A review of our last article

- C.Bild–D.A. Deckert–H.Ruhl. Phys. Rev. D, **99** (2019)
- Dirac's derivation of LAD equation: unjustified Gauss-Stokes theorem, Taylor expansion, limit to a point
- rigid spherical shell, radious ϵ, uniform continuous charge distribution; world tube
- Basic assumption: the electromagnetic fields outside the world tube of the shell and the point charge at the centre are equal.
- Equation with time delay ϵ.

Problems

1. the world tube breaks down for accelerations higher than $\frac{1}{\epsilon}$
2. the basic assumption is not valid

Invalidity is proved by Distribution Theory.

Spacetime formulae

T. Matolcsi: *Spacetime without Reference Frames*, Minkowski Institute Press, 2020

- Spacetime points $x, y \dots$ spacetime vectors $\boldsymbol{x}, \boldsymbol{y}, \dots$
- If x, y are spacetime points then $x - y$ is a spacetime vector
- $\boldsymbol{x} \cdot \boldsymbol{y} \sim x_k y^k, \quad \boldsymbol{L} \cdot \boldsymbol{x} \sim L^{ik} x_k, \qquad \boldsymbol{x} \otimes \boldsymbol{y} \sim x^i y^k$
- absolute velocity $\boldsymbol{u}$, $\boldsymbol{u} \cdot \boldsymbol{u} = -1$,
- Spacetime differentiation $\mathrm{D} \sim \partial_k, \qquad \mathrm{D} \cdot \boldsymbol{T} \sim \partial_k T^{ik}$

Distributions in spacetime

The electromagnetic quantites are considered Distributions

Pole taming (Gelfand)

- $\rho(\boldsymbol{q}) := |\boldsymbol{q}|$ $(\boldsymbol{q} \in \mathbf{S})$
- $\frac{1}{\rho^{2+m}}$ is not locally integrable if m is a positive integer
- $\left(\mathrm{tm}\frac{1}{\rho^{2+m}} \;\middle|\; \psi\right) := \int \frac{\psi(\boldsymbol{q}) - T_\psi^{(m-1)}(\boldsymbol{q})}{|\boldsymbol{q}|^{2+m}} \; \boldsymbol{dq}$
- $T_\psi^{(m-1)}$ is
  - on the support of ψ: the Taylor polynomial of order $(m-1)$ at zero of ψ
  - outside the support of ψ: zero

Electrostatics

An important notion

- Charge distributions **extraneous** to each other: disjoint supports
- Electric field **extraneous** to a charge distribution: its producing charge distribution is extraneous to the one in question

Electrostatic energies (by Jackson)

- Point charge e at space point q_e, extraneous potential V, extraneous electric energy: $eV(q_e)$; charge density ρ, extraneous potential, extraneous electric energy density: ρV
- Point charges, self-energy: the work done by transporting $e_1 \ldots, e_n$ from the infinity to their places $q_1, \ldots, q_n$
- 1. step: no work for the first charge, the self-energy is zero
- 2. step: $\frac{1}{4\pi}\frac{e_1 e_2}{4\pi|q_2-q_1|} = \frac{1}{2}(e_1 V_2(q_1) + e_2 V_1(q_2))$
- n. step:

$$\frac{1}{2}\sum_{k\neq i=1}^{n} \frac{e_k e_i}{4\pi|q_k - q_i|} = \frac{1}{2}\sum_{k=1}^{n}(e_k(V - V_k)(q_k))$$

- More and more particles with smaller and smaller charges together $\rightarrow$ continuous charge distribution

Without a serious objection, $\frac{1}{2}\rho V$ can be accepted as the self-energy density of a charge density ρ in its own potential V

Electrostatic energies (by Jackson)

- $\rho V = (\mathrm{div}\boldsymbol{E})V = \mathrm{div}(\boldsymbol{E}V) - \boldsymbol{E}\cdot\mathrm{grad}\,V =$
  $= \mathrm{div}(\boldsymbol{E}V) + |\boldsymbol{E}|^2 \qquad (\mathrm{div}\boldsymbol{E} = \rho)$
- The total self-energy (Gauss theorem)

$$\frac{1}{2}\int \rho V = \frac{1}{2}\int |\boldsymbol{E}|^2$$

- 'Electric self-energy density' $= \frac{1}{2}|\boldsymbol{E}|^2$
- For a point charge: the electric self-energy is infinite

Comments

ρV is zero where ρ is zero but $|\boldsymbol{E}|^2$ is not zero there

It is unjustified to consider $\frac{1}{2}|\boldsymbol{E}|^2$ the self-energy density of a static electric field $\boldsymbol{E}$. It has the only physical meaning that its integral over all the space is the total self-energy

Amazing and shocking:
first step "the electric self-energy of a point charge is zero"
conclusion "the electric self-energy of a point charge is infinite"

Electric self-energy of a point charge

- The infinite self-electric energy: three times incorrect reasoning:
    1. $\frac{1}{2}\rho V$ does not make sense for a point charge
    2. $\frac{1}{2}|\boldsymbol{E}|^2$ is not the self-energy density even in the continuous case
    3. Gauss theorem is not applicable
- $\frac{1}{2}|\boldsymbol{E}|^2$ is not locally integrable
- $\frac{1}{2}|\boldsymbol{E}|^2$ has a pole at the space point where the charge is
- $\frac{1}{2}\mathrm{tm}|\boldsymbol{E}|^2$, mathematical construction; it could have the only physical meaning: its 'integral' is the self-energy of the point charge
- $\frac{1}{2}\mathrm{tm}|\boldsymbol{E}|^2$ does have this physical meaning: the self-energy is zero:

$$\left(\frac{1}{2}\mathrm{tm}|\boldsymbol{E}|^2 \;\middle|\; 1\right) = 0$$

Electrostatic forces

- Point charge e at space point q_e, extraneous electric field $\boldsymbol{E}$, extraneous force: $e\boldsymbol{E}(q_e)$; charge density ρ, extraneous electric field $\boldsymbol{E}$, extraneous force density: $\rho\boldsymbol{E}$
- A point charge does not act on itself: its self-force is zero
- Point charges, self-force: $\sum_{k=1}^{n} e_k(\boldsymbol{E} - \boldsymbol{E}_k)(q_k)$
- More and particles with smaller and smaller charges together $\rightarrow$ continuous charge distribution

Without a serious objection, $\rho\boldsymbol{E}$ can be accepted as the self-force density of the charge density ρ in its own electric field $\boldsymbol{E}$

- 'Electrostatical stress tensor' $\boldsymbol{P} := -\boldsymbol{E} \otimes \boldsymbol{E} + \frac{1}{2}|\boldsymbol{E}|^2 \mathbf{1}_{\mathsf{S}}$
  $-\mathrm{div}\boldsymbol{P} = \rho\boldsymbol{E}$ $\quad$ ($\mathrm{div}\boldsymbol{E} = \rho$, $\boldsymbol{E}$ is produced by ρ)

Comment

$\rho\boldsymbol{E}$ is zero where ρ is zero but $\boldsymbol{P}$ is not zero there:

It is unjustified to consider $\boldsymbol{P}$ a real stress tensor of a static electric field. It has the only physical meaning that its negative divergence is the self-force density

Electrostatic self-force of a point charge

- The fictitious stress tensor $\boldsymbol{P} := -\boldsymbol{E} \otimes \boldsymbol{E} + \frac{1}{2}|\boldsymbol{E}|^2 \mathbf{1}_{\mathsf{S}}$ is not locally integrable
- $\boldsymbol{P}$ has a pole at the space point where the charge is
- $\mathrm{tm}\boldsymbol{P}$, mathematical construction; it could have the only physical meaning: its negative divergence is the self-force of the point charge
- $\mathrm{tm}\boldsymbol{P}$ does have this physical meaning: the self-force is zero:

$$-\mathrm{div}(\mathrm{tm}\boldsymbol{P}) = 0$$

Beyond statics

- Point charge e, velocity $\boldsymbol{v}$, extraneous electric and magnetic field $\boldsymbol{E}$ and $\boldsymbol{B}$, extraneous force: $e(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})$
- Charge density ρ, velocity field $\boldsymbol{v}$, extraneous electric and magnetic fields, extraneous force density: $\rho(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})$
- A non-inertial point charge acts on itself, self-force: $\boldsymbol{f}_s$
- Point charges, self-force:

$$\sum_{k=1}^{n} e_k\left((\boldsymbol{E}-\boldsymbol{E}_k)+\boldsymbol{v}_k\times(\boldsymbol{B}-\boldsymbol{B}_k)\right)+\sum_{k=1}^{n}\boldsymbol{f}_{s,k}$$

- More and more particles with smaller and smaller charges together $\rightarrow$ continuous charge distribution:

$$\rho(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})+?$$

$? \neq 0$ most probably

> Contrary to electrostatics, *it is questionable* and even more than questionable that $\rho(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})=\rho\boldsymbol{E}+\boldsymbol{i}\times\boldsymbol{B}$ is the self-force density of ρ in its own fields $\boldsymbol{E}$ and $\boldsymbol{B}$

Spacetime formulation; continuous case

- Absolute current density $\boldsymbol{j} \sim (\rho, \boldsymbol{i})$
- Electromagnetic field $\boldsymbol{F} \sim (\boldsymbol{E}, \boldsymbol{B})$
- $\boldsymbol{F} \cdot \boldsymbol{j} \sim (\rho \boldsymbol{E}, \rho \boldsymbol{E} + \boldsymbol{i} \times \boldsymbol{B})$; extraneous absolute force density

> *It is more than questionable* that $\boldsymbol{F} \cdot \boldsymbol{j}$ is the absolute force density of $\boldsymbol{j}$ in its own field $\boldsymbol{F}$

- 'Energy-momentum tensor' $\boldsymbol{T} := -\boldsymbol{F} \cdot \boldsymbol{F} - \frac{1}{4}(\mathrm{Tr}\boldsymbol{F} \cdot \boldsymbol{F})\mathbf{1}$
- The time-time component of $\boldsymbol{T}$ is $\frac{1}{2}(|\boldsymbol{E}|^2 + |\boldsymbol{B}|^2)$
- The space-space component of $\boldsymbol{T}$ is $-\boldsymbol{E} \otimes \boldsymbol{E} + \frac{1}{2}|\boldsymbol{E}|^2 \mathbf{1}_{\mathsf{S}} + \ldots$

> *It is unjustified* to consider $\boldsymbol{T}$ a real energy-momentum tensor

- $-\mathrm{D} \cdot \boldsymbol{T} = \boldsymbol{F} \cdot \boldsymbol{j}$ $\qquad (\mathrm{D} \cdot \boldsymbol{F} = \boldsymbol{j},$ $\boldsymbol{F}$ is produced by $\boldsymbol{j})$

> *It is more than questionable* that $\boldsymbol{T}$ has the physical meaning that its negative spacetime divergence is the absolute self-force density

Spacetime formulation; point charge

- Existence of a point particle in spacetime: world line (one dimensional submanifold)
- World line function r, world line $\mathrm{Ran}\, r$, Lebesgue measure $\lambda_{\mathrm{Ran}\, r}$
- A point charge e, world line function r, spacetime current: $e\dot{r}\lambda_{\mathrm{Ran}\, r}$
- **Given** r, retarded proper time: $\boldsymbol{s}_r(x)$, $x - r(\boldsymbol{s}_r(x))$ is future-lightlike
- A point charge e, **given** r, produced electromagnetic potential (Liénard–Wiechert)

$$x \mapsto \frac{e}{4\pi} \frac{\dot{r}(\boldsymbol{s}_r(x))}{-\dot{r}(\boldsymbol{s}_r(x)) \cdot (x - r(\boldsymbol{s}_r(x)))}$$

- Electromagnetic field $\boldsymbol{F}[r]$ is some functional of the **given** r
- Fictitious energy-momentum tensor $\boldsymbol{T}[r] := -\boldsymbol{F}[r] \cdot \boldsymbol{F}[r] - \frac{1}{4}(\mathrm{Tr}\boldsymbol{F}[r] \cdot \boldsymbol{F}[r])\mathbf{1}$ is some functional of the **given** r

Spacetime formulation; point charge

- $\boldsymbol{T}[r]$ is not locally integrable
- $\boldsymbol{T}[r]$ is not differentiable on the world line
- Nevertheless, $\boldsymbol{T}[r]$ can be expounded, in a convenient sense, in powers of the 'radial distance' from the world line
- $\boldsymbol{T}[r]$ has a pole in 'radial distance' on the world line
- $\mathrm{tm}\,\boldsymbol{T}[r]$, mathematical construction; it could have the only physical meaning: its negative spacetime divergence is the self-force of the point charge but even this meaning is questionable
- It is proved that

$$-\mathrm{D}\cdot\mathrm{tm}\,\boldsymbol{T}[r] = \frac{1}{4\pi}\frac{2e^2}{3}(\dot{r}\wedge\dddot{r})\cdot\dot{r}\lambda_{\mathrm{Ran}\,r}$$

On the LAD equation

- We obtained the usual self-force for a **given** world line function
- It is unjustified to put this self-force in a Newtonian-like equation to obtain the LAD equation which would serve **to determine** the world line function
- Fundamental problem: both electrodynamics and mechanics in their known forms are theories of **action**:
  – the Maxwell equations define the electromagnetic field $\boldsymbol{F}$ produced by a **given** world line function r of a particle
  – the Newtonian equation defines the world line function r of a particle in a **given** force (e.g. the Lorentz force in an extraneous electromagnetic field $\boldsymbol{F}$)
- At present we have not a well working theory of **interaction** which would define both $\boldsymbol{F}$ and r together

On the LAD equation

- Usual conception: the Newtonian equation for a point charge under the action of a given force $\boldsymbol{f}$, is replaced with the balance equation

$$-\mathrm{D}\cdot(\mathcal{T}_m[r]+\mathcal{T}_e[\boldsymbol{F}])+\boldsymbol{f}(r,\dot{r})\lambda_{\mathrm{Ran}\,r}=0$$

- It seems evident that $\mathcal{T}_m[r]=m\dot{r}\otimes\dot{r}\lambda_{\mathrm{Ran}\,r}$ and then

$$m\ddot{r}\lambda_{\mathrm{Ran}\,r}=\boldsymbol{f}(r,\dot{r})\lambda_{\mathrm{Ran}\,r}-\mathrm{D}\cdot\mathcal{T}_e[F]$$

$$\mathrm{D}\cdot\boldsymbol{F}=\dot{r}\lambda_{\mathrm{Ran}\,r},\qquad \mathrm{D}\wedge\boldsymbol{F}=0$$

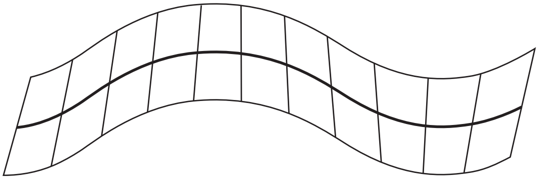
  would describe that r and $\boldsymbol{F}$ **determine** each other mutually
- It is not sure that this is a good system of equations with a convenient $\mathcal{T}_e[\boldsymbol{F}]$
- It is sure that $\mathcal{T}_e[F]$ cannot be replaced with $\mathrm{tm}\,\boldsymbol{T}[r]$ obtained for a **given** r
- The LAD equation ($\mathcal{T}_e[\boldsymbol{F}]$ replaced with $\mathrm{tm}\,\boldsymbol{T}[r]$) is a misconception; its pathological properties are not surprising

On the self-force

- To get the usual self-force, the unjustified use of Gauss-Stokes theorem, Taylor expansion, limit to a point are ruled out by pole taming
- The usual formula of the self-force can be possibly accepted only for a given spacetime existence of a point charge
- If the force $\boldsymbol{f}$ is necessary for bringing about a given r without radiation i.e. $m\ddot{r} = \boldsymbol{f}(r, \dot{r})$ would valid then besides $\boldsymbol{f}$, the force $-\frac{1}{4\pi}\frac{2e^2}{3}(\dot{r} \wedge \dddot{r}) \cdot \dot{r}$, opposite to the self-force, must be applied for obtaining the desired r. An actual example is an elementary particle revolved in a cyclotron

Equation, equality

- Equation is a definition
- Equality is a statement
- LAD ‘equation’? No: runaway solutions
- LAD ‘equality’? No: see C.J.Eliezer, Mathematical Proceedings of the Cambridge Philosophical Society, Vol 39. Nr 3. 1943



Sugárzási visszahatás

Wigner Fizikai Kutatóközpont, Részecske és Magfizikai Intézet

Budapest, 2020. november 6.

Major approaches

- Nonlocal modifications of the electromagnetic part, the Maxwell equations, [1, 2, 3].
- Dissipative modifications of the mechanical part, the Newton equation [4, 5, 6, 7].
- A suitable interpretation of the LAD equation, e.g. excluding particular solutions, [8, 9, 10].
- Application of continuum charge distributions instead of point charges. This is the method of the classical papers of Lorentz, Abraham and Dirac, too, [11, 12, 13, 8]. There are two main aspects of this strategy:
  - One may improve the classical theory, with the identification and elimination of the mathematical problems, [14, 15],
  - One may modify the point charge model with the help of quantum mechanics, or with various renormalization procedures, [16, 17, 18, 19, 20, 21].

Related issues

- The recent experiments of radiation reaction related phenomena, see, e.g. [22, 23, 24], open the way toward the verification of the mentioned theories.
- There are less rigorous, but more applicable (?) approaches, [25, 26, 27].

M. Born and L. Infeld.
Foundations of the new field theory.
Proceedings of the Royal Society of London. Series A, 144(852):425–451, 1934.

F. Bopp.
Eine lineare theorie des elektrons.
Annalen der Physik, 430(5):345–384, 1940.

M. K.-H. Kiessling.
Force on a point charge source of the classical electromagnetic field.
Physical Review D, 100(6):065012, 2019.
arXiv:1907.11239v4, with erratum.

G.W. Ford and O'Connell.
Radiation reaction in electrodynamics and the elimination of runaway solutions.
Physics Letters A, 157(4-5):217–220, 1991.

J. Verhás.

White areas on the map of applying non-equilibrium thermodynamics: on the self accelerating electron.
Atti dell'Accademia Peloritana dei Pericolanti, Classe di Scienze Fisiche, Matematiche e Naturali, Suppl. I., 86:1–7, 2008.

J. Polonyi.
Effective dynamics of a classical point charge.
Annals of Physics, 342:239–263, 2014.

M. Formanek, A. Steinmetz, and J. Rafelski.
Radiation reaction friction: I. Resistive material medium.
arXiv:2004.09634, 2020.

P. A. M. Dirac.
Classical theory of radiating electrons.
Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences, 167(929):148–169, 1938.

H. Spohn.
The critical manifold of the Lorentz-Dirac equation.
EPL (Europhysics Letters), 50(3):287, 2000.

Matolcsi T., Fülöp T., and Weiner M.
Second order equation of motion for electromagnetic radiation back-reaction.
Journal of Modern Physics, 32(27):1750147(18), 2017.
arXiv:1207.0428.

H.A. Lorentz.
La théorie électromagnétique de Maxwell et son application aux corps mouvants.
Archives Neerlandaises des Sciences Exactes et Naturelles, 25:363–552, 1892.

Max Abraham.
Theorie der Elektrizität, Vol. II: Elektromagnetische Theorie der Strahlung.
Teubner, Leipzig, 1905.

H.A. Lorentz.
The Theory of Electrons.
Teubner, Leipzig, 1909.

S. Parrott.
Relativistic electrodynamics and differential geometry.
Springer, 2012.

C. Bild, D.-A. Deckert, and H. Ruhl.
Radiation reaction in classical electrodynamics.
Physical Review D, 99(9):096001, 2019.

E. J. Moniz and D.H. Sharp.
Absence of runaways and divergent self-mass in nonrelativistic quantum electrodynamics.
Physical Review D, 10(4):1133, 1974.

E. J. Moniz and D.H. Sharp.
Radiation reaction in nonrelativistic quantum electrodynamics.
Physical Review D, 15(10):2850, 1977.

C. R. Galley, A. K. Leibovich, and I. Z. Rothstein.
Finite size corrections to the radiation reaction force in classical electrodynamics.
Physical review letters, 105(9):094802, 2010.

P. Forgács, T. Herpay, and P. Kovács.

Comment on "Finite Size Corrections to the Radiation Reaction Force in Classical Electrodynamics".
Physical review letters, 109(2):029501, 2012.

C. R. Galley, A.K. Leibovich, and I.Z. Rothstein.
Galley, leibovich, and rothstein reply.
Physical Review Letters, 109(2):029502, 2012.

J. Polonyi.
The Abraham–Lorentz force and electrodynamics at the classical electron radius.
International Journal of Modern Physics A, 34(15):1950077, 2019.

T.N. Wistisen, A. Di Piazza, H. V. Knudsen, and U. I. Uggerhoj.
Experimental evidence of quantum radiation reaction in aligned crystals.
Nature communications, 9(1):1–6, 2018.

K. Poder, M. Tamburini, G. Sarri, A. Di Piazza, S. Kuschel, C.D. Baird, K. Behm, S. Bohlen, J.M. Cole, D.J. Corvan, and +other 14 authors.
Experimental signatures of the quantum nature of radiation reaction in the field of an ultraintense laser.

Physical Review X, 8(3):031004, 2018.

T.G. Blackburn.
Radiation reaction in electron–beam interactions with high-intensity lasers.
Reviews of Modern Plasma Physics, 4(1):1–37, 2020.

L. D. Landau and E. M. Lifshitz.
The Classical Theory of Fields (Course of Theoretical Physics, vol. 2).
Pergamon Press, Oxford, 2th edition, 1959.

I.V. Sokolov.
Renormalization of the Lorentz-Abraham-Dirac equation for radiation reaction force in classical electrodynamics.
Journal of Experimental and Theoretical Physics, 109(2):207–212, 2009.

D.B. Zot'ev.
Critical remarks on Sokolov's equation of the dynamics of a radiating electron.
Physics of Plasmas, 23(9):093302, 2016.