Kinetic Equation Approach to Graphene in Strong Fields

Biplab Mahato

Institute of theoretical Physics, University of Wroclaw
IMPRS "Many particle systems in structured media", MPIPKS, Dresden

Introduction

Motivation

- Schwinger predicted creation of electron-positron pairs from vacuum in strong external electric field. [Schwinger J., Phys. Rev. (1951)]

- Schwinger predicted creation of electron-positron pairs from vacuum in strong external electric field. [Schwinger J., Phys. Rev. (1951)]
- Experimental realisation of this novel phenomenon is still afar.

$$E_c = \frac{m^2 c^3}{e\hbar} \approx 1.32 \times 10^{18} Vm^{-1}$$

- Schwinger predicted creation of electron-positron pairs from vacuum in strong external electric field. [Schwinger J., Phys. Rev. (1951)]
- Experimental realisation of this novel phenomenon is still afar.

$$E_c = \frac{m^2 c^3}{e\hbar} \approx 1.32 \times 10^{18}\, Vm^{-1}$$

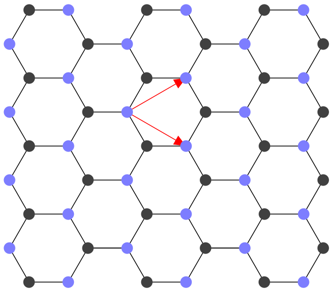
- Many condensed matter systems exhibit behaviour (analogous) known in high energy physics in the low energy regime. (due to vanishing or small band gap)

- Schwinger predicted creation of electron-positron pairs from vacuum in strong external electric field. [Schwinger J., Phys. Rev. (1951)]
- Experimental realisation of this novel phenomenon is still afar.

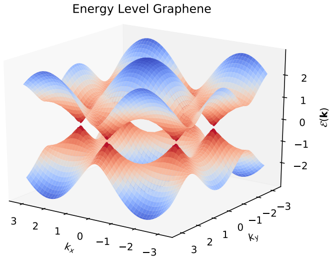
$$E_c = \frac{m^2 c^3}{e\hbar} \approx 1.32 \times 10^{18} Vm^{-1}$$

- Many condensed matter systems exhibit behaviour (analogous) known in high energy physics in the low energy regime. (due to vanishing or small band gap)
- Is there a condensed matter analog to Schwinger pair production?

- Single layer of Carbon atoms arranged in a hexagonal lattice.



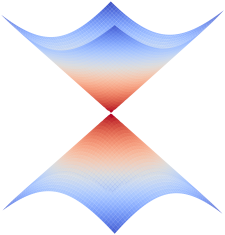
- Single layer of Carbon atoms arranged in a hexagonal lattice.
- Two bands touches each other at Dirac Points.



Introduction

Why Graphene?

- Single layer of Carbon atoms arranged in a hexagonal lattice.
- Two bands touches each other at Dirac Points.
- Theory around Dirac points look exactly like the theory of massless Dirac particles[Novoselov et al. Nature (2005)].



$$\mathcal{E} = v_F|\mathbf{q}|$$

Effect of External Electric Field

- Parallels between Schwinger process in QED vacuum and Graphene

QED	Graphene
Dirac Sea	Fermi Sea
Electron-Positron pairs	Electron-Hole pairs

Effect of External Electric Field

- Parallels between Schwinger process in QED vacuum and Graphene

QED	Graphene
Dirac Sea	Fermi Sea
Electron-Positron pairs	Electron-Hole pairs

- Proposed in the paper [Dora B., Moessner R.,Phys Rev B., (2010)]

$$n(\vec{p},t) = \Theta(p_x)\Theta(eEt - p_x)\exp\left(-\frac{\pi v_F p_y^2}{\hbar eE}\right)$$

$$N(t) = \frac{2eE}{\pi^2 v_F \hbar}\sqrt{\frac{v_F eEt^2}{\hbar}} \approx E^{3/2}$$

Effect of External Electric Field

- Parallels between Schwinger process in QED vacuum and Graphene

QED	Graphene
Dirac Sea	Fermi Sea
Electron-Positron pairs	Electron-Hole pairs

- Proposed in the paper [Dora B., Moessner R.,Phys Rev B., (2010)]

$$n(\vec{p}, t) = \Theta(p_x)\Theta(eEt - p_x)\exp\left(-\frac{\pi v_F p_y^2}{\hbar e E}\right)$$

$$N(t) = \frac{2eE}{\pi^2 v_F \hbar}\sqrt{\frac{v_F e E t^2}{\hbar}} \approx E^{3/2}$$

- Different stages

Classical	Kubo	Schwinger/Kibble-Zurek
$t \ll \frac{h}{W}$	$\frac{h}{W} \ll t \ll \sqrt{\frac{\hbar}{v_F e E}}$	$\sqrt{\frac{\hbar}{v_F e E}} \ll t \ll \frac{\hbar}{eaE}$
$j \approx Et$	$j \approx E$	$j \approx tE^{3/2}$

Kinetic Equation Approach

Formalism[1]

[1]Based on works by Smolyansky S. A., Blaschke D., Schmidt S. ...,e.g. see [Smolyansky et. al. Particles (2020); 2004.03759]

Kinetic Equation Approach

Formalism[1]

- Hamiltonian

$$H(t) = v_F \frac{1}{L^2} \sum_{\vec{p}} \Psi^\dagger(\vec{p}, t) \vec{P} \vec{\sigma} \Psi(\vec{p}, t)$$

[1] Based on works by Smolyansky S. A., Blaschke D., Schmidt S. ...,e.g. see [Smolyansky et. al. Particles (2020); 2004.03759]

Kinetic Equation Approach

Formalism[1]

- Hamiltonian

$$H(t) = v_F \frac{1}{L^2} \sum_{\vec{p}} \Psi^\dagger(\vec{p}, t)\vec{P}\vec{\sigma}\Psi(\vec{p}, t)$$

- Diagonalise the Hamiltonian using unitary transformation to go to the quasiparticle picture.

$$\Psi \to U\Psi = \Phi = \begin{pmatrix} a(\vec{p}, t) \\ b^\dagger(-\vec{p}, t) \end{pmatrix}$$

[1] Based on works by Smolyansky S. A., Blaschke D., Schmidt S. ...,e.g. see [Smolyansky et. al. Particles (2020); 2004.03759]

Kinetic Equation Approach

Formalism[1]

- Hamiltonian

$$H(t) = v_F \frac{1}{L^2} \sum_{\vec{p}} \Psi^\dagger(\vec{p}, t) \vec{P} \vec{\sigma} \Psi(\vec{p}, t)$$

- Diagonalise the Hamiltonian using unitary transformation to go to the quasiparticle picture.

$$\Psi \rightarrow U\Psi = \Phi = \begin{pmatrix} a(\vec{p}, t) \\ b^\dagger(-\vec{p}, t) \end{pmatrix}$$

- Distribution functions

$$f_e(\vec{p}, t) = \langle a^\dagger(\vec{p}, t) a(\vec{p}, t) \rangle \qquad f_h(\vec{p}, t) = \langle b^\dagger(-\vec{p}, t) b(-\vec{p}, t) \rangle$$

[1] Based on works by Smolyansky S. A., Blaschke D., Schmidt S. ...,e.g. see [Smolyansky et. al. Particles (2020); 2004.03759]

Kinetic Equation Approach

Formalism[1]

- Hamiltonian

$$H(t) = v_F \frac{1}{L^2} \sum_{\vec{p}} \Psi^\dagger(\vec{p}, t) \vec{P} \vec{\sigma} \Psi(\vec{p}, t)$$

- Diagonalise the Hamiltonian using unitary transformation to go to the quasiparticle picture.

$$\Psi \to U\Psi = \Phi = \begin{pmatrix} a(\vec{p}, t) \\ b^\dagger(-\vec{p}, t) \end{pmatrix}$$

- Distribution functions

$$f_e(\vec{p}, t) = \langle a^\dagger(\vec{p}, t) a(\vec{p}, t) \rangle \qquad f_h(\vec{p}, t) = \langle b^\dagger(-\vec{p}, t) b(-\vec{p}, t) \rangle$$

- Equation of motion: $\mathrm{i}\frac{d}{dt}\Psi = v_F \vec{P}\vec{\sigma}\Psi$

[1] Based on works by Smolyansky S. A., Blaschke D., Schmidt S. ...,e.g. see [Smolyansky et. al. Particles (2020); 2004.03759]

Kinetic Equation

Numerical Solutions

- Kinetic Equation

$$\dot{f}(\vec{p},t) = \frac{1}{2}\lambda(\vec{p},t)\int_{t_0}^{t} dt'\lambda(\vec{p},t')(1-2f(\vec{p},t'))\cos\theta(t,t')$$

where $\lambda(\vec{p},t) = ev_F^2\dfrac{E_1P_2 - E_2P_1}{\varepsilon(\vec{p},t)^2}$, $\theta(t,t') = \frac{2}{\hbar}\int_{t'}^{t} dt''\varepsilon(\vec{p},t'')$ and $\varepsilon(\vec{p},t) = v_F\sqrt{P_1^2+P_2^2}$

- Kinetic Equation

$$\dot{f}(\vec{p},t) = \frac{1}{2}\lambda(\vec{p},t)\int_{t_0}^{t} dt'\lambda(\vec{p},t')(1-2f(\vec{p},t'))\cos\theta(t,t')$$

where $\lambda(\vec{p},t) = ev_F^2 \dfrac{E_1P_2 - E_2P_1}{\varepsilon(\vec{p},t)^2}$, $\theta(t,t') = \frac{2}{\hbar}\int_{t'}^{t} dt''\varepsilon(\vec{p},t'')$ and $\varepsilon(\vec{p},t) = v_F\sqrt{P_1^2 + P_2^2}$

- The above Integral equation is equivalent to the following set of Ordinary Differential Equations

$$\dot{f} = \frac{1}{2}\lambda u \qquad \dot{u} = \lambda(1-2f) - \frac{2\varepsilon}{\hbar}v \qquad \dot{v} = \frac{2\varepsilon}{\hbar}u$$

- These can be solved numerically for any given external electric field E.

Results

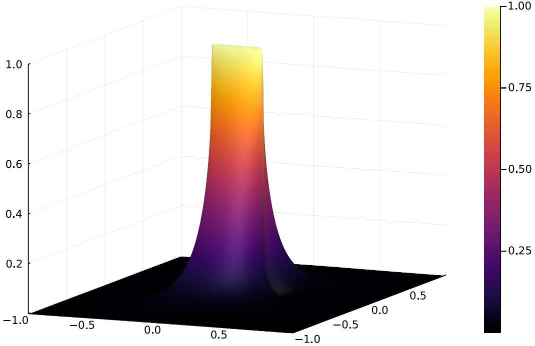
The numerical method works for all kind of external field model

Results

The numerical method works for all kind of external field model

- Sauter Pulse

$$E(t) = E_0 \cosh^{-2}(\kappa t)$$

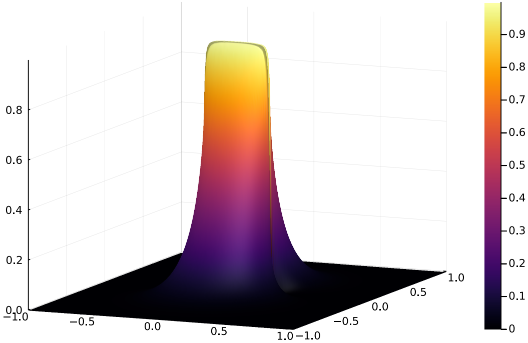


Results

The numerical method works for all kind of external field model

- Gaussian Pulse

$$E(t) = E_0 e^{-t^2/2\tau^2}$$



The created pair align (in momentum space) along the direction of the applied electric field

Approximate Solution[2]

[2] [Blaschke et al., (2022); 2201.10594]

Approximate Solution[2]

- Low Density Approximation $f \ll 1$

$$f(t) = \left(\frac{1}{2}\int_{-\infty}^{t} dt'\lambda(t')\cos\theta(t',-\infty)\right)^2 + \left(\frac{1}{2}\int_{-\infty}^{t} dt'\lambda(t')\sin\theta(t',-\infty)\right)^2$$

- Effective mass Approximation
    - Replace momentum with its time average

$$P^2 \rightarrow < P(t)^2 >= p^2 + e^2 < A(t)^2 >= p^2 + e^2 m_*^2 v_F^2$$

    - $\lambda_*(\vec{p},t) = -\frac{ev_F^2}{\varepsilon_*^2(\vec{p})}E(t) := \Lambda(p)E(t)$

[2] [Blaschke et al., (2022); 2201.10594]

Approximate Solution[2]

- Low Density Approximation $f \ll 1$

$$f(t) = \left(\frac{1}{2}\int_{-\infty}^{t} dt'\lambda(t')\cos\theta(t',-\infty)\right)^2 + \left(\frac{1}{2}\int_{-\infty}^{t} dt'\lambda(t')\sin\theta(t',-\infty)\right)^2$$

- Effective mass Approximation
  - Replace momentum with its time average

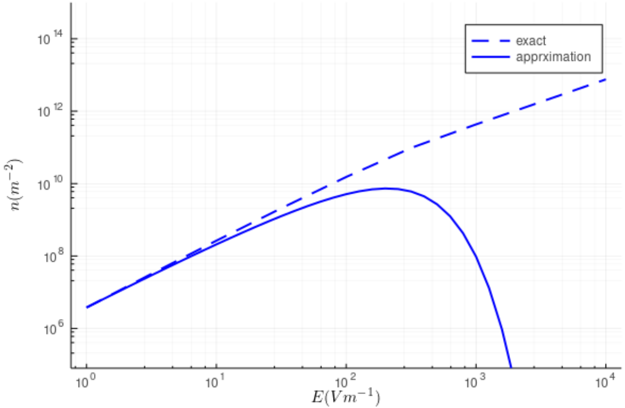
$$P^2 \rightarrow < P(t)^2 >= p^2 + e^2 < A(t)^2 >= p^2 + e^2 m_*^2 v_F^2$$

  - $\lambda_*(\vec{p},t) = -\frac{ev_F^2}{\varepsilon_*^2(\vec{p})}E(t) := \Lambda(p)E(t)$
- Distribution functions can directly be computed via

$$f(\vec{p},t) = \Lambda^2(\vec{p})\left(\frac{1}{2}\int_{-\infty}^{t} dt'\lambda_*(t')\cos\theta_*(t',-\infty)\right)^2$$

[2] [Blaschke et al., (2022); 2201.10594]

Comparison



Slope $= 1.52 \approx \frac{3}{2}$

The Approximate solution deviates from the exact solution for large external fields. $\frac{2\pi m_* v_F^2}{\hbar\kappa} < 1$ or $E_0 < \frac{\hbar\kappa^2}{2\pi e v_F}$

Summary and Outlook

- Graphene can be a testing ground to gain insight about the structure of QED vacuum.
- Recently some experimental signature of this process was seen in graphene superlattices. [Berdyugin et. al., Science, 2022]

Future Works

- Include back-reaction and collision terms
- Gapped system [S. P. Gavrilov, D. M. Gitman, Phys. Rev. D,1996]

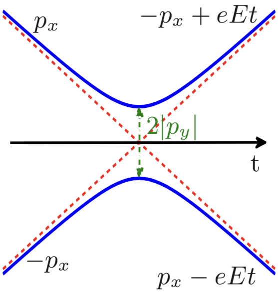
$$\langle j(t)\rangle \approx tE^{\frac{d+1}{2}} \exp\left(\frac{-\pi\Delta^2}{ev_F E\hbar}\right)$$

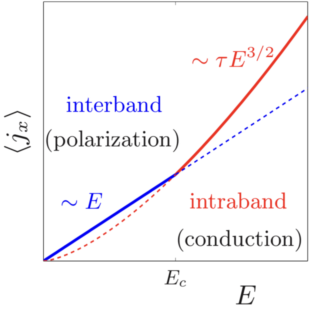
Thank You

Supplimentary slide

1[Dora B., Moessner R.,Phys Rev B., 2010]

Constant Electric field E





$$n(\vec{p}, t) = \Theta(p_x)\Theta(eEt - p_x)\exp\left(-\frac{\pi v_F p_y^2}{\hbar eE}\right)$$

$$N(t) = \frac{2eE}{\pi^2 v_F \hbar}\sqrt{\frac{v_F eEt^2}{\hbar}} \approx E^{3/2} t$$

Comparison with other methods

[Panferov A., EPJ WoC 204,06008 (2019); 1901.01395]