Adiabatic Quantum Computing

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ELTE

Computing = Statistical Mechanics

Ising formulations of many NP problems

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We provide Ising formulations for many NP-complete and NP-hard problems, including all of Karp's 21 NP-complete problems. This collects and extends mappings to the Ising model from partitioning, covering and satisfiability. In each case, the required number of spins is at most cubic in the size of the problem. This work may be useful in designing adiabatic quantum optimization algorithms.

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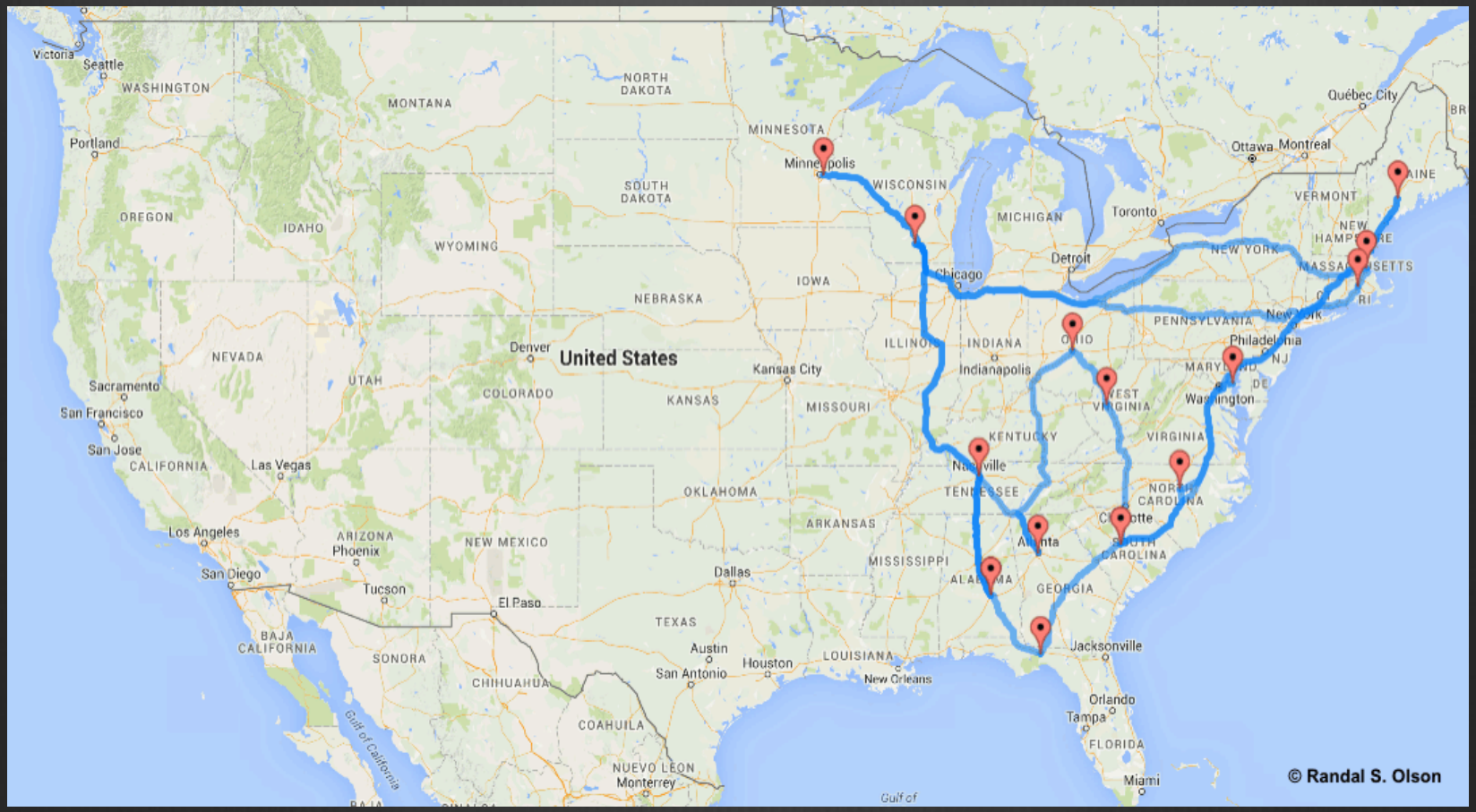
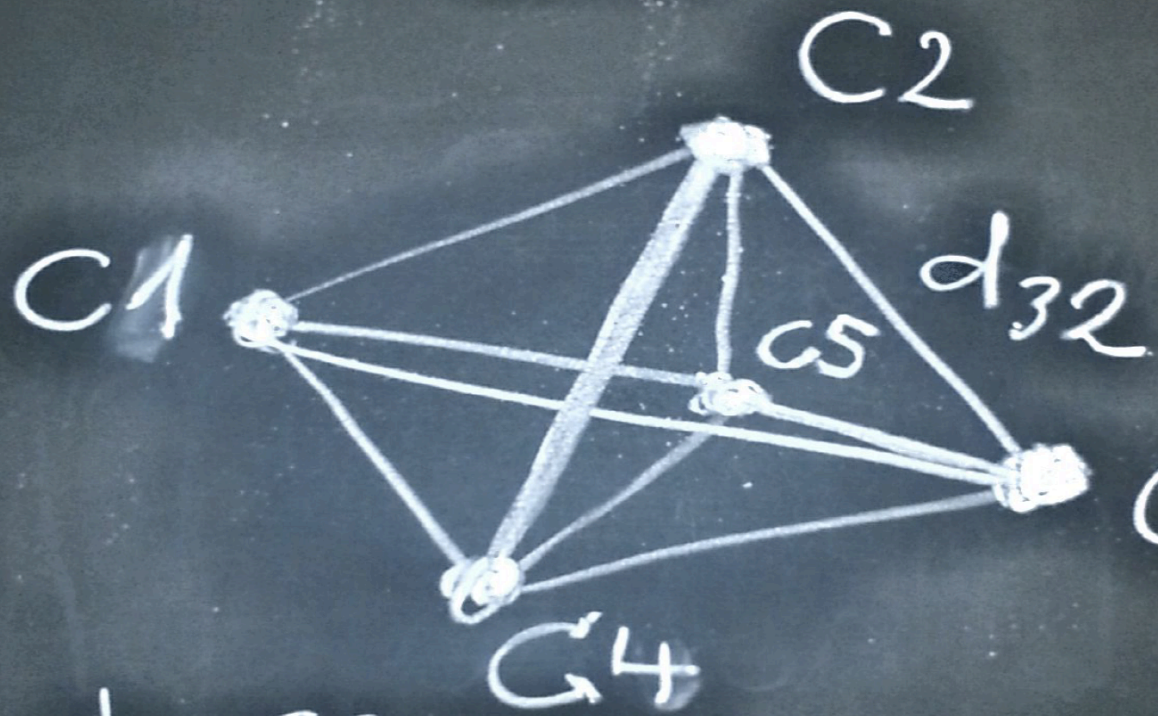
Road trip stopping at major U.S. landmarks



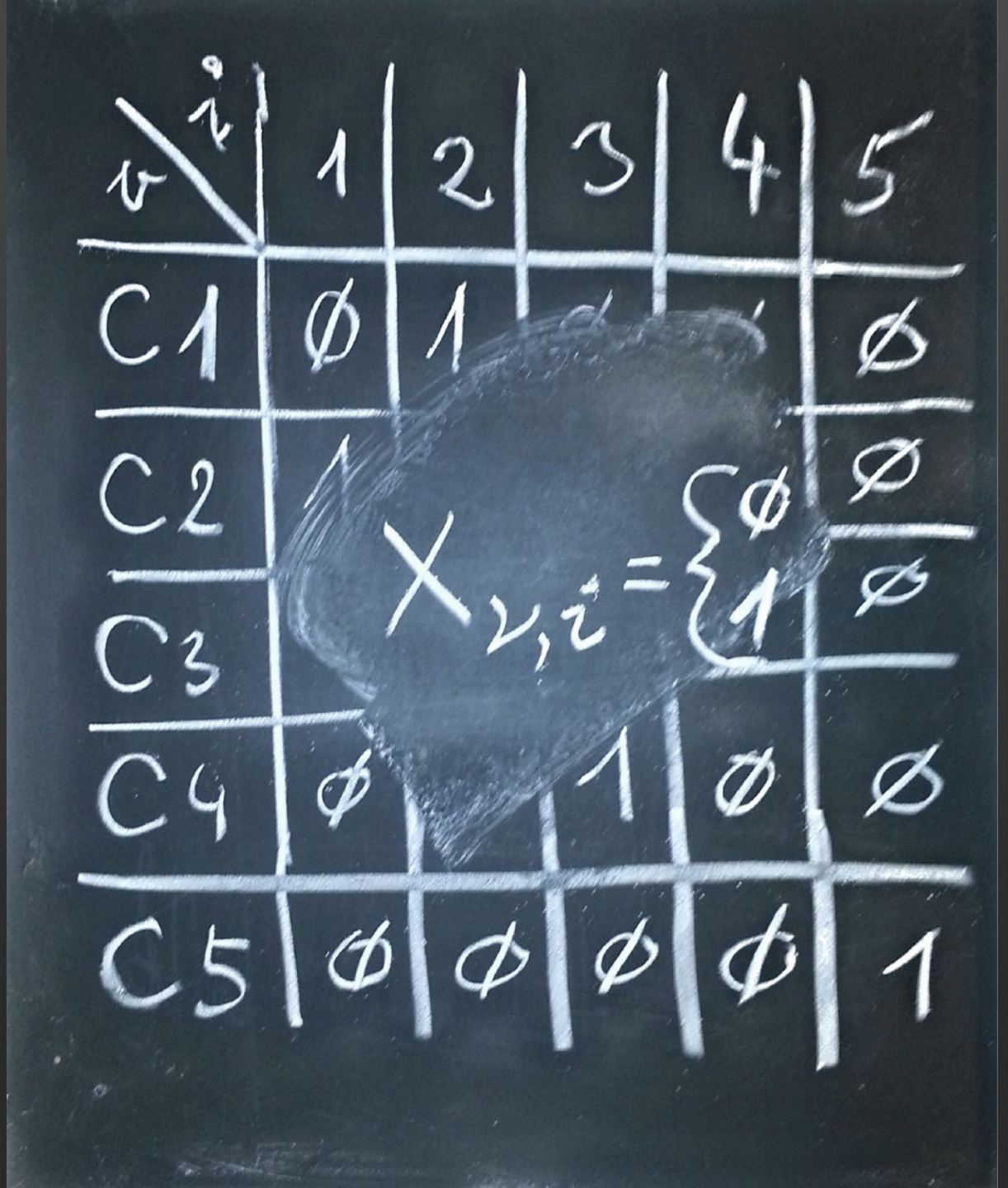
TSP

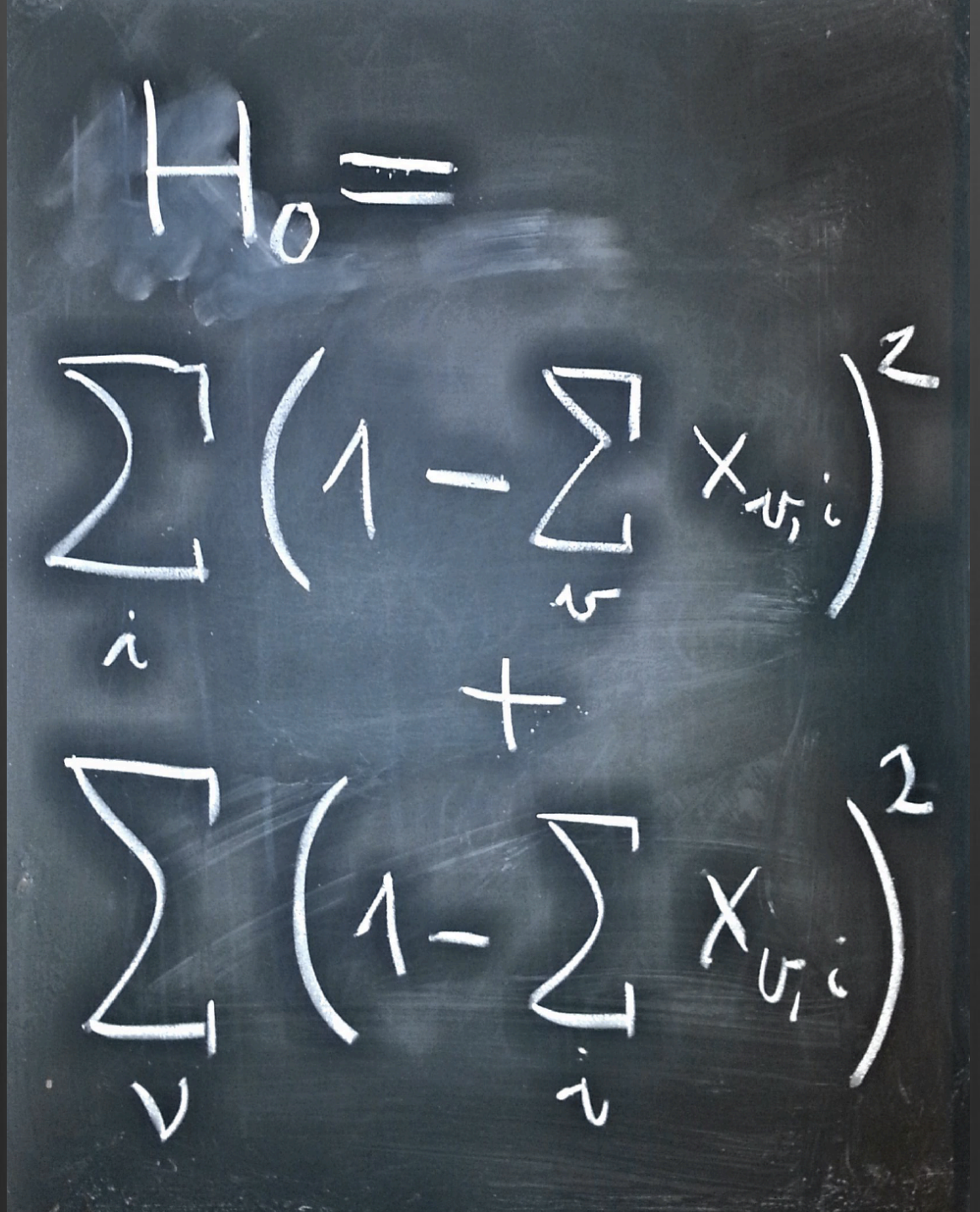


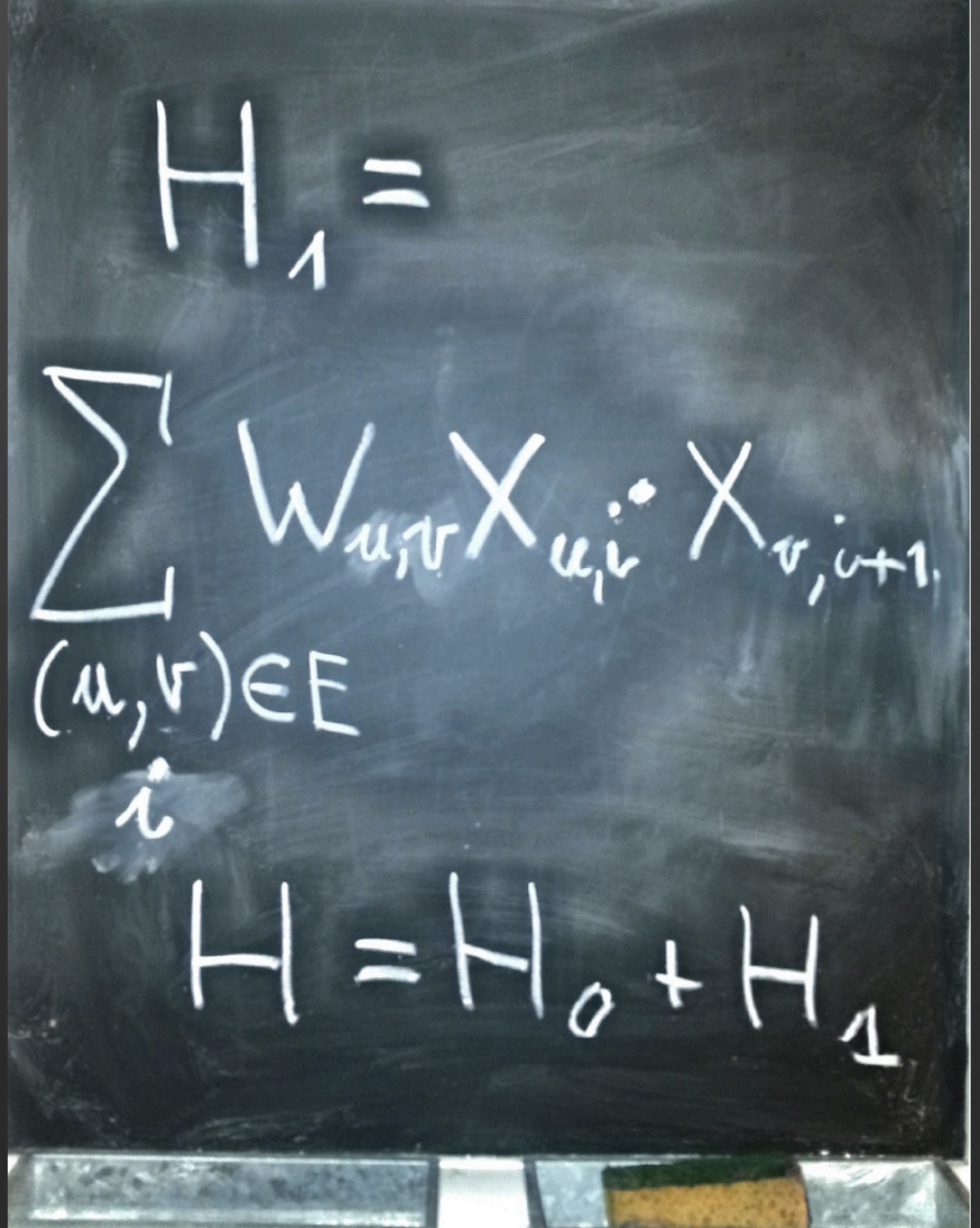
	C2 ⋯
C3 ⋮	d_{32} ⋮

$W_{32} = d_{32}$


v
i
1
2
3
4
5
C1
C2
C3
C4
C5


v \ i
1 2 3 4 5
C1 0 1 0
C2 0 0
C3 1 0
C4 0 1 0 0
C5 0 0 0 0 1
$X_{v,i} = \begin{cases} 0 \\ 1 \end{cases}$


$H_0 =$
$\sum_i \left(1 - \sum_\nu x_{\nu,i}\right)^2$
$+$
$\sum_\nu \left(1 - \sum_i x_{\nu,i}\right)^2$


$H_1 =$
$\sum_{(u,v)\in E} \sum_i W_{u,v} X_{u,i} \cdot X_{v,i+1}$
$H = H_0 + H_1$

$$\min_{\{\underline{\underline{x}}\}} H(\underline{\underline{x}})$$

$$\underline{\underline{x}} = ?$$

Optimization!

7.1. Hamiltonian Cycles and Paths

Let $G = (V, E)$, and $N = |V|$. The graph can either be directed or undirected; our method of solution will not change. The Hamiltonian path problem is as follows: starting at some node in the graph, can one travel along an edge, visiting other nodes in the graph, such that one can reach every single node in the graph without ever returning to the same node twice? The Hamiltonian cycles problem asks that, in addition, the traveler can return to the starting point from the last node he visits. Hamiltonian cycles is a generalization of the famous Königsberg bridge problem [24], and is NP-complete [18].

Without loss of generality, let us label the vertices $1, \ldots, N$, and take the edge set (uv) to be directed – i.e., the order uv matters. It is trivial to extend to undirected graphs, by just considering a directed graph with (vu) added to the edge set whenever (uv) is added to the edge set. Our solution will use N^2 bits $x_{v,i}$, where v represents the vertex and i represents its order in a prospective cycle. Our energy will have three components. The first two things we require are that every vertex can only appear once in a cycle, and that there must be a j^{th} node in the cycle for each j. Finally, for the nodes in our prospective ordering, if $x_{u,j}$ and $x_{v,j+1}$ are both 1, then there should be an energy penalty if $(uv) \notin E$. Note that $N + 1$ should be read as 1, in the expressions below, if we are solving the cycles problem. These are encoded in the Hamiltonian:

$$H = A\sum_{v=1}^{n}\left(1 - \sum_{j=1}^{N} x_{v,j}\right)^2 + A\sum_{j=1}^{n}\left(1 - \sum_{v=1}^{N} x_{v,j}\right)^2 + A\sum_{(uv)\notin E}\sum_{j=1}^{N} x_{u,j}x_{v,j+1}. \tag{56}$$

$A > 0$ is a constant. It is clear that a ground state of this system has $H = 0$ only if we have an ordering of vertices where each vertex is only included once, and adjacent vertices in the cycle have edges on the graph – i.e., we have a Hamiltonian cycle.

Traveling Salesman

$$H_B = B\sum_{(uv)\in E} W_{uv}\sum_{j=1}^{N} x_{u,j}x_{v,j+1}.$$

Adiabatic Quantum Computing

Quantum Computation by Adiabatic Evolution

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MIT CTP # 2936 quant-ph/0001106

Abstract

We give a quantum algorithm for solving instances of the satisfiability problem, based on adiabatic evolution. The evolution of the quantum state is governed by a time-dependent Hamiltonian that interpolates between an initial Hamiltonian, whose ground state is easy to construct, and a final Hamiltonian, whose ground state encodes the satisfying assignment. To ensure that the system evolves to the desired final ground state, the evolution time must be big enough. The time required depends on the minimum energy difference between the two lowest states of the interpolating Hamiltonian. We are unable to estimate this gap in general. We give some special symmetric cases of the satisfiability problem where the symmetry allows us to estimate the gap and we show that, in these cases, our algorithm runs in polynomial time.

1 Introduction

Adiabatic theorem

$$i\hbar \frac{\partial \Psi}{\partial t} = H(t)\Psi$$

$$H(t)\psi_k(t) = E_k(t)\psi_k(t)$$

$$\Psi = \sum_k C_k(t)\psi_k(t)\exp\left[-\frac{i}{\hbar}\int_{t_0}^{t} E_k(t')dt'\right]$$

$$\dot{C}_b(t) = -\sum_k C_k(t)\exp\left\{\frac{i}{\hbar}\int_{t_0}^{t}[E_b(t') - E_k(t')]dt'\right\}\left\langle \psi_b \middle| \frac{\partial \psi_k}{\partial t}\right\rangle.$$

$$\left\langle \frac{\partial \psi_k}{\partial t} \middle| \psi_k \right\rangle + \left\langle \psi_k \middle| \frac{\partial \psi_k}{\partial t} \right\rangle = \alpha_k^*(t) + \alpha_k(t) = 0$$

$$\alpha_b(t) = \langle \psi_b | \partial \psi_b / \partial t \rangle = \mathrm{i}\beta_b(t)$$

$$C_k'(t) = C_k(t) \exp\left[\mathrm{i} \int_{t_0}^{t} \beta_k(t') \mathrm{d}t' \right]$$

$$\dot{C}_b(t) = \sum_{k \neq b} \frac{C_k(t)}{\hbar \omega_{bk}(t)} \left(\frac{\partial H}{\partial t} \right)_{bk} \exp\left[\mathrm{i} \int_{t_0}^{t} \omega_{bk}(t') \mathrm{d}t' \right]$$

$$\omega_{bk}(t) = \frac{E_b(t) - E_k(t)}{\hbar}, \quad b \neq k.$$

$$C_k = \delta_{ka}$$

$$C_b(t) = \hbar^{-1} \int_{t_0}^{t} \mathrm{d}t' \omega_{ba}^{-1}(t') \left(\frac{\partial H(t')}{\partial t'} \right)_{ba} \exp\left[\mathrm{i} \int_{t_0}^{t'} \omega_{ba}(t'') \mathrm{d}t'' \right],$$

$$C_b(t) \simeq (\mathrm{i}\hbar)^{-1} \omega_{ba}^{-2} \left(\frac{\partial H}{\partial t} \right)_{ba} \{ \exp[\mathrm{i}\omega_{ba}(t - t_0)] - 1 \}$$

$$\hat{H}_{tr}(s) = (1-s)\hat{H}_{ini} + s\hat{H}_{fin},$$

$$i\hbar\partial_t\psi(t) = \hat{H}_{tr}(t/T_A)\psi(t) = \left[(1-t/T_A)\hat{H}_{ini} + (t/T_A)\hat{H}_{fin}\right]\psi(t),$$

$$E_0(0)\psi(0) = \hat{H}_{ini}\psi(0). \qquad E_0(T_A)\psi(T_A) = \hat{H}_{fin}\psi(T_A),$$

$$g_{min} = \min_{s\in[0,1]}\{E_1(s) - E_0(s)\} \qquad \mathcal{E} = \max_{s\in[0,1]}\langle\psi_0(s) \mid \hat{H}_{fin} - \hat{H}_{ini} \mid \psi_0(s)\rangle.$$

$$T_A \gg \mathcal{E}\hbar/g_{min}^2$$

$$H_{ini}(\sigma_1^x, ..., \sigma_N^x) = -\Delta\sum_i \sigma_i^x,$$

$$H_{fin} = \sum_i h_i\sigma_i^z + \sum_{ij} J_{ij}\sigma_i^z\sigma_j^z,$$

D-Wave

LETTER

doi:10.1038/nature10012

Quantum annealing with manufactured spins

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Many interesting but practically intractable problems can be reduced to that of finding the ground state of a system of interacting spins; however, finding such a ground state remains computationally difficult[1]. It is believed that the ground state of some naturally occurring spin systems can be effectively attained through a process called quantum annealing[2,3]. If it could be harnessed, quantum annealing might improve on known methods for solving certain types of problem[4,5]. However, physical investigation of quantum annealing has been largely confined to microscopic spins in condensed-matter systems[6–12]. Here we use quantum annealing to find the ground state of an artificial Ising spin system comprising an array of eight superconducting flux quantum bits with programmable spin–spin couplings. We observe a clear signature of quantum annealing, distinguishable from classical thermal annealing through the temperature dependence of the time at which the system dynamics freezes. Our implementation can be configured *in situ* to realize a wide variety of different spin networks, each of which can be monitored as it moves towards a low-energy configuration[13,14]. This programmable artificial spin network bridges the gap between the theoretical study of ideal isolated spin networks and the experimental investigation of bulk magnetic samples. Moreover, with an increased number of spins, such a system may provide a practical physical means to implement a quantum algorithm, possibly allowing more-effective approaches to solving certain classes of hard combinatorial optimization problems.

Physically interesting in their own right, systems of interacting spins also have practical importance for quantum computation[15]. One widely studied example is the Ising spin model, where spins may take on one of two possible values: up or down along a preferred axis. Many seemingly unrelated yet important hard problems, in fields ranging from artificial intelligence[16] to zoology[17], can be reformulated as the problem of finding the lowest energy configuration, or ground state, of an Ising spin system.

Quantum annealing has been proposed as an effective way for finding such a ground state[2–5]. To implement a processor that uses quantum annealing to help solve difficult problems, we would need a programmable quantum spin system in which we could control individual spins and their couplings, perform quantum annealing and then determine the state of each spin. Until recently, physical investigation of quantum annealing has been confined to configurations achievable in condensed-matter systems, such as molecular nanomagnets[6–10] or bulk solids with quantum critical behaviour[11,12]. Unfortunately, these systems cannot be controlled or measured at the level of individual spins, and are typically investigated through the measurement of bulk properties. They are not programmable. Nuclear magnetic resonance techniques have been used to demonstrate a quantum annealing algorithm on three quantum spins[18]. Recently, three trapped ions were used to perform a quantum simulation of a small, frustrated Ising spin system[19].

One possible implementation of an artificial Ising spin system involves superconducting flux quantum bits[20–28] (qubits). We have implemented such a spin system, interconnected as a bipartite graph, using an *in situ* reconfigurable array of coupled superconducting flux qubits[14]. The device fabrication is discussed in Methods and in Supplementary Information. The simplified schematic in Fig. 1a shows two superconducting loops in the qubit, each subject to an external flux bias Φ_{1x} or Φ_{2x}, respectively. The device dynamics can be modelled as a quantum mechanical double-well potential with respect to the flux, Φ_1, in loop 1 (Fig. 1b). The barrier height, δU, is controlled by Φ_{2x}. The energy difference between the two minima, $2h$, is controlled by Φ_{1x}. The two lowest energy states of the system, corresponding to clockwise or anticlockwise circulating current in loop 1, are labelled $|\downarrow\rangle$ and $|\uparrow\rangle$, with flux localized in the left- or the right-hand well (Fig. 1b), respectively. If we consider only these two states (a valid restriction at low temperature), the qubit dynamics is equivalent to those of an Ising spin, and we treat the qubits as such in what follows. Qubits (spins) are

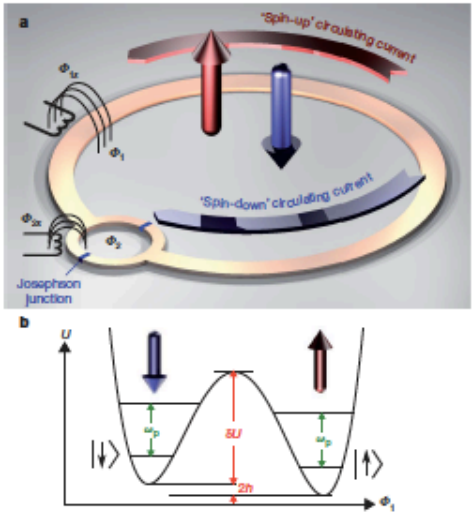
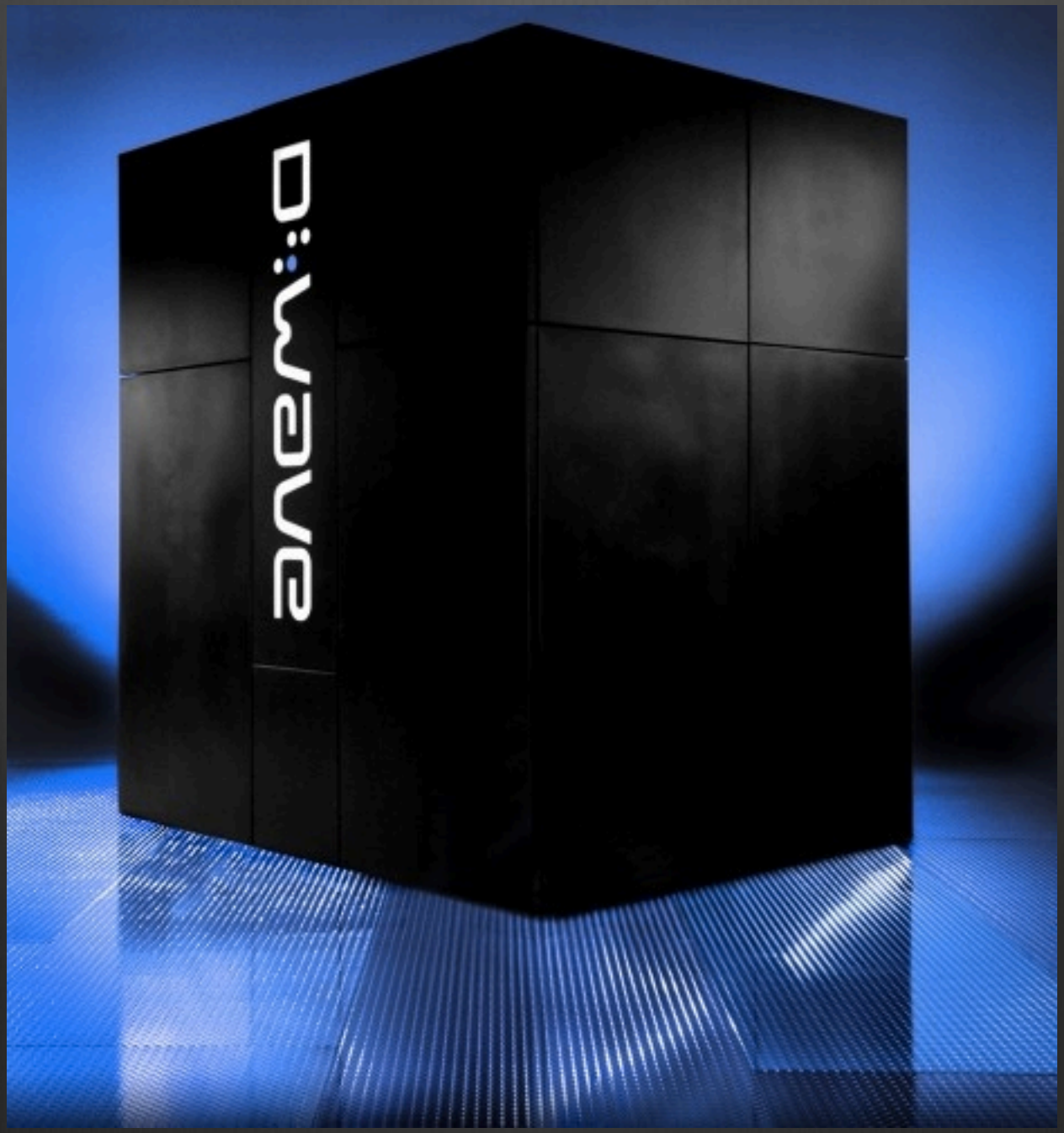
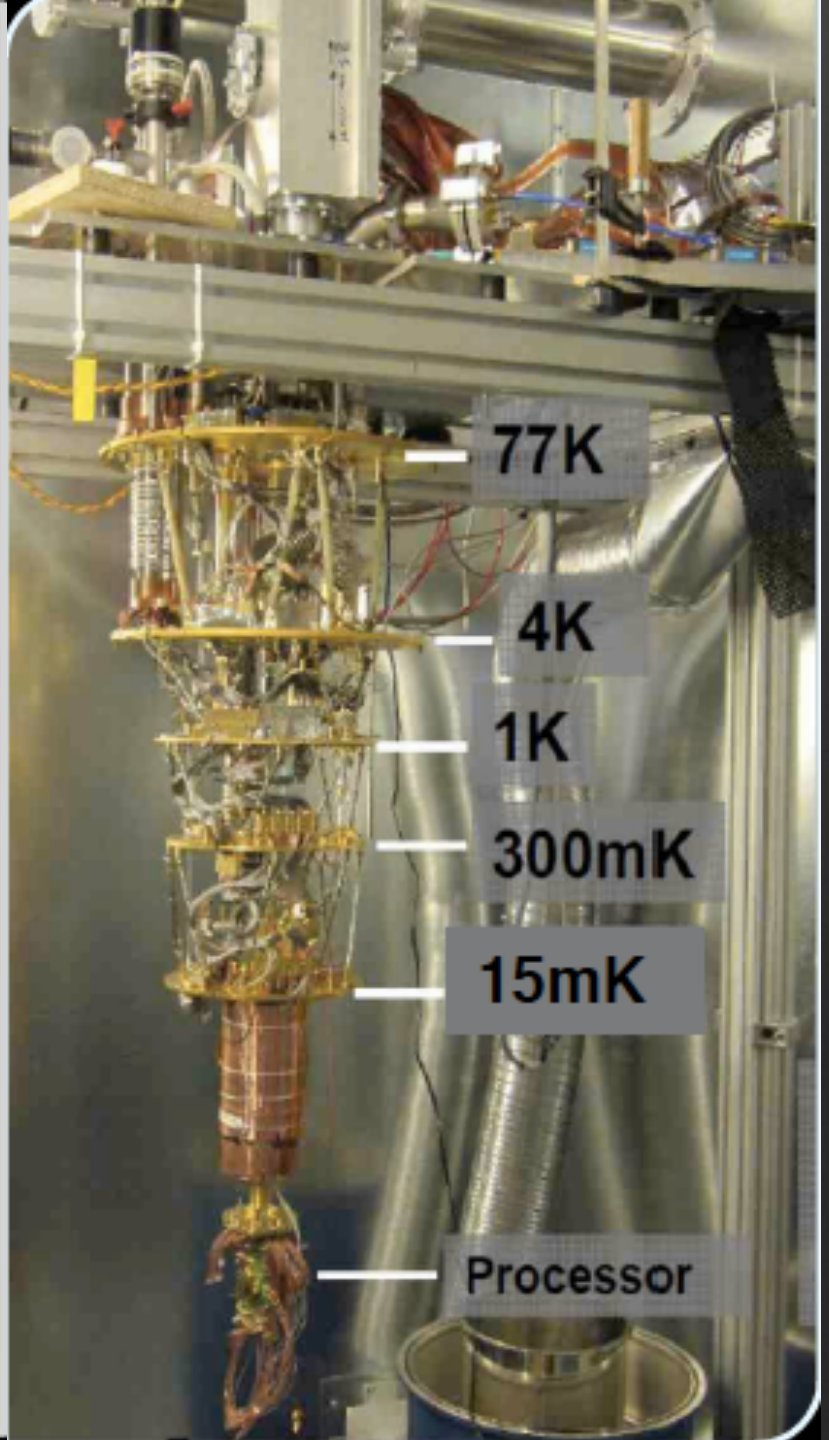
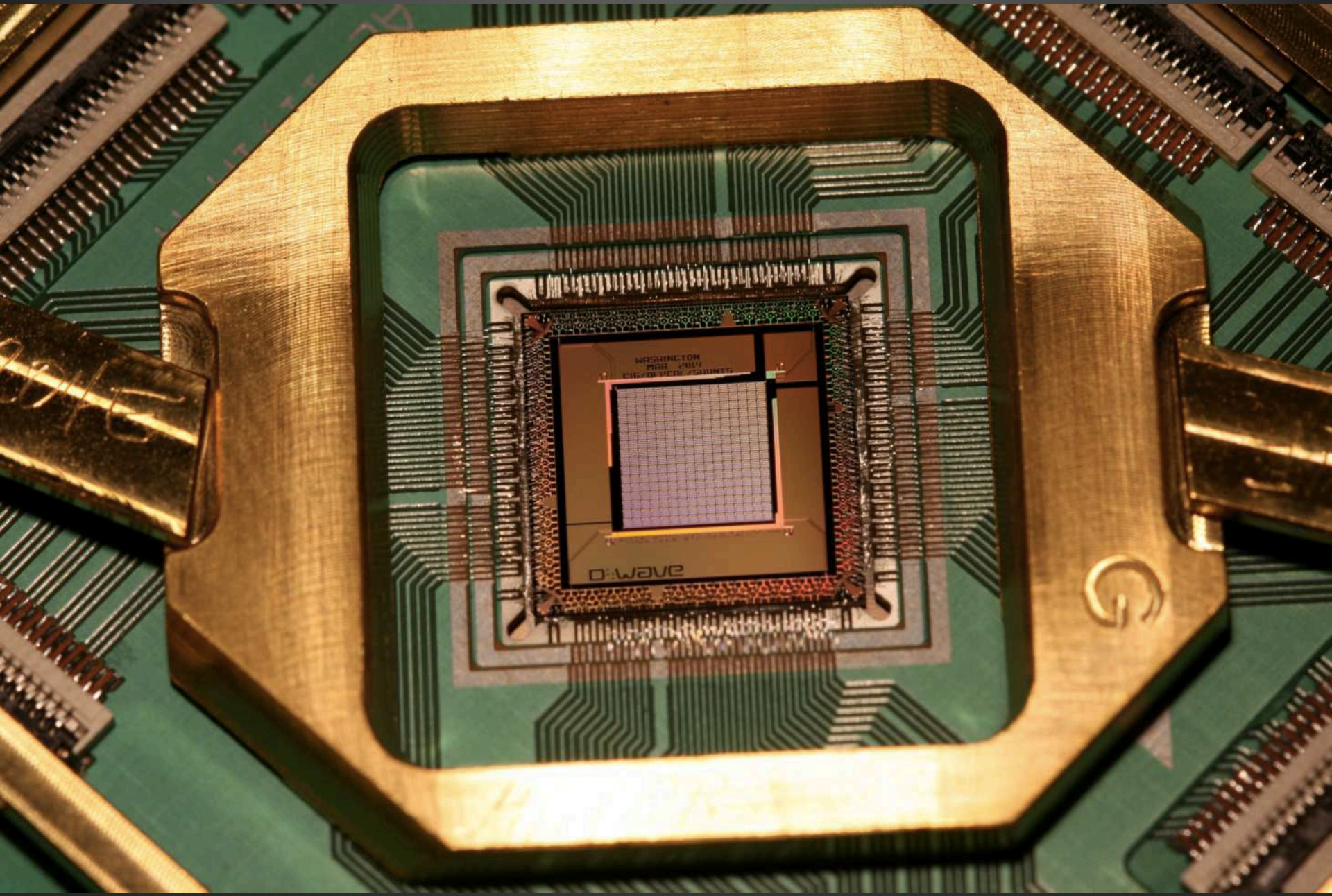


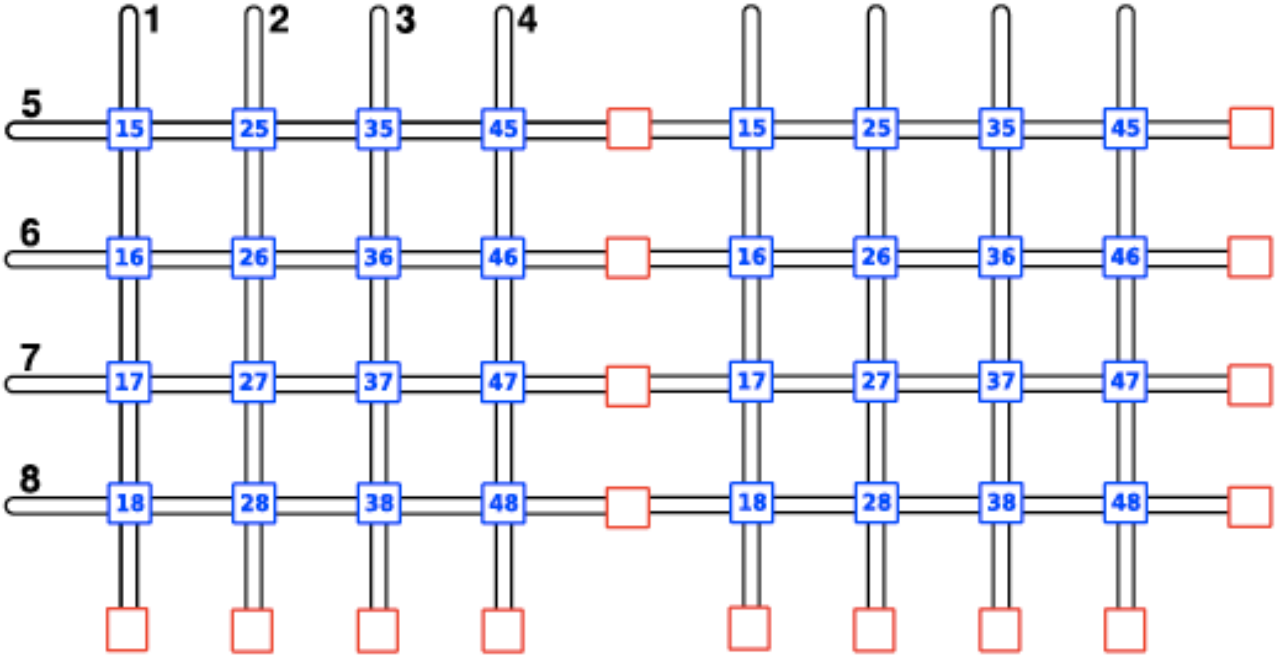
Figure 1 | Superconducting flux qubit. **a**, Simplified schematic of a superconducting flux qubit acting as a quantum mechanical spin. Circulating current in the qubit loop gives rise to a flux inside, encoding two distinct spin states that can exist in a superposition. **b**, Double-well potential energy diagram and the lowest quantum energy levels corresponding to the qubit. States $|\uparrow\rangle$ and $|\downarrow\rangle$ are the lowest two levels, respectively. The intra-well energy spacing is ω_p. The measurement detects magnetization, and does not distinguish between, say, $|\uparrow\rangle$ and excited states within the right-hand well. In practice, these excitations are exceedingly improbable at the time the state is measured.

[1]D-Wave Systems Inc., 100-4401 Still Creek Drive, Burnaby, British Columbia V5C 6G9, Canada. [2]Department of Natural Sciences, University of Agder, Post Box 422, NO-4604 Kristiansand, Norway. [3]Department of Physics, Simon Fraser University, Burnaby, British Columbia V5A 1S6, Canada.


D-Wave


77K
4K
1K
300mK
15mK
Processor


D-Wave


1
2
3
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8
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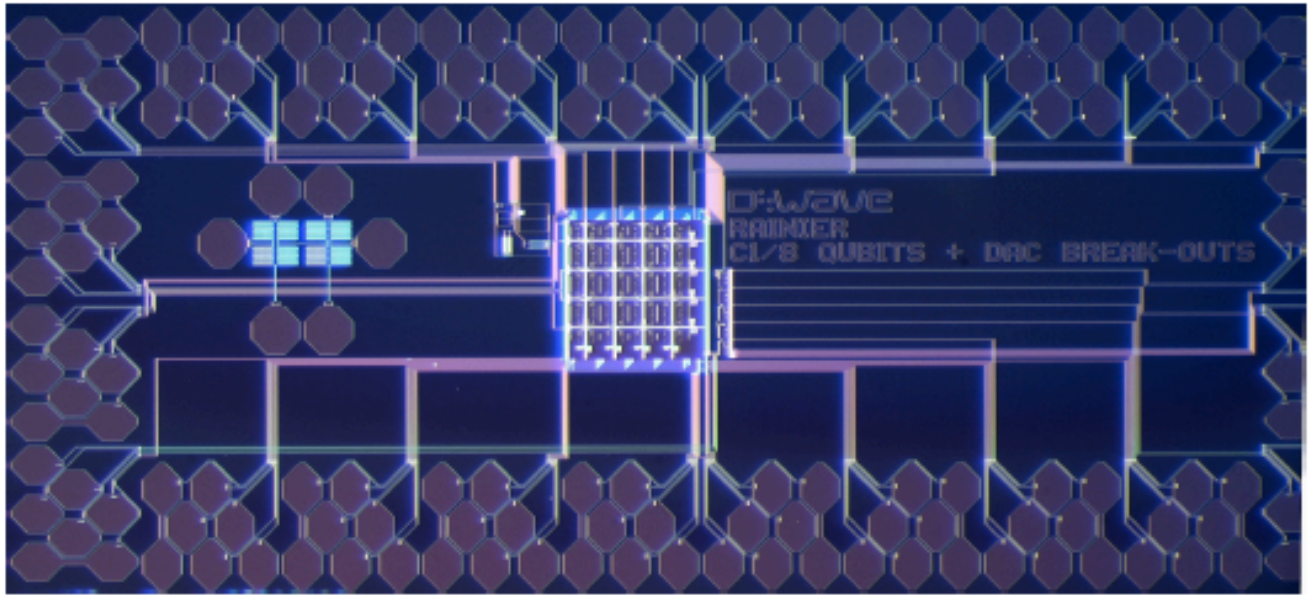
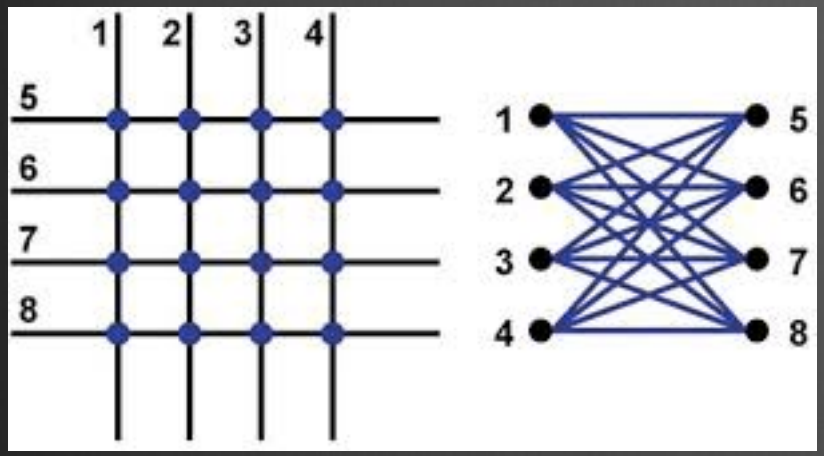

D-WAVE
RAINIER
C1/8 QUBITS + DAC BREAK-OUTS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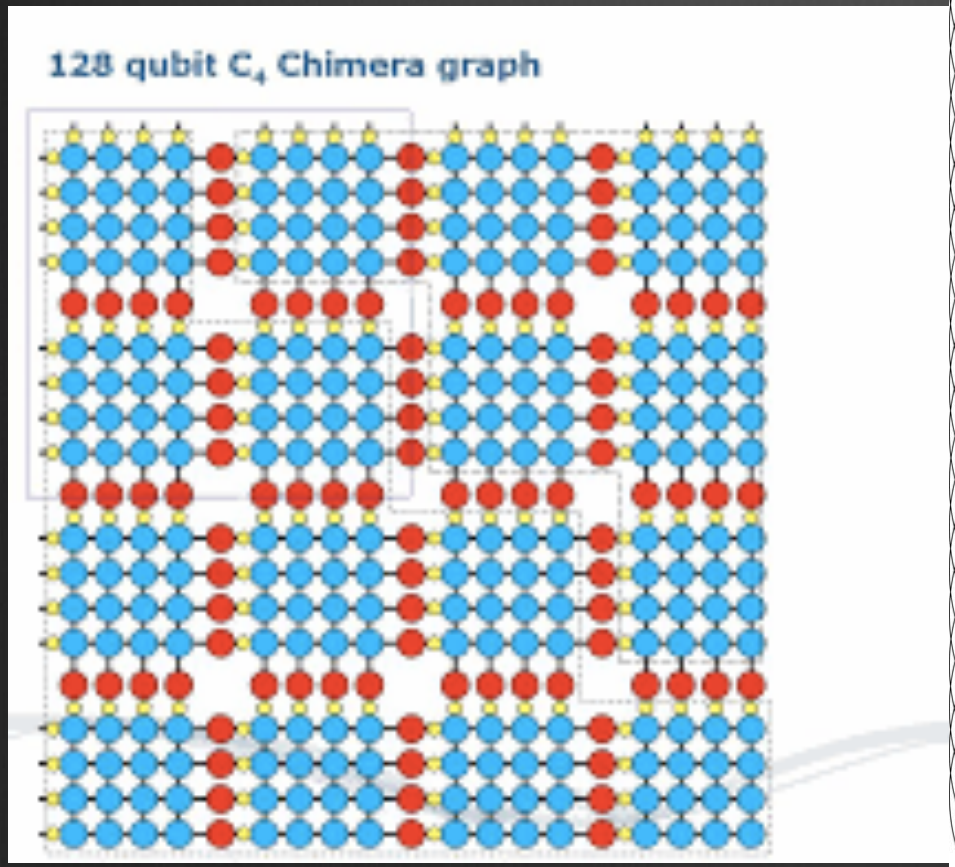
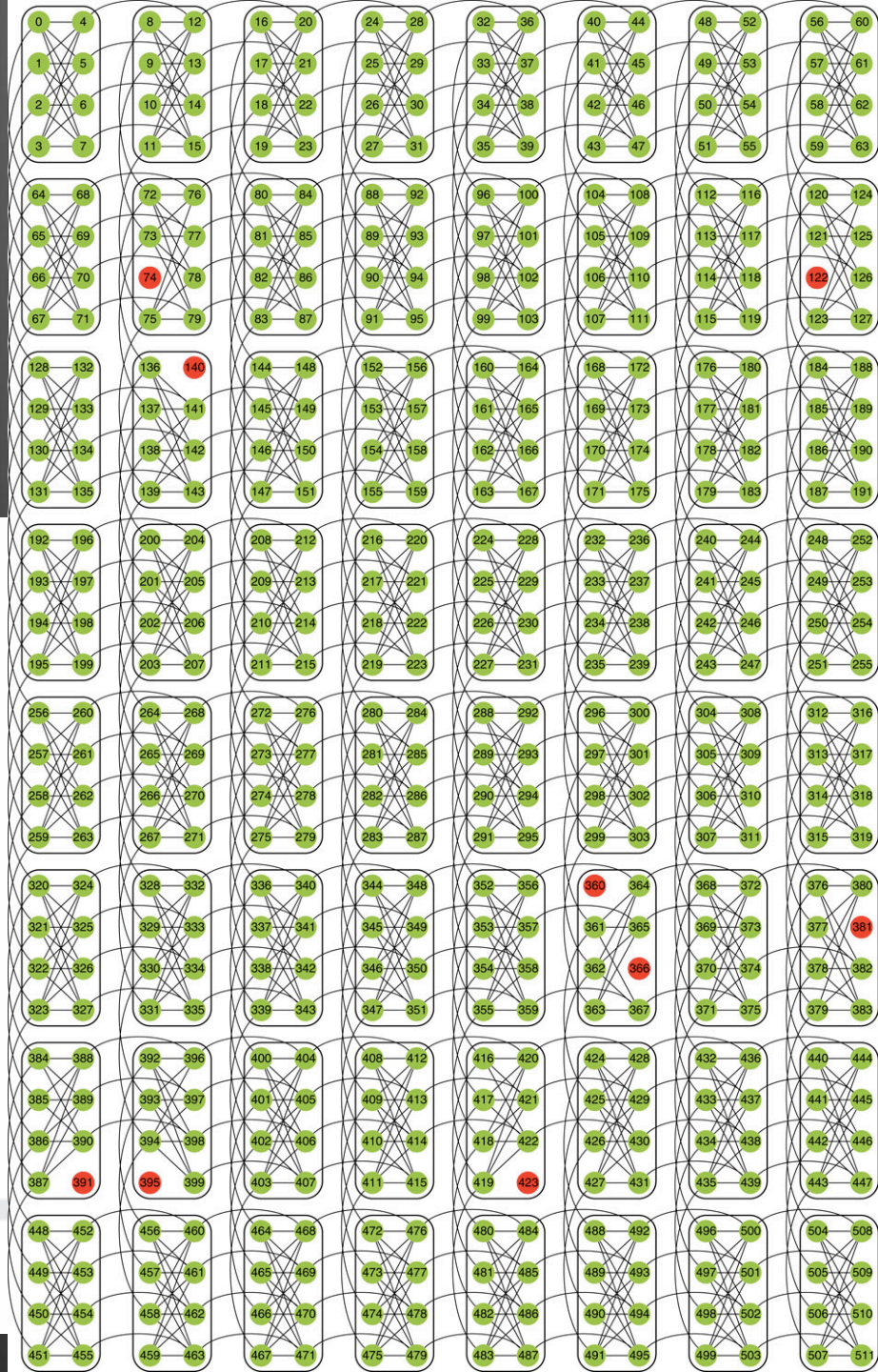

128 qubit C_4 Chimera graph



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Adiabatic Quantum Computation Is Equivalent to Standard Quantum Computation*

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Wim van Dam‡
Julia Kempe§
Zeph Landau¶
Seth Lloyd‖
Oded Regev§

Abstract. The model of adiabatic quantum computation is a relatively recent model of quantum computation that has attracted attention in the physics and computer science communities. We describe an efficient adiabatic simulation of any given quantum circuit. This implies that the adiabatic computation model and the standard circuit-based quantum computation model are polynomially equivalent. Our result can be extended to the physically realistic setting of particles arranged on a two-dimensional grid with nearest neighbor interactions. The equivalence between the models allows one to state the main open problems in quantum computation using well-studied mathematical objects such as eigenvectors and spectral gaps of Hamiltonians.

Key words. quantum computation, adiabatic computation, nearest neighbor interactions

AMS subject classifications. 81P68, 68Q05

DOI. 10.1137/080734479

1. Introduction. The model of quantum computation has been thoroughly investigated in the last two decades and is by now a well-established one [47]. In this

*Published electronically November 5, 2008. This paper originally appeared in *SIAM Journal on Computing*, Volume 37, Number 1, 2007, pages 166–194. This work was performed by an employee of the U.S. Government or under U.S. Government contract. The U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes. Copyright is owned by SIAM to the extent not limited by these rights.
http://www.siam.org/journals/sirev/50-4/73447.html

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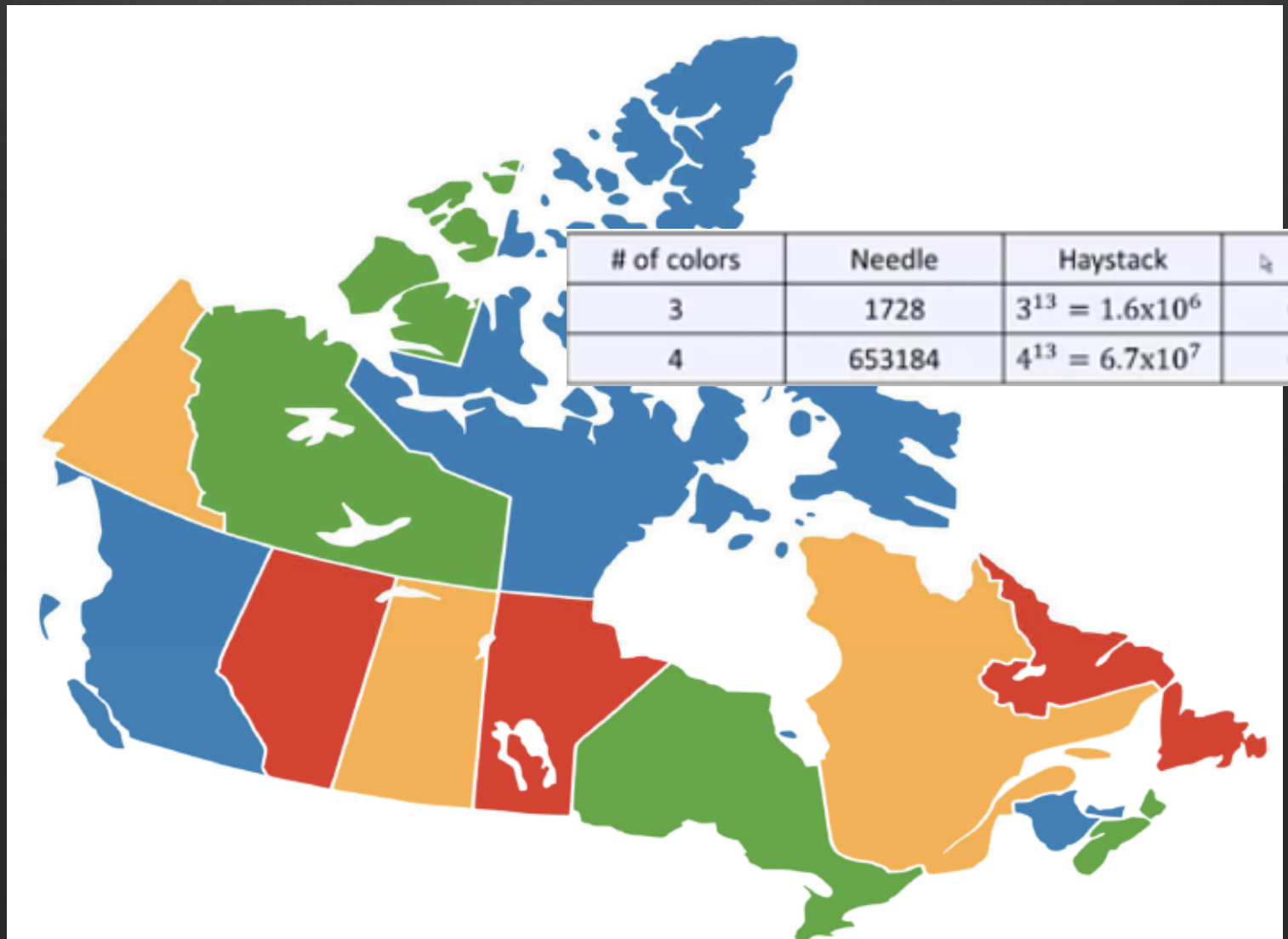
Programming D-Wave

Map coloring problem



# of colors	Needle	Haystack	N/H
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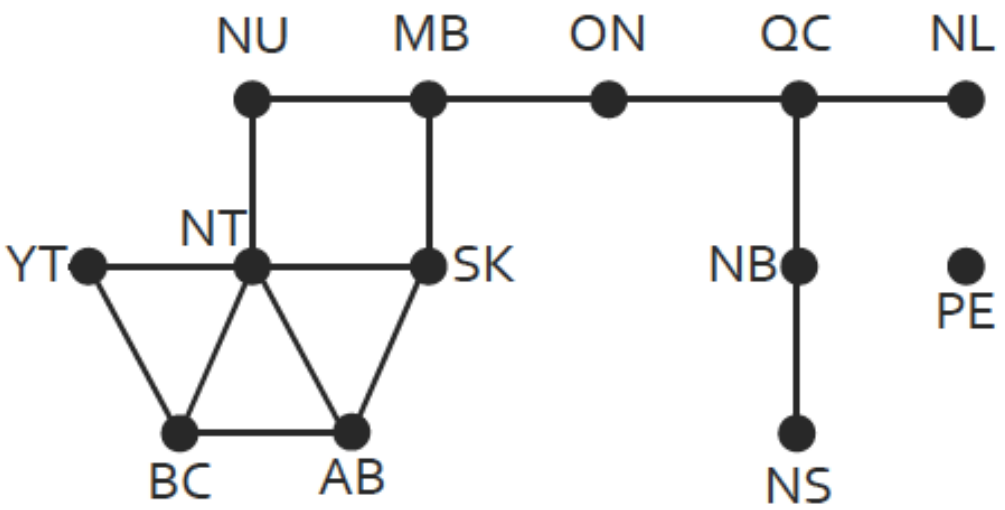
4	653184	$4^{13} = 6.7\text{x}10^{7}$	0.0097



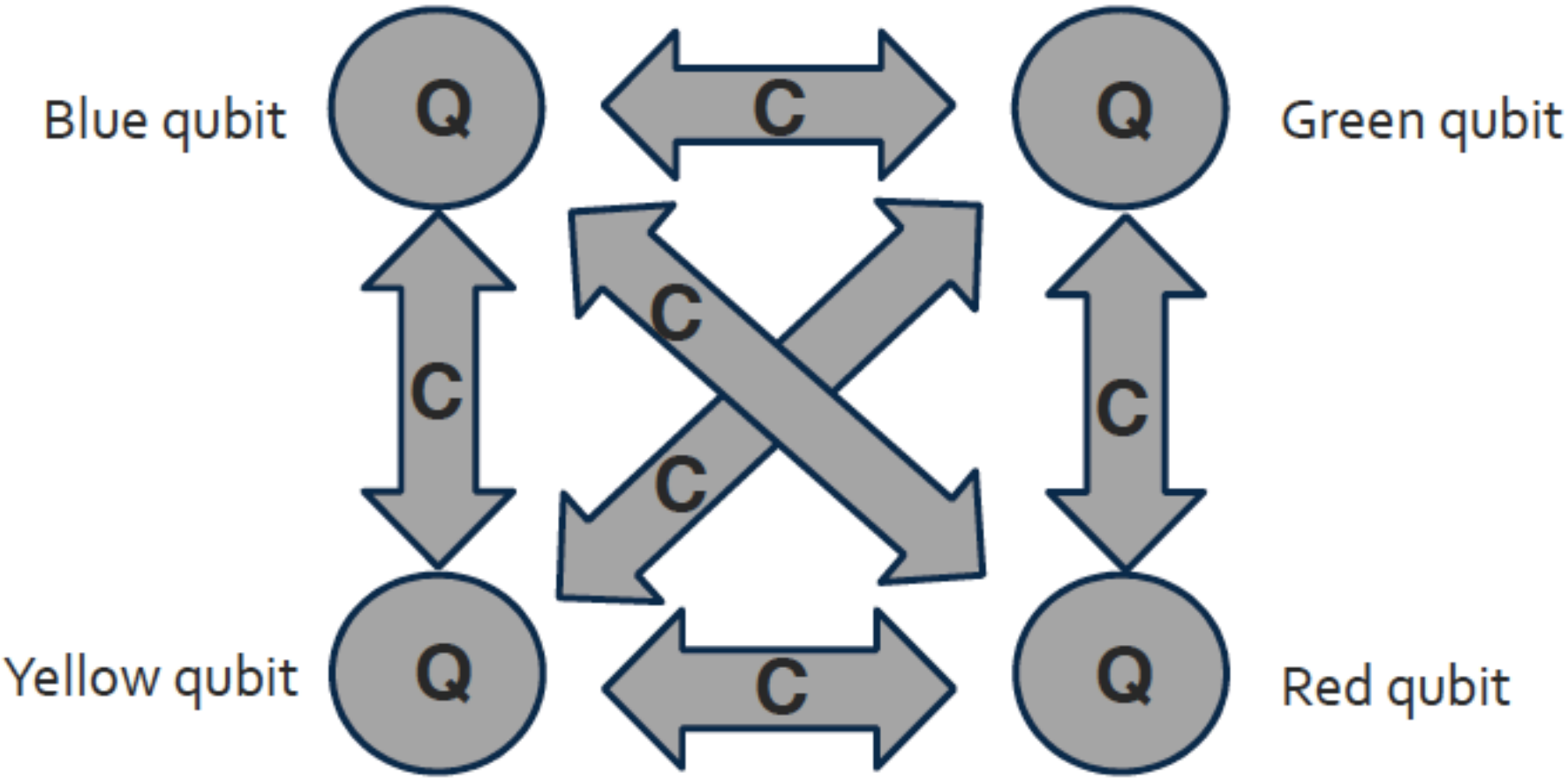
AB Alberta
BC British Columbia
MB Manitoba
NB New Brunswick
NL Newfoundland and Labrador
NS Nova Scotia
NT Northwest Territories
NU Nunavut
ON Ontario
PE Prince Edward Island
QC Quebec
SK Saskatchewan
YT Yukon

$$H = A\sum_{v}\left(1-\sum_{i=1}^{n}x_{v,i}\right)^{2} + A\sum_{(uv)\in E}\sum_{i=1}^{n}x_{u,i}x_{v,i}.$$

Objectives

$$O(\mathbf{a}, \mathbf{b}; \mathbf{q}) = \sum_{i=1}^{N} a_i q_i + \sum_{<i,j>} b_{ij} q_i q_j$$

q_1	q_2	$O(\mathbf{a}, \mathbf{b}; \mathbf{q})$
0	0	0
0	1	a_2
1	0	a_1
1	1	$a_1 + a_2 + b_{12}$

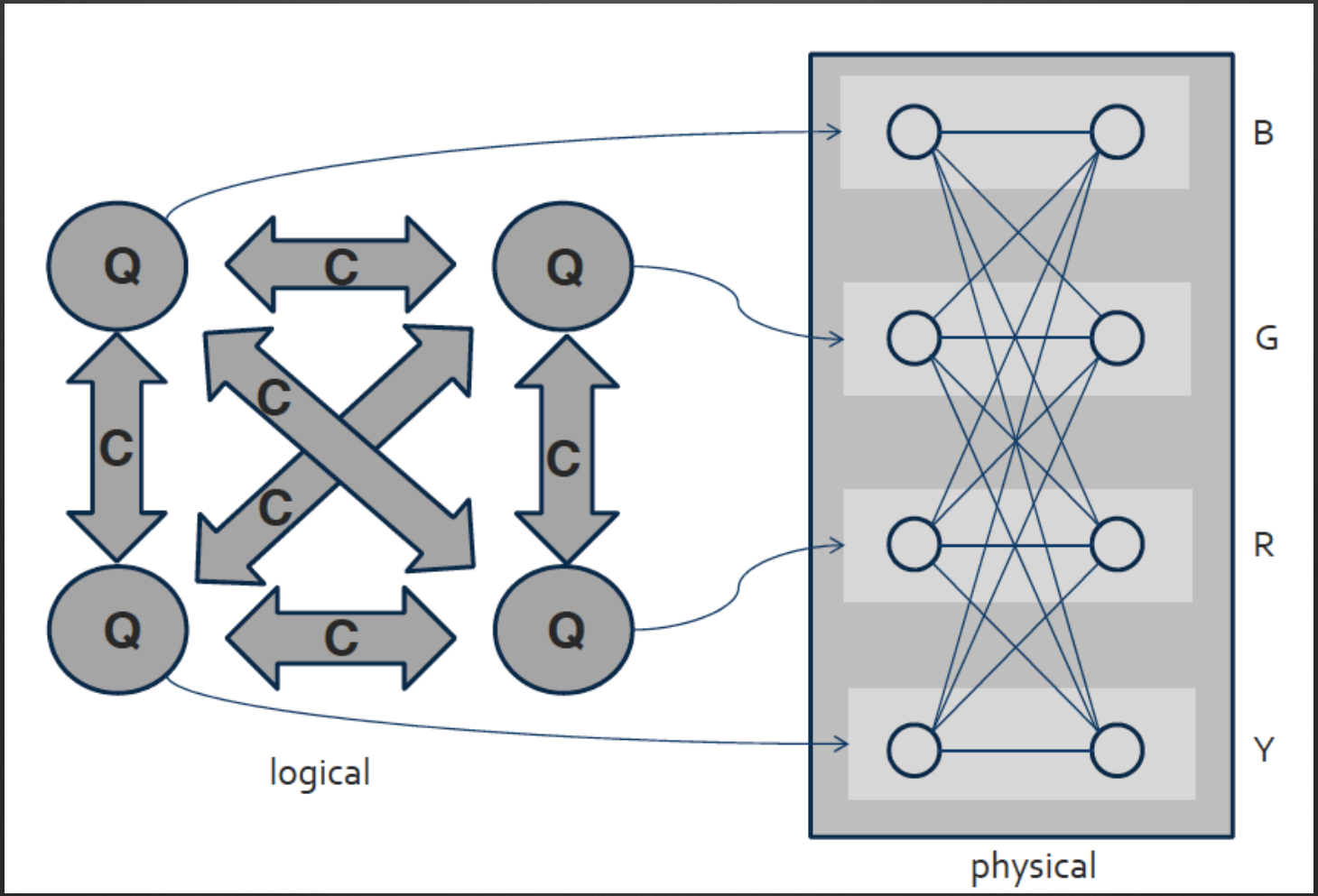


Objective : $O(q_b, q_g, q_r, q_y) = (q_b + q_g + q_r + q_y - 1)^2 \cong$

$$-1(q_b + q_g + q_r + q_y)$$

$$+2(q_b q_g + q_b q_r + q_b q_y + q_g q_r + q_g q_y + q_r q_y)$$

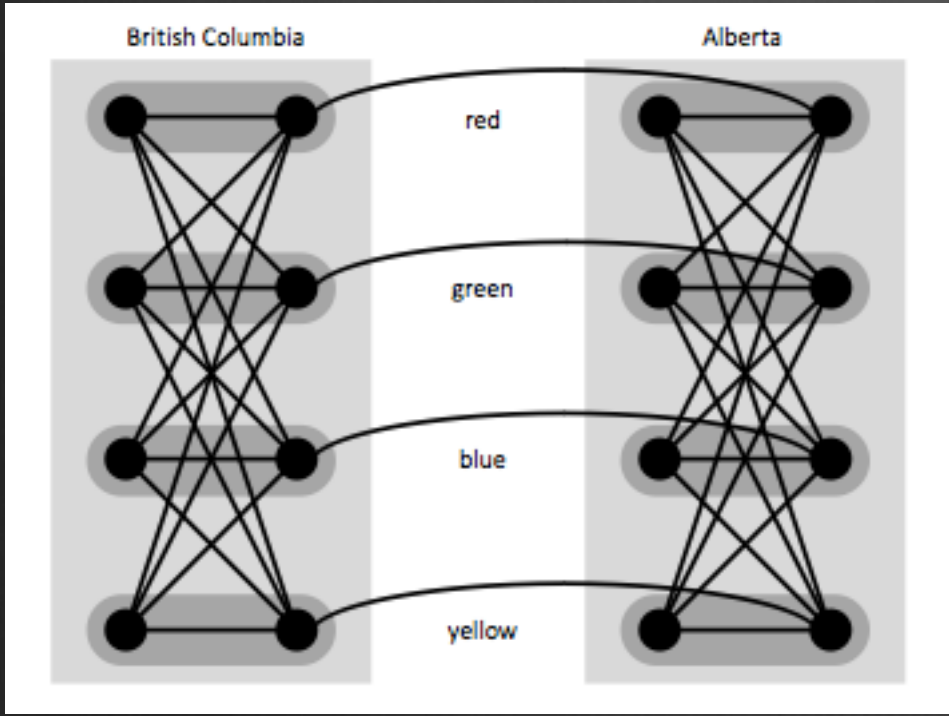
Maping onto the unit cell



Code

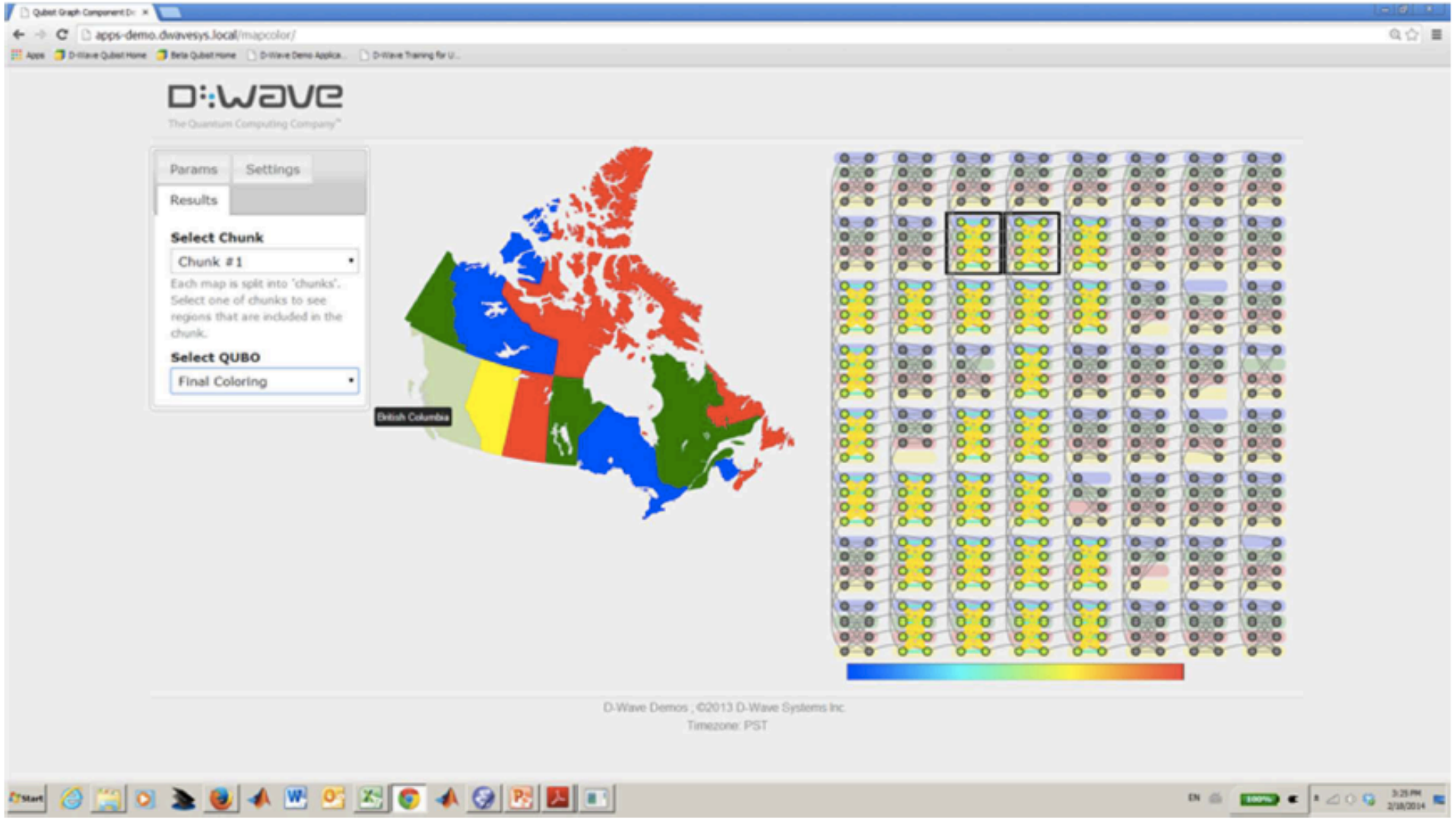
```
/* STEP 1: turn on one of C qubits */
/* Handle weights                  */
for (i=0; i<C; ++i)
  {
    weight[DW_QUBIT(row,col,'L',i)] += -0.5;
    weight[DW_QUBIT(row,col,'R',i)] += -0.5;
  }
/* Handle strengths */
for (i=0; i<C; ++i)
  for (j=0; j<C; ++j)
    if (i != j)
      strength[DW_INTRACELL_COUPLER(row,col,i,j)] += 1;
```

Neighbors and cloning



	0	1	2	3	4
0	NL	ON	MB	SK	AB
1	PE	QC	NU	NT	AB
2		NB	NS	NT	BC
3				YT	BC

GUI


apps-demo.dwavesys.local/mapcolor/
D:Wave
The Quantum Computing Company™
Params
Settings
Results
Select Chunk
Chunk #1
Each map is split into 'chunks'. Select one of chunks to see regions that are included in the chunk.
Select QUBO
Final Coloring
British Columbia
D-Wave Demos , ©2013 D-Wave Systems Inc.
Timezone: PST

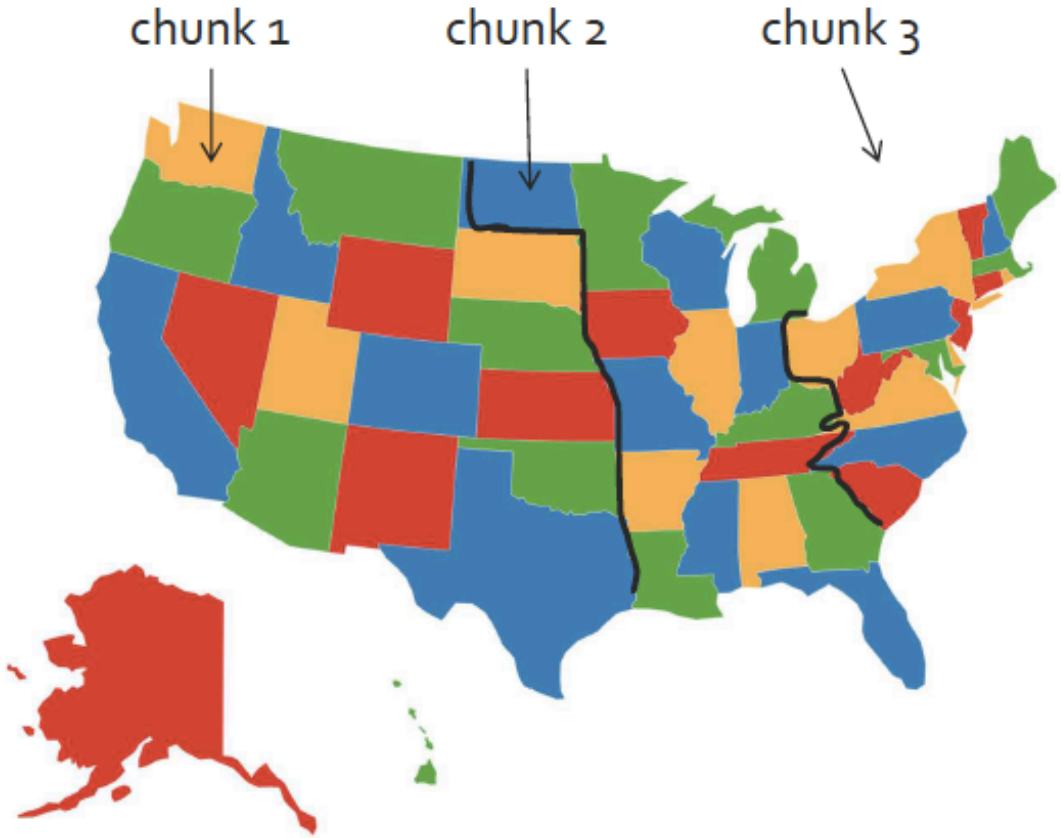
Chunks

Divide the US map into chunks.

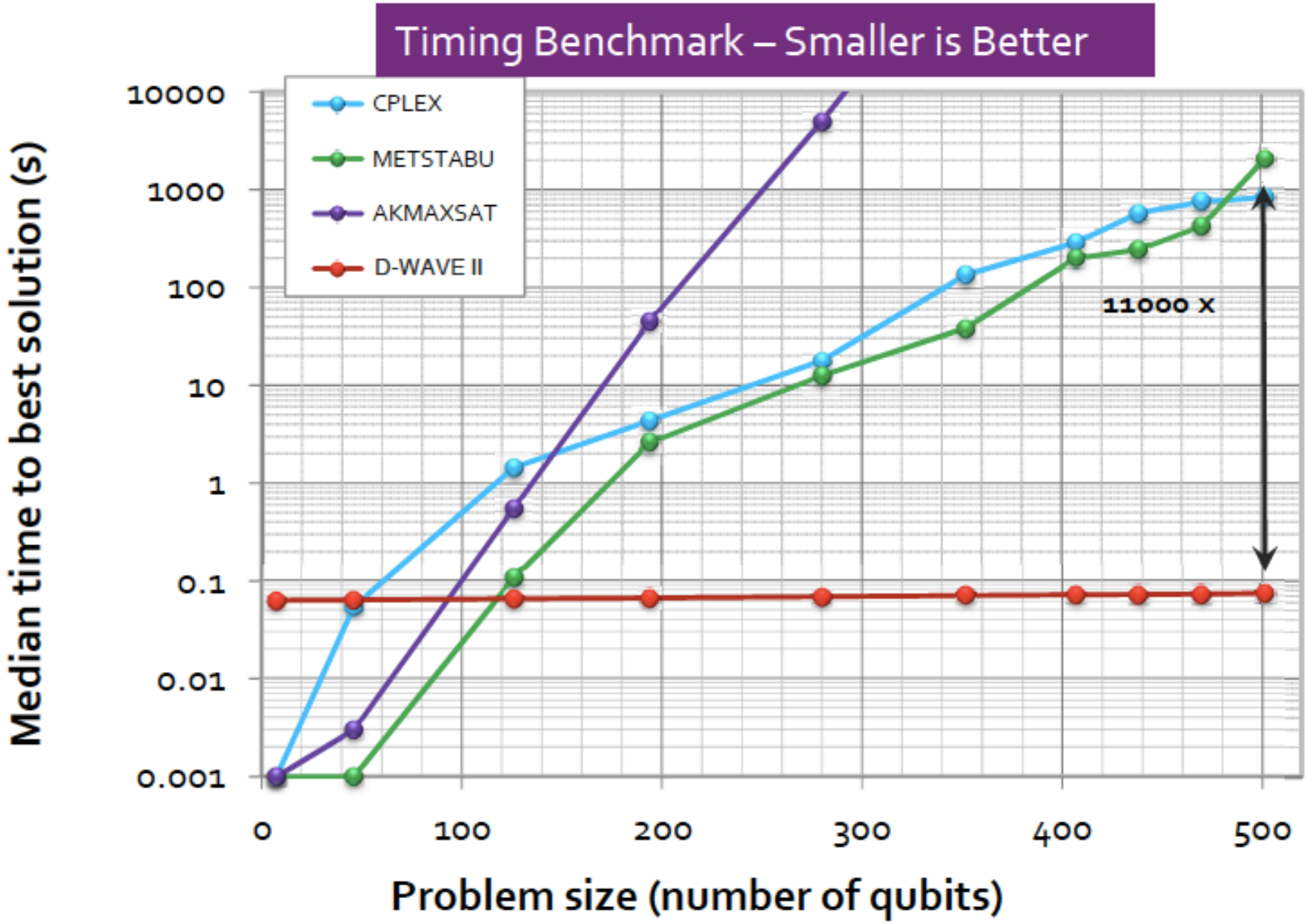
Process the first chunk and get valid colorings for the first chunk of states.

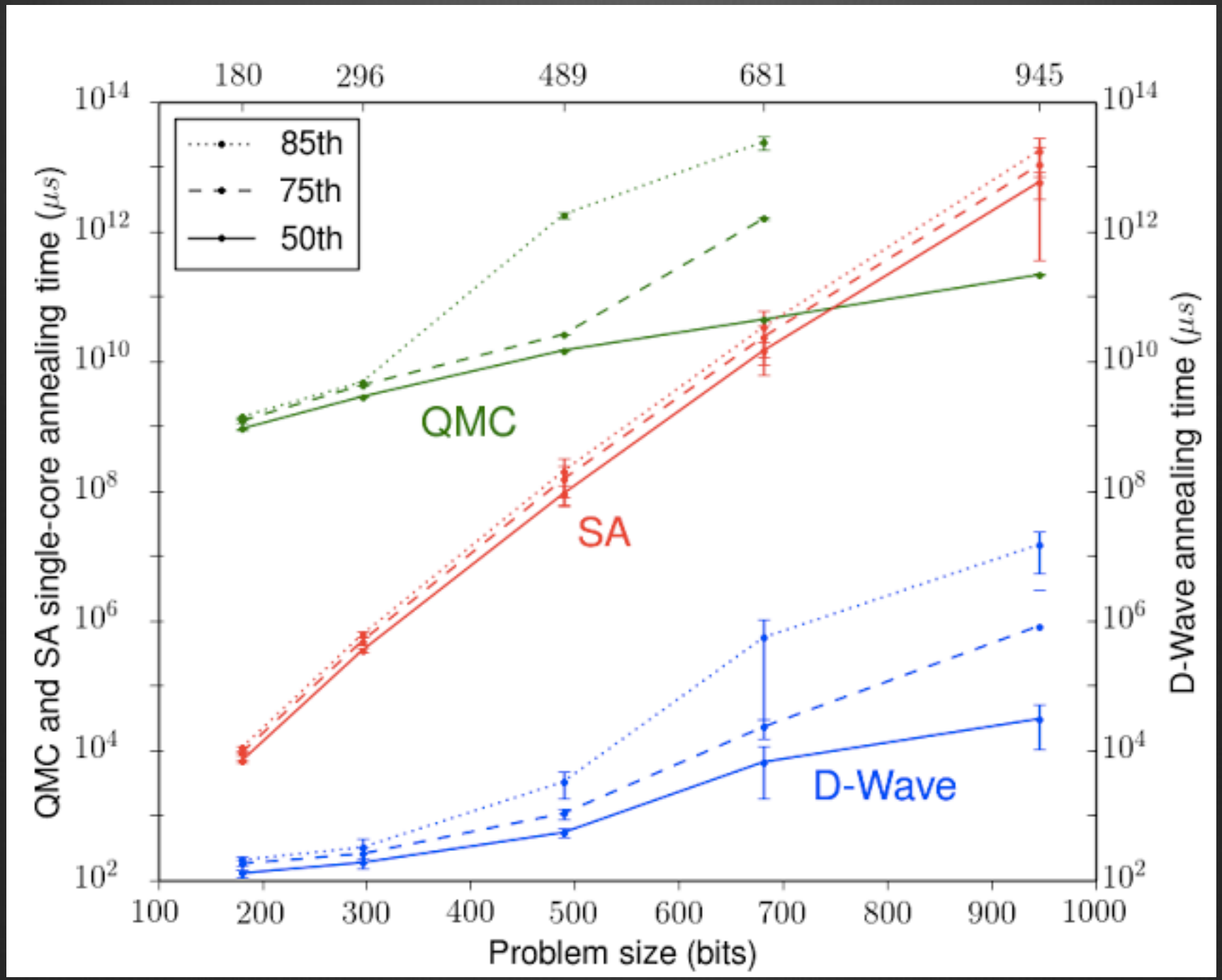
Use these colorings to *bias* the second chunk.

Repeat.



# of colors	Needle	Haystack	N/H
3	0	$3^{49} = 2.4\text{x}10^{23}$	0
4	25623183458304	$4^{49} = 3.2\text{x}10^{29}$	$8\text{x}10^{-17}$


Timing Benchmark – Smaller is Better
CPLEX
METSTABU
AKMAXSAT
D-WAVE II
11000 X
10000
1000
100
10
1
0.1
0.01
0.001
0
100
200
300
400
500
Median time to best solution (s)
Problem size (number of qubits)

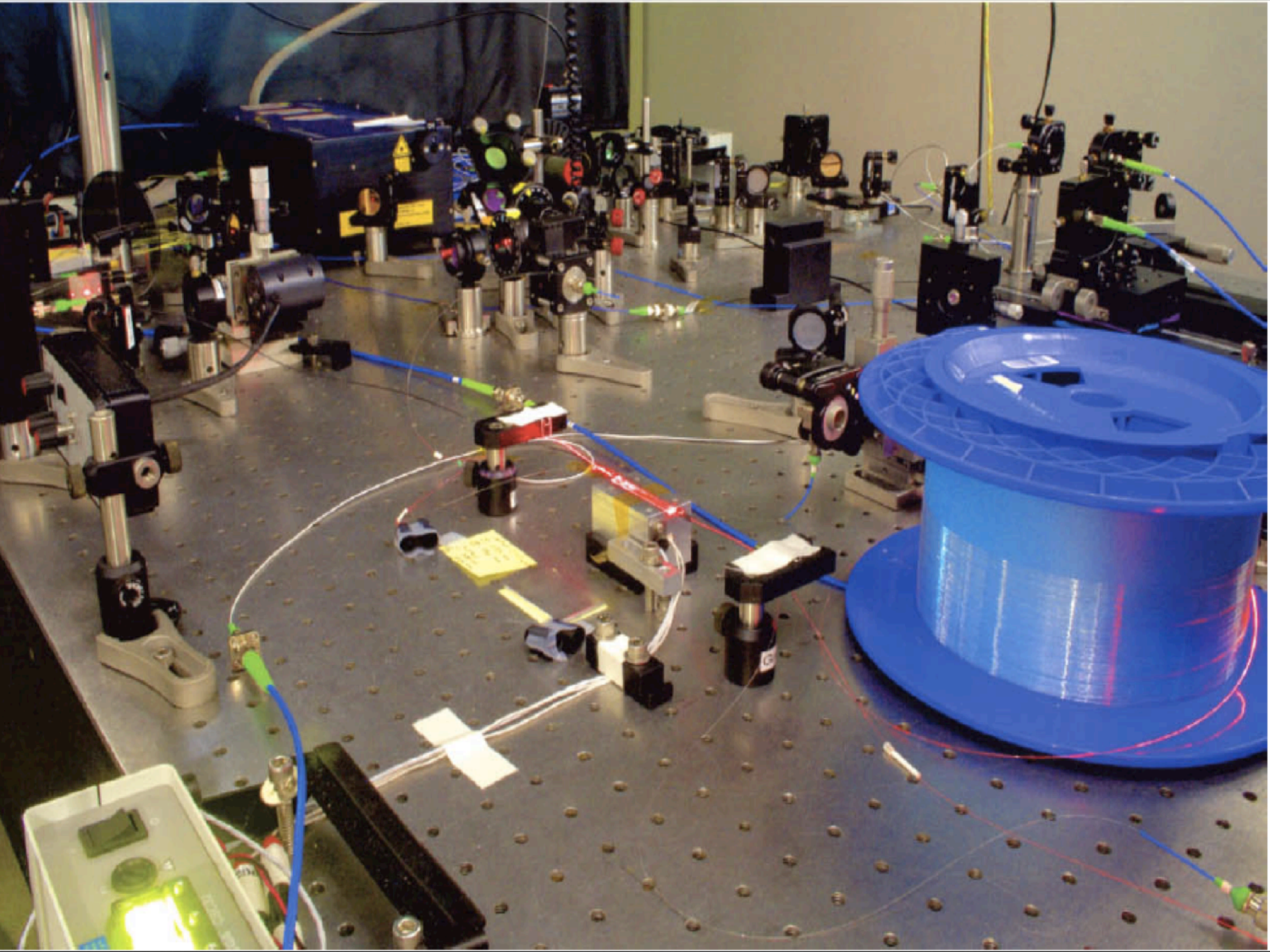

180
296
489
681
945
85th
75th
50th
QMC
SA
D-Wave
QMC and SA single-core annealing time (μs)
D-Wave annealing time (μs)
Problem size (bits)
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
100
200
300
400
500
600
700
800
900
1000

Machine Learning: Binary Classification

- Traditional algorithm recognized car about 84% of the time
- Google/D-Wave Qboost algorithm implemented to recognize a car (cars have big shadows!)
- "Quantum Classifier" was more accurate (94%) and more efficient
- Ported quantum classifier back to traditional computer, more accurate and fewer CPU cycles (less power)!

D-Wave
The Quantum Computing Company™

Build your own …



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ARTICLE

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A poor man's coherent Ising machine based on opto-electronic feedback systems for solving optimization problems

Fabian Böhm [1], Guy Verschaffelt [1] & Guy Van der Sande [1]

Coherent Ising machines (CIMs) constitute a promising approach to solve computationally