Thermodynamics of rarefied gases

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What are the plasmas from a continuum perspective?

Ionized gases.

One utilization: fusion reactor, ionized rarefied gases.

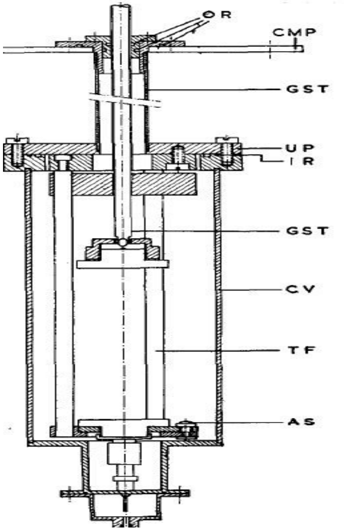
What are the rarefied gases from a continuum perspective?

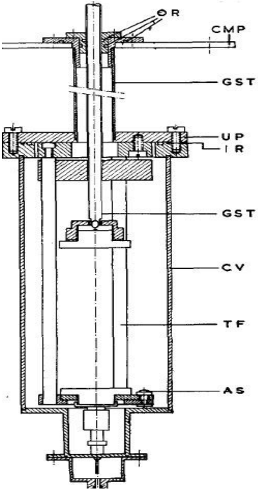
Low-pressure states of gases.

Problem: Navier-Stokes-Fourier equations are not valid.

Experimental arrangement

1. There is some gas in a vessel.
2. Under controlled pressure.
3. On controlled temperature.
4. Excited by a constant frequency.
5. Incident signal is measured on the detector.





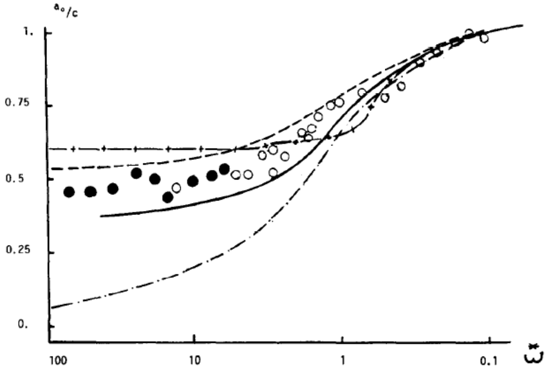


Fig. 1. The phase speed versus frequency. Experimental data: ● Mayer and Sessler; ○ Greenspan.
Theoretical results:
—·— Navier-Stokes;
----- Woods-Troughton;
-+-+- Anile-Pluchino;
—— Lebon-Cloot.

Reason: the presence of ballistic effects.

Solution: extending the Navier-Stokes-Fourier system.

Electrodynamic part is neglected.

Tool: irreversible thermodynamics.

Aspects of ballistic propagation

- Ballistic propagation: goes with speed of sound.
    - Rational Extended Thermodynamics (RET): phonon hydro, **non-interacting particles**, particle-wall interaction. **ONLY** for low temperature, rarefied or small systems...
    - Continuum theory: elastic wave propagation. It can be used even in **room temperature** case.

Speed of sound $\Rightarrow$ mechanical coupling!

- Experiments
    - heat conduction: rarefied phonon gas
    - acoustics: rarefied gases

Generalization of fluid dynamics: Meixner's theory

- Balances: mass, energy, momentum $\rightarrow$
  $\dot{\rho} + \rho \partial^i v^i = 0,$
  $\rho \dot{e} + \partial^i q^i = -P^{ij} \partial^i v^j,$
  $\rho \dot{v}^i + \partial^j P^{ij} = 0,$
  and $P^{ij} = \Pi^{ij} + p\delta^{ij}$ (static (p) and dynamic (Π^{ij}) pressure).
- entropy density: $s(e, \rho, Q^{ij}) = s_{eq}(e, \rho) - \frac{m_1}{2} Q^{ij} Q^{ij},$
- entropy current: $J^i = q^i / T$, classical! $\rightarrow$
- NO coupling!

Constitutive equations: generalized Navier-Stokes

$$q + \lambda \partial_x T = 0,$$
$$\tau_\Pi \dot{\Pi} + \Pi + \nu \partial_x v = 0.$$

Q^{ij}: pressure!

Heat conduction? Coupling? Not enough.

Generalization of fluid dynamics: RET

Arima et al. (2014),
derived from kinetic theory for polyatomic gases:

$$\partial_t F + \partial_k F^k = 0, \text{ mass balance}$$
$$\partial_t F^i + \partial_k F^{ik} = 0, \text{ momentum balance}$$
$$\partial_t F^{ij} + \partial_k F^{ijk} = P^{ij},$$
$$\partial_t G^{ii} + \partial_k G^{iik} = 0, \text{ energy balance}$$
$$\partial_t G^{ppi} + \partial_k G^{ppik} = Q^{ppi}$$

Reason: energy is the trace of pressure!

Constitutive equations (1D), linearized, coupled!

$$\tau_q \dot{q} + q + \lambda \partial_x T + a T_0 \partial_x \Pi = 0,$$
$$\tau_\Pi \dot{\Pi} + \Pi + \nu \partial_x v + \frac{\nu}{1 + c_v^*} \partial_x q = 0.$$

Generalization of fluid dynamics: NET + IV

Non-equilibrium thermodynamics with internal variables

Balances +

- entropy density: $s(e,\rho,q^i,Q^{ij}) = s_e(e,\rho) - \frac{m_1}{2} q^i q^i - \frac{m_2}{2} Q^{ij} Q^{ij}$
- entropy current: $J^i = \boxed{b^{ij} q^j}$ $\to$ coupling!

Constitutive equations (1D), linearized, coupled!

$$\tau_q \dot{q} + q + \lambda \partial_x T + \varepsilon \partial_x \Pi = 0,$$
$$\tau_\Pi \dot{\Pi} + \Pi + \nu \partial_x v + \eta \partial_x q = 0.$$

Q^{ij}: pressure = Meixner's theory!

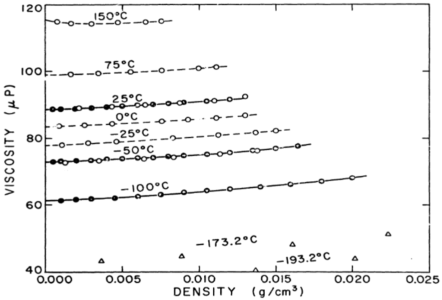
"Ballistic generalization": **thermodynamic equivalence** between phonon and real gases!

Rarefied gases: density dependence.

Cornerstone of rarefaction: density dependence

- Viscosity
  - Reality: same viscosity at 1 and $< 10^{-4}$ atm? $\rightarrow$ Experiments!
  - RET: constant, e.g., $\nu = p\tau_2 \rightarrow \tau_2 \sim \frac{1}{\rho}$.
    (Only for?) Maxwell molecules.
    Density dependence: Enskog correction.
  - Continuum theory: $\nu \sim \rho \rightarrow$ role of kinematic viscosity!

Experiments: Gracki et al, 1969.



Cornerstone of rarefaction: density dependence

Assumptions:

- Viscosity
  - RET: constant, e.g., $\nu = p\tau_2 \to \tau_2 \sim \frac{1}{\rho}$.
  - Continuum theory: $\nu \sim \rho \to$ role of kinematic viscosity!
- Thermal conductivity
  - Reality: same conductivity at 1 and $< 10^{-4}$ atm?
  - RET: constant, i.e., $\lambda \sim \rho\tau_1$.
  - Continuum theory: $\lambda \sim \frac{1}{\rho}$ condition of ballistic effects!
- Relaxation time
  - Expectation: rarefaction leads to non-classical the behavior
  - RET: $\tau \sim \frac{1}{\rho}$
  - Continuum theory: $\tau \sim \frac{1}{\rho}$
- Coupling parameters
  - RET: constrained and can not be adjusted!
  - Continuum theory: free, $\square_{12} \sim \frac{1}{\rho}$, $\square_{21} \sim \rho$ (reciprocity)

Final benchmark: experiments

Scaling of $\frac{\omega}{p}$?!
Kinetic theory: strict expectation.
After calculating the dispersion relations...

1. Classical and generalized NSF theory: $\frac{\omega}{p}$ dependence is obtained!
   Only for constant material parameters and ideal gas.
2. These should not be necessary!

Common point: to evaluate the measurements,
ω **and** p **must be given separately!**

SEE THE EXPERIMENTS!

Experimental arrangement

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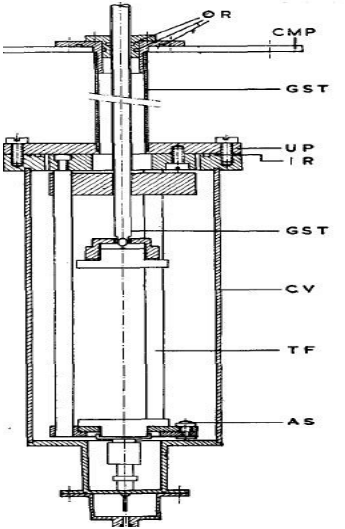
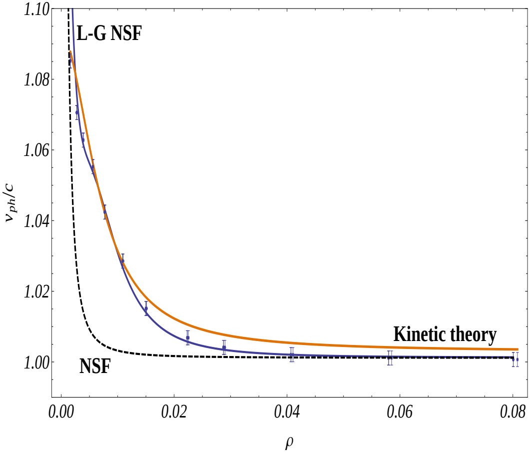
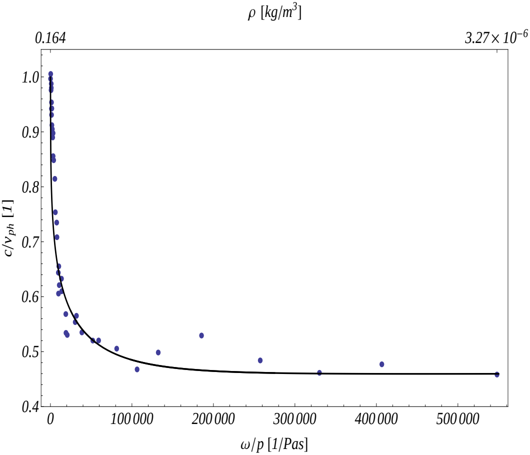


Experiments - Rhodes (1946)

- If ω and p are given separately.
- Using the same temperature with varying the density.



Experiments - Mayer and Sessler (1957)



Outlook

- The generalized theories effectively characterize the experiments.
- Unified framework for thermo-mechanical phenomena.
- Electrodynamics is missing. Future work.

Thank you for your kind attention!

About kinetic theory - phonon hydrodynamics

Momentum series expansion + truncation closure

$$u_{\langle i_1 i_2 \dots i_N \rangle} = \int k n_{\langle i_1 \dots i_n \rangle} f dk.$$

$$\frac{\partial u_{\langle n \rangle}}{\partial t} + \frac{n^2}{4n^2-1} c \frac{\partial u_{\langle n-1 \rangle}}{\partial x} + c \frac{\partial u_{\langle n+1 \rangle}}{\partial x} = \begin{cases} 0 & n = 0 \\ -\frac{1}{\tau_R} u_{\langle 1 \rangle} & n = 1 \\ -\left(\frac{1}{\tau_R} + \frac{1}{\tau_N} \right) u_{\langle n \rangle} & 2 \leq n \leq N \end{cases}$$

It requires at least N=30 momentum equations to **approximate** the real propagation speeds!

Coupling between the heat flux and the pressure!

Ballistic-conductive system, tested on NaF experiments!

$$\begin{aligned} \rho c \partial_t T + \partial_x q &= 0, \\ \tau_q \partial_t q + q + \lambda \partial_x T + \kappa \partial_x Q &= 0, \\ \tau_Q \partial_t Q + Q + \kappa \partial_x q &= 0. \end{aligned}$$

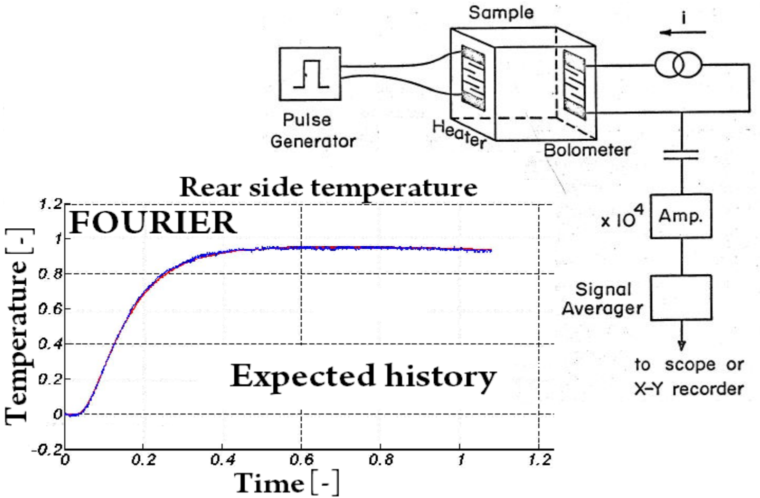
Properties of the structure

- Thermo-mechanical coupling:
  $\boxed{Q^{ij}}$ → current density of the heat flux → $\boxed{\text{pressure}}$
- Ballistic-conductive:
  $\tau_q \tau_Q \partial_{ttt} T + (\tau_q + \tau_Q)\partial_{tt} T + \partial_t T = a \partial_{xx} T + (\kappa^2 + \tau_Q)\partial_{txx} T$

B.C. vs P.H.: κ is free!

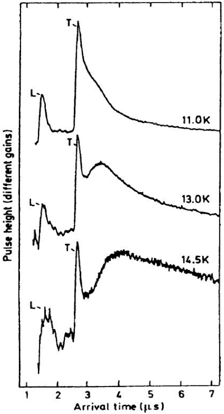
Heat pulse experiment I

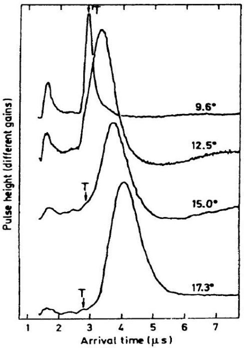
Arrangement for solids



Heat pulse experiment II

Not the expected results at low temperatures!
Presence of second sound and ballistic propagation!





Jackson, Walker and McNelly (1968): NaF experiments

The ballistic-conductive model - Solutions