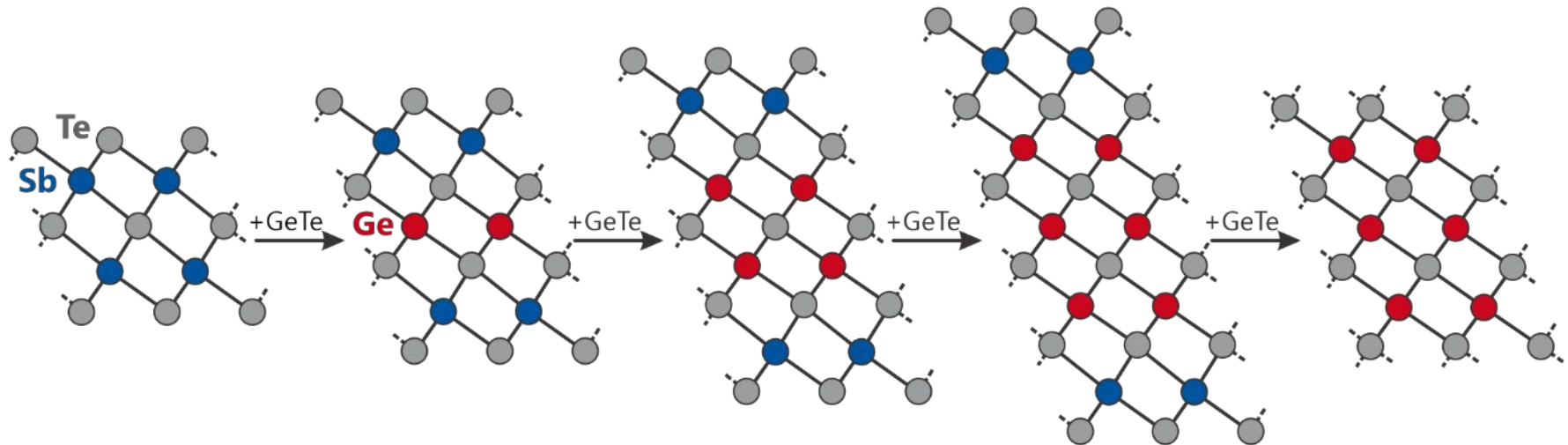
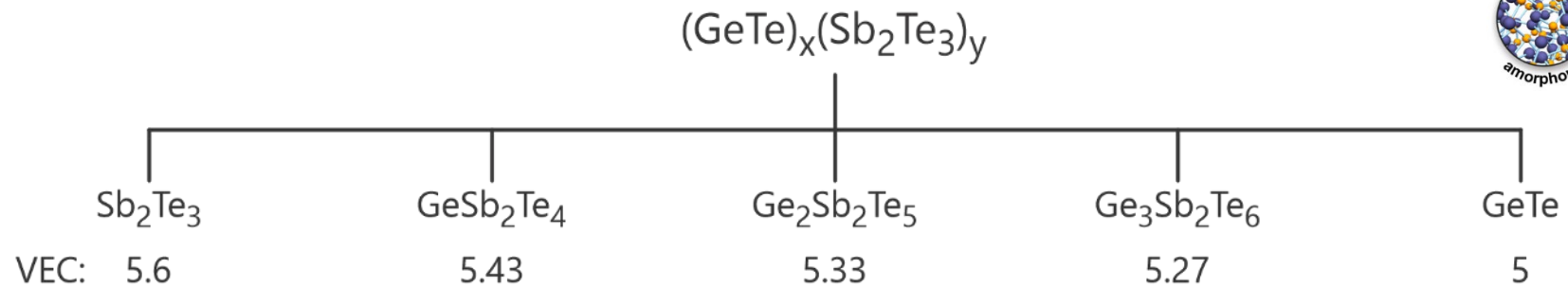
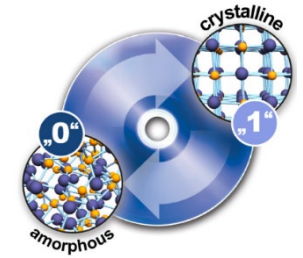
COBI

(almost cubic) GeTe, a phase-change prototype



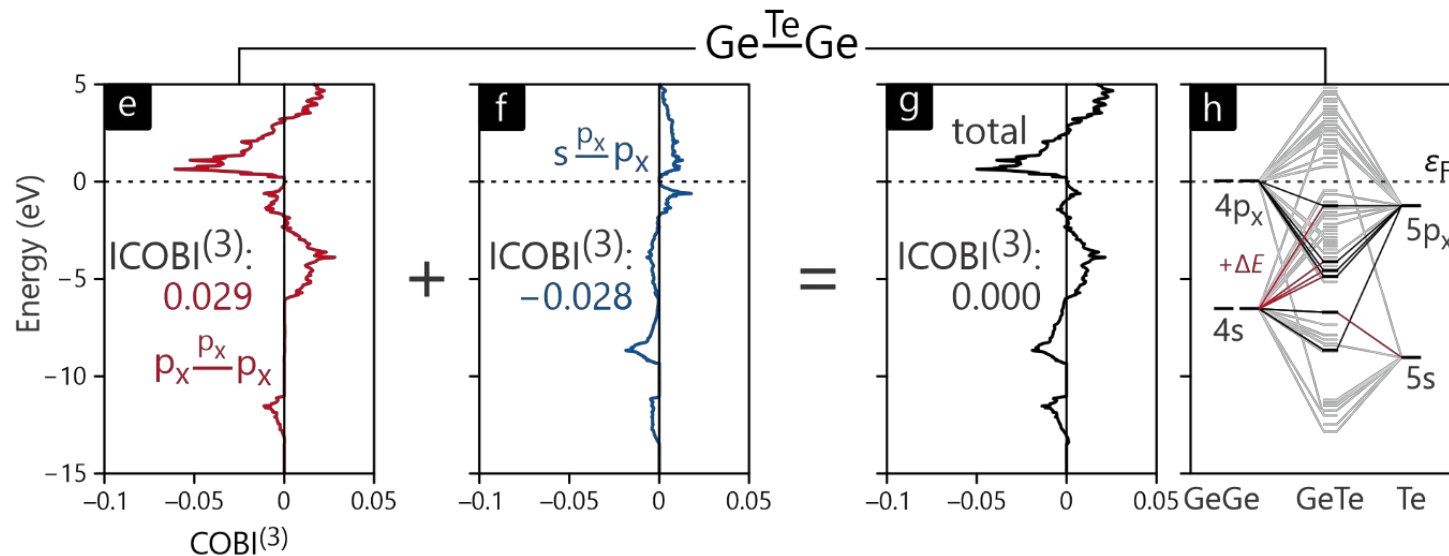
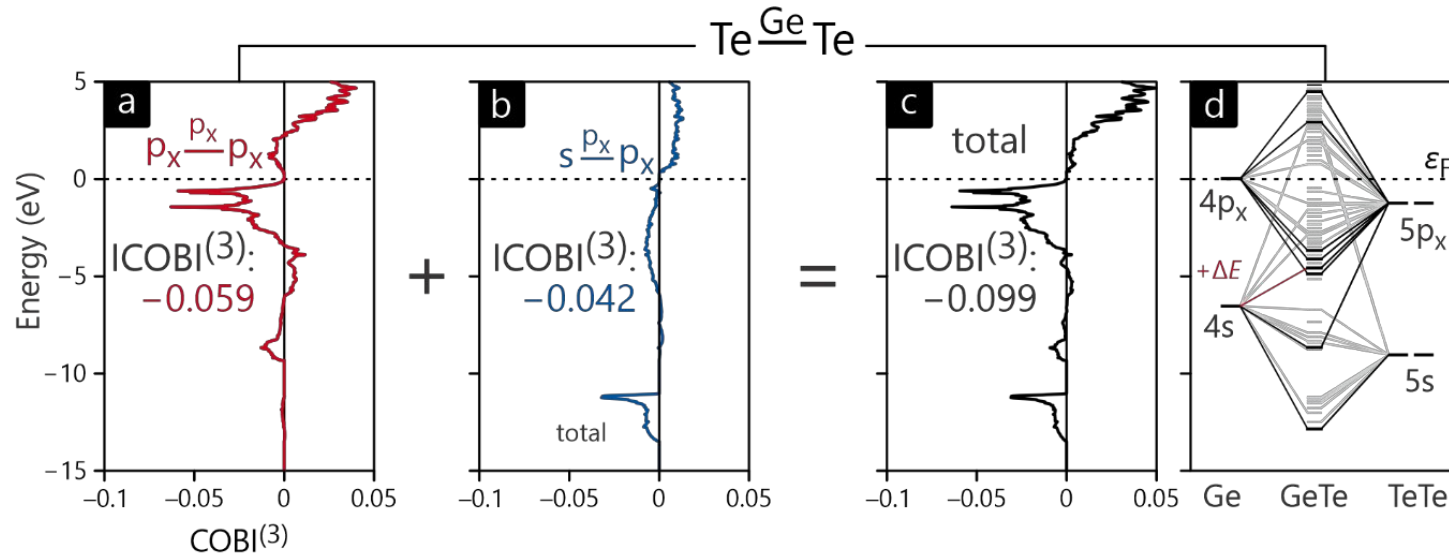
Synthesis: W. Klemm, G. Frischmuth, *Z. Anorg. Allg. Chem.* **1934**, 218, 249
Structure: K. Schubert, H. Fricke, *Z. Naturforsch.* **1951**, 6a, 781; J. Goldak, C. S. Barrett, D. Innes, W. Youdelis, *J. Chem. Phys.* **1966**, 44, 3323; T. Chattopadhyay, J. X. Boucherle, H. G. von Schnering, *J. Phys. C: Solid State Phys.* **1987**, 20, 1431

"GST" phase-change materials family



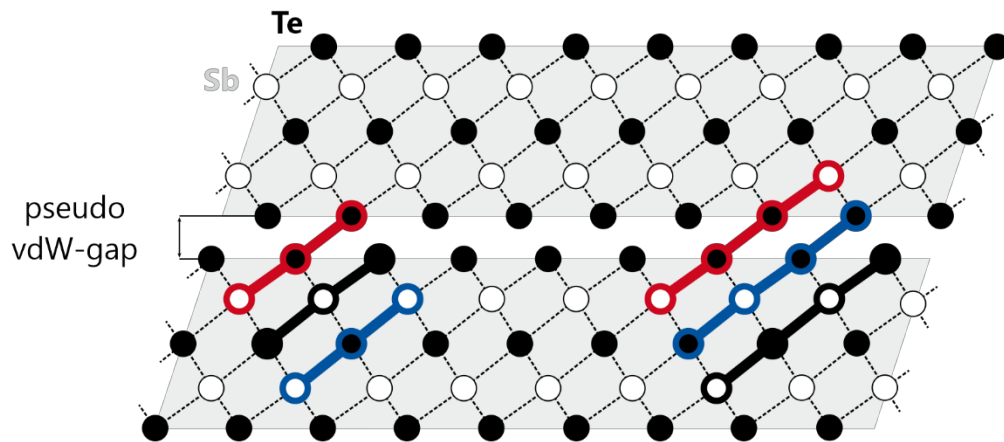
property portfolio: extraordinarily large optical dielectric constant / unusually large electrical conductivity / large Grüneisen parameter / **multiple-emission events in atom-probe tomography** / **12% too small van-der-Waals gaps**

Violating the 8 - N rule yields **multicenter bonding**

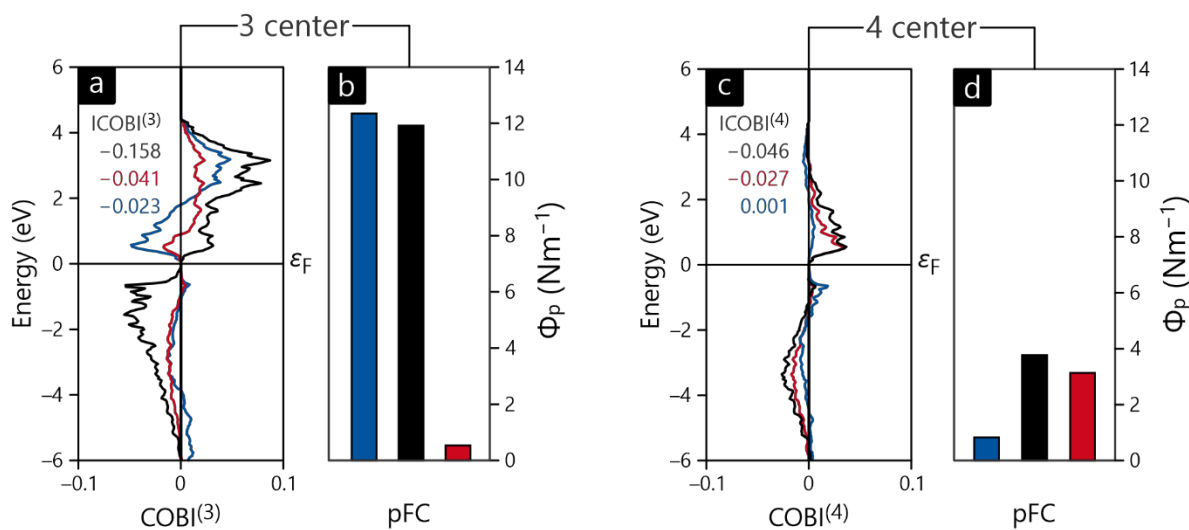


Ge^{2+} ($4s^2$ configuration) induces multicenter bonding, Te^{2-} (noble-gas shell) does not

Multicenter bonding closes the vdW gap (Sb_2Te_3)



ChemRxiv™



“The Orbital Origins of Chemical Bonding in Phase-Change Materials”

Summary

Chemical-bonding analysis (for H₂, solid Te, ferromagnetic Fe, phase-change materials) provides physical understanding.

Modern computational materials science operates with plane waves and pseudopotentials, so *chemistry is hidden*.

LOBSTER digests plane-wave electronic structures and performs an analytical nonlocal-to-local *unitary transformation* (projection).

By directly working with the *wave function* (*not* with the density), atomic charges are both more accurate and faster to achieve; unexpected orbital mixing (say, Pb sp) is automatically detected.

The Crystal Orbital Bond Index, **COBI**, a solid-state variant of István Mayer's bond index, detects three- and more-center interactions; for *any* given material (or molecule).

Metavalency simply is *multicenter bonding* for electron-rich solids.

Emperor Karl and the Aachen Cathedral



Karl der Große
747–814 AD

