Margaret Island Symposium on Vacuum Structure, Particles, and Plasmas 2022

Dynamic Electromagnetic Response of Viscous QGP in Heavy Ion Collisions: Magnetic Fields

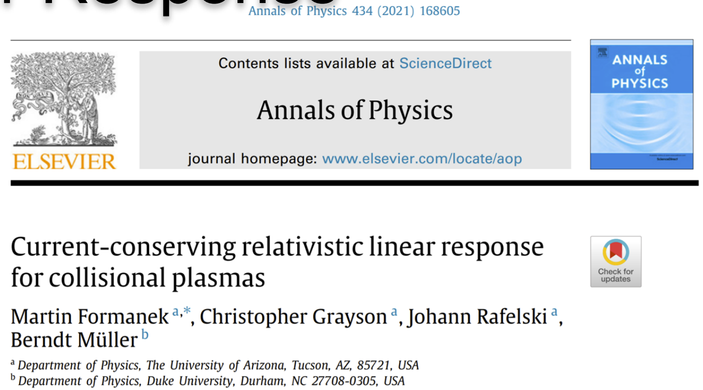
Submitted Manuscript: C. Grayson, M. Formanek, B. Müller and J. Rafelski. "Dynamic Magnetic Response of Quark-Gluon Plasma to Electromagnetic Fields"*

**Duke University*

arXiv: 2204.14186

THE UNIVERSITY OF ARIZONA

Current-Conserving Relativistic Linear Response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Current-conserving relativistic linear response for collisional plasmas

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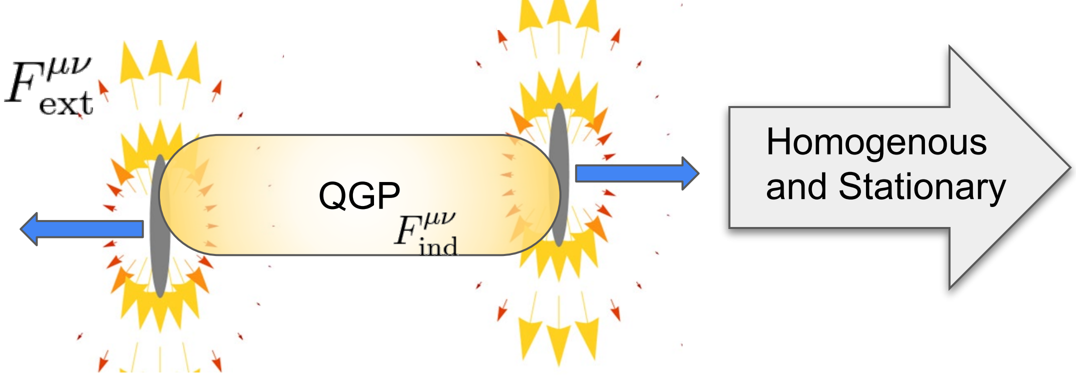
Applying previous work to QGP

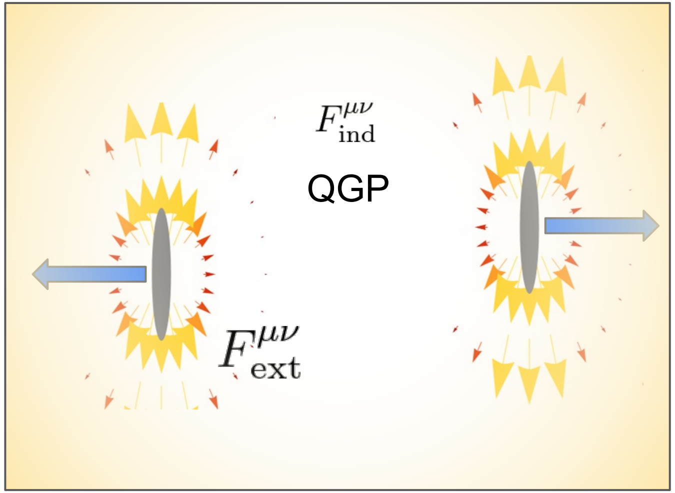
M. Formanek, C. Grayson, J. Rafelski and B. Müller, Annals Phys. 434, 168605 (2021)

Presents the polarization tensor for relativistic collisional plasmas.

Electromagnetic Perturbations $\leftrightarrow$ QGP Plasma

→ Calculate induced electromagnetic fields of QGP due to colliding heavy Ion fields.





Dynamic Viscous QGP Response in Strong EM Fields

Forthcoming publication: [arXiv: 2204.14186]

Electromagnetic Perturbations ↔ QCD Plasma

<u>We present</u>:

The electromagnetic response of viscous QGP considering the full space- and time- dependece of perturbing heavy ion fields. *(only constant and ideal conductivity previously studied)*

- *The inclusion of collisions allows for a finite dynamic conductivity of QGP.*
- *The corrected collision term generates a polarization tensor which conserves current and consequently energy.*

Dynamic Magnetic Response of Quark-Gluon Plasma to Electromagnetic Fields

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Berndt Müller§
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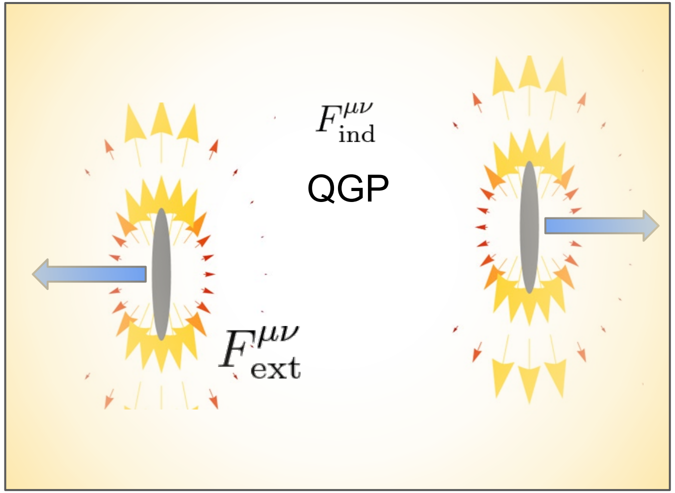
(Dated: 4/29/2022)

We investigate the electromagnetic response of a viscous quark-gluon plasma in the framework of the relativistic Boltzmann equation with current conserving collision term. Our formalism incorporates dissipative effects at all orders in linear response to the electromagnetic field while accounting for the full space- and time-dependence of the perturbing fields. As an example, we consider the collision of two nuclei in a stationary, homogeneous quark-gluon plasma. We show that for large collision energies the induced magnetic fields are governed by the response of quark-gluon plasma along the light-cone. In this limit we derive an analytic expression for the magnetic field along the beam axis between the receding nuclei and show that its strength varies only weakly with collision energy for $\sqrt{s_{\rm NN}} \geq 30\,{\rm GeV}$.



Linear Response from the Vlasov-Boltzmann Equation

Using the Vlasov-Boltzmann equation for each quark flavor one can calculate the induced electromagnetic current in linear response and identify the polarization tenso $\Pi^{\mu}_{\nu}(k)$ M. Formanek, C. Grayson, J. Rafelski and B. Müller, Annals Phys. 434, 168605 (2021)

$$(p\cdot\partial)f(x,p)+qF^{\mu\nu}p_{\nu}\frac{\partial f(x,p)}{\partial p^{\mu}}=(p\cdot u)\kappa\left(f_{\mathrm{eq}}(p)\frac{n(x)}{n_{\mathrm{eq}}}-f(x,p)\right)$$

κ is the QGP strong damping parameter and is obtained from the inverse of the (collisional) relaxation time $1/\tau$.

Where the current due to up, down, and strange quarks is,

$$\tilde{j}^{\mu}_{\text{ind}}(k)=2N_c\int\frac{d^4p}{(2\pi)^4}4\pi\delta_{+}(p^2-m^2)p^{\mu}\sum_{u,d,s}q_f(\tilde{f}_f(k,p)-\tilde{f}_{\bar{f}}(k,p))$$

The first order response of a medium to fields can be described in fourier space using a simple relation,

$$\tilde{j}^{\mu}_{\mathrm{ind}}(k)=\Pi^{\mu}_{\nu}(k)\tilde{A}^{\nu}(k)$$

k is the 4 vector $k^{\mu}=(\omega,\mathbf{k})$

Ohm's Law

$$\mathbf{J}=\sigma_0\mathbf{E}$$

(R. Starke and G. A. H. Schober, Int. J. Mod. Phys. D 25, 16400

Where $\Pi^{\mu}_{\nu}(k)$ Is the polarization tensor $\tilde{j}^{\mu}_{\mathrm{ind}}(k)$ is the fourier transformed induced current in the medium and $\tilde{A}^{\nu}(k)$ is the fourier transformed perturbing 4-potential of the colliding ions.

$$\sigma_0\equiv\text{“Satic Conductivity”}$$

Linear Response from the Vlasov-Boltzmann Equation

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Using this current one can calculate the polarization tensor using the method shown in the previous talk,

$$\begin{aligned}j^{\mu}_{\rm ind}(x)&=2N_c\int(dp)p^{\mu}\sum_{u,d,s}q_f(\delta f_f(x,p)-\delta f_{\bar{f}}(x,p))\\&=4N_c\int(dp)p^{\mu}\sum_{u,d,s}q_f^2\delta f(x,p)\\&=4N_Qe^2\int(dp)p^{\mu}\delta f(x,p)\end{aligned}$$

$$N_Q\equiv N_c\sum_f(q_f/e)^2=2$$

- Enhancement due to quarks and color

$$\tilde{j}^{\mu}_{\rm ind}(k)=\Pi^{\mu}_{\nu}(k)\tilde{A}^{\nu}(k)$$

The polarization tensor is identical besides a change in the Debye mass due to the extra degrees of freedom.

$$m_{D\,(\rm EM)}^{2}=\sum_{u,d,s}q_f^2T^2\frac{N}{3}=C_{\rm em}T^2=2\frac{e^2T^2}{3}$$

Electromagnetic Polarization Tensor: Infinite Medium

The polarization tensor for an infinite homogeneous plasma is composed of two independent response functions which can be found by projecting onto ***k*** and considering longitudinal and transverse polarization functions.

$$\Pi^{\mu}_{\nu}(\omega, \boldsymbol{k}) = \begin{bmatrix} -\frac{|\mathbf{k}|^2}{\omega^2}\Pi_{\parallel} & 0 & 0 & \frac{|\mathbf{k}|}{\omega}\Pi_{\parallel} \\ 0 & \Pi_{\perp} & 0 & 0 \\ 0 & 0 & \Pi_{\perp} & 0 \\ -\frac{|\mathbf{k}|}{\omega}\Pi_{\parallel} & 0 & 0 & \Pi_{\parallel} \end{bmatrix}_{rest}$$

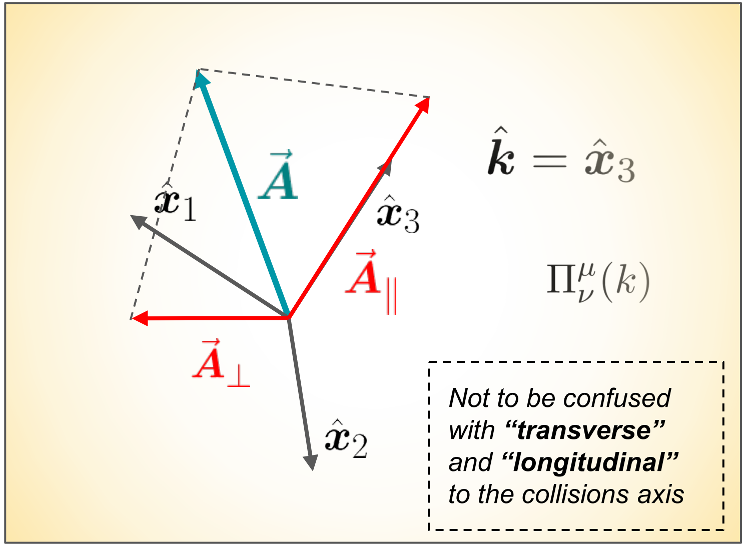
Calculated in the ultrarelativisitic (p >>m) limit

$$\Pi_{\parallel}(\omega, |\boldsymbol{k}|) = m_D^2 \frac{\omega^2}{\boldsymbol{k}^2}\left(1 - \frac{\omega\Lambda}{2|\boldsymbol{k}| - i\kappa\Lambda}\right), \qquad \Lambda \equiv \ln \frac{\omega + i\kappa + |\boldsymbol{k}|}{\omega + i\kappa - |\boldsymbol{k}|}$$

$$\Pi_{\perp}(\omega, |\boldsymbol{k}|) = \frac{m_D^2 \omega}{4|\boldsymbol{k}|}\left(\Lambda\left(\frac{(\omega + i\kappa)^2}{\boldsymbol{k}^2} - 1\right) - \frac{2(\omega + i\kappa)}{|\boldsymbol{k}|}\right)$$

Where m_D is the electromagnetic Debye mass of the plasma

$$m_{D\,(\mathrm{EM})}^{2} = \sum_{u,d,s} q_f^2 T^2 \frac{N}{3} = C_{\mathrm{em}} T^2 = 2\frac{e^2 T^2}{3}$$



*Here we have chosen **k** to point along the third component **x_3**.*

H.A. Weldon, Phys. Rev. D 26 (1982)

Estimating Quark Collision Term

To get an estimate of κ we multiply the parton-quark transport cross section by the parton density, (P. Danielewicz and M. Gyulassy, Phys. Rev. D 31, 1985) (Mrowczynski, Acta Phys. Polon 1988)

$$\kappa(T) = \frac{10}{17\pi}(9N_f + 16)\zeta(3)\alpha_s^2 \ln\left(\frac{1}{\alpha_s}\right) T$$

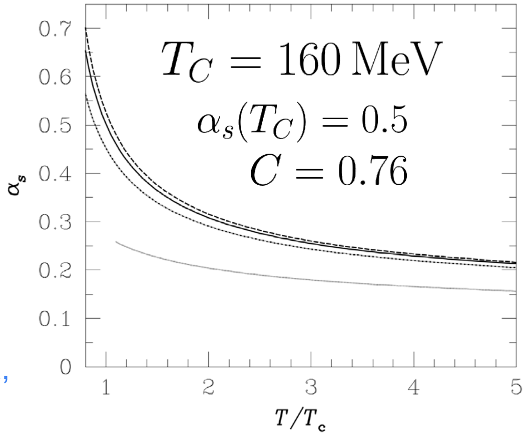
Where we model how the strong coupling varies with temperature using a fit.

$$\alpha_s(T) \approx \frac{\alpha_s(T_C)}{1 + C\ln(T/T_C)}$$

$$\alpha_s = g^2/4\pi$$

(Hadrons and Quark Gluon plasma, Letessier, J.,Rafelski 2002)

QCD, hadronic structure and high temperature



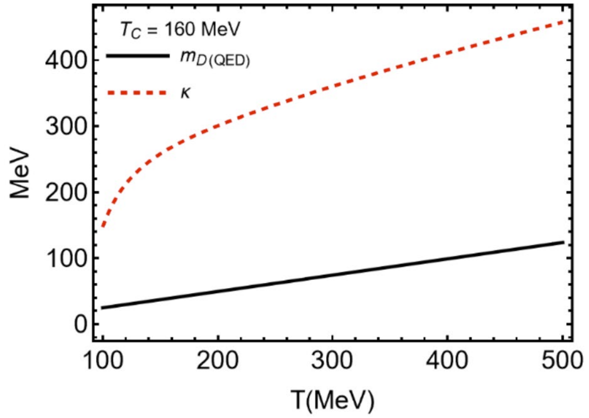


FIG. 2: Plot of the QED debye mass and the QCD dampening rate κ as a function of temperature

Plasma Oscillation Frequency

$$\omega_p^{\pm} = -\frac{i\kappa}{2} \pm \sqrt{\frac{m_D^2}{3} - \frac{\kappa^2}{4}}$$

If $\kappa_{(QCD)} > \frac{2}{\sqrt{3}} m_{D(EM)}$ then plasma oscillations are overdamped

Comparison with Lattice Results

$$C_{\text{em}} = \sum_{u,d,s} q_f^2 = \frac{6}{9} e^2$$

Kappa can also be compared to lattice conductivity calculations

<u>Static conductivity</u>

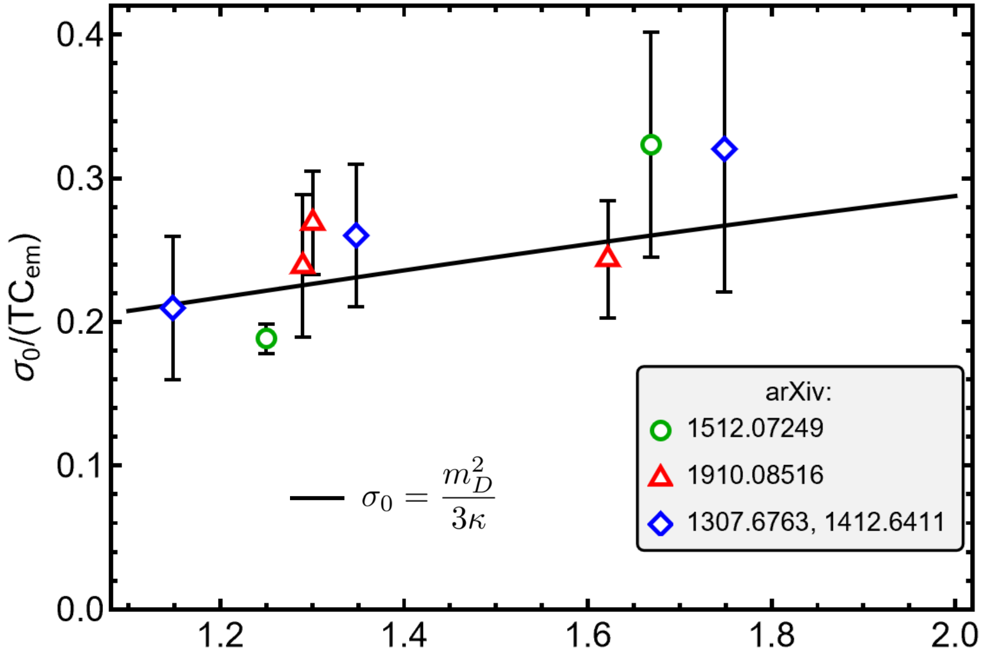
$$\sigma_0 = \frac{m_D^2}{3\kappa}$$

$$m_{D\,(\text{EM})}^{2} = 2\frac{e^2 T^2}{3}$$

"Debye mass"

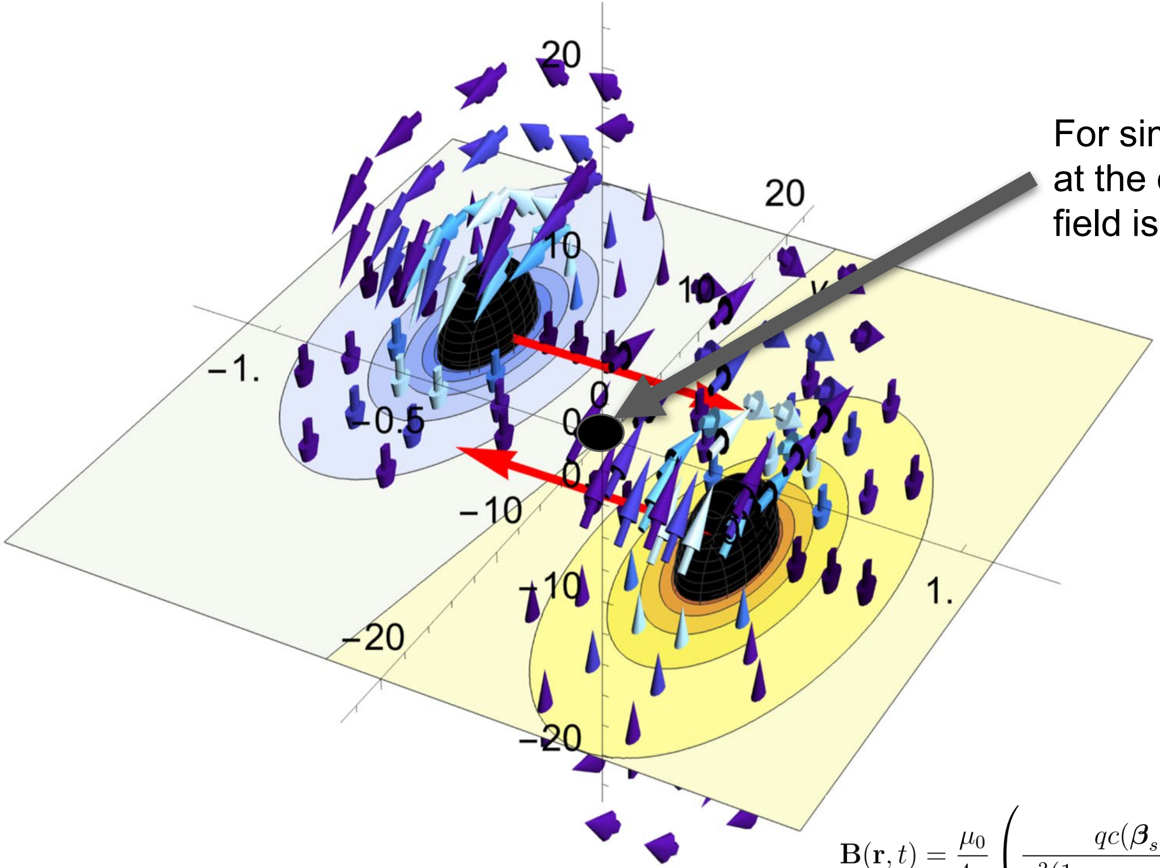
$$\kappa(T) = \frac{10}{17\pi}(9N_f + 16)\zeta(3)\alpha_s^2 \ln\left(\frac{1}{\alpha_s}\right) T$$

"QCD damping parameter"

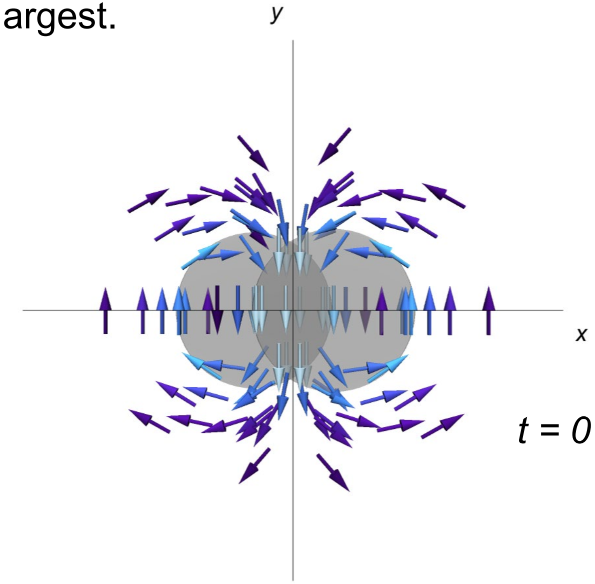


A. Amato, G. Aarts, C. Allton, P. Giudice, S. Hands and J. I. Skullerud, Phys. Rev. Lett. **111**, 172001 (2013) [arXiv:1307.6763 [hep-lat]].
G. Aarts, C. Allton, A. Amato, P. Giudice, S. Hands and J. I. Skullerud, JHEP **02**, 186 (2015) [arXiv:1412.6411 [hep-lat]].
B. B. Brandt, A. Francis, B. Jäger and H. B. Meyer, Phys. Rev. D **93**, 054510 (2016) [arXiv:1512.07249 [hep-lat]].
N. Astrakhantsev, V. V. Braguta, M. D'Elia, A. Y. Kotov, A. A. Nikolaev and F. Sanfilippo, Phys. Rev. D **102**, 054516 (2020) [arXiv:1910.08516 [hep-lat]].

Magnetic field at the Center of Colliding Ions



For simplicity we present the magnetic field at the center of QGP where the magnetic field is largest.



$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\left(\frac{qc(\boldsymbol{\beta}_s \times \mathbf{n}_s)}{\gamma^2(1-\mathbf{n}_s\cdot\boldsymbol{\beta}_s)^3|\mathbf{r}-\mathbf{r}_s|^2} + \frac{q\mathbf{n}_s \times \left(\mathbf{n}_s \times \left((\mathbf{n}_s - \boldsymbol{\beta}_s) \times \dot{\boldsymbol{\beta}}_s\right)\right)}{(1-\mathbf{n}_s\cdot\boldsymbol{\beta}_s)^3|\mathbf{r}-\mathbf{r}_s|}\right)_{t_r}$$

Self-consistent Fields

In order to calculate the induced charge and field using the longitudinal and transverse response functions one must project vector perturbations onto ***k***

$$\widetilde{A}_{\|} = \frac{\boldsymbol{k}\cdot\widetilde{\boldsymbol{A}}}{|\boldsymbol{k}|}, \quad \widetilde{\boldsymbol{A}}_{\perp} = \widetilde{\boldsymbol{A}} - \widetilde{A}_{\|}\hat{\boldsymbol{k}}$$

Thus are 4-vectors in the different coordinates are,

$$\widetilde{A}^{\mu}(\omega,\boldsymbol{k}) = (\widetilde{\phi},\ \widetilde{\boldsymbol{A}}) \Rightarrow (\widetilde{\phi},\ \widetilde{\boldsymbol{A}}_{\perp},\ \widetilde{A}_{\|})$$

Then solving maxwell's equations in projected space one find the self consistent potentials,

$$\widetilde{\phi}(\omega,\boldsymbol{k}) = \frac{\widetilde{\rho}_{\text{ext}}(\omega,\boldsymbol{k})}{\varepsilon_0(\boldsymbol{k}^2-\omega^2)\left(\Pi_{\|}/(\omega^2\varepsilon_0)+1\right)}$$

$$\widetilde{\boldsymbol{A}}_{\perp}(\omega,\boldsymbol{k}) = \frac{\mu_0\widetilde{\boldsymbol{j}}_{\perp\text{ext}}(\omega,\boldsymbol{k})}{\boldsymbol{k}^2-\omega^2-\mu_0\Pi_{\perp}} \qquad \widetilde{A}_{\|} = \frac{\omega}{|\boldsymbol{k}|}\widetilde{\phi}$$

The self consistent potentials can be used to calculate the self consistent fields.

$$\widetilde{\boldsymbol{B}}(\omega,\boldsymbol{k}) = i\boldsymbol{k}\times\widetilde{\boldsymbol{A}}_{\perp}$$
$$\widetilde{\boldsymbol{E}}(\omega,\boldsymbol{k}) = -i\boldsymbol{k}\widetilde{\phi} + i\omega\widetilde{\boldsymbol{A}}$$

Then we fourier transform the fields back to position space

$$B_y(x) = \int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot x}\widetilde{B}_y(k) = \int\frac{d^4k}{(2\pi)^4}e^{-i\omega t+i\boldsymbol{k}\cdot\boldsymbol{x}}\widetilde{B}_y(\omega,\boldsymbol{k})$$

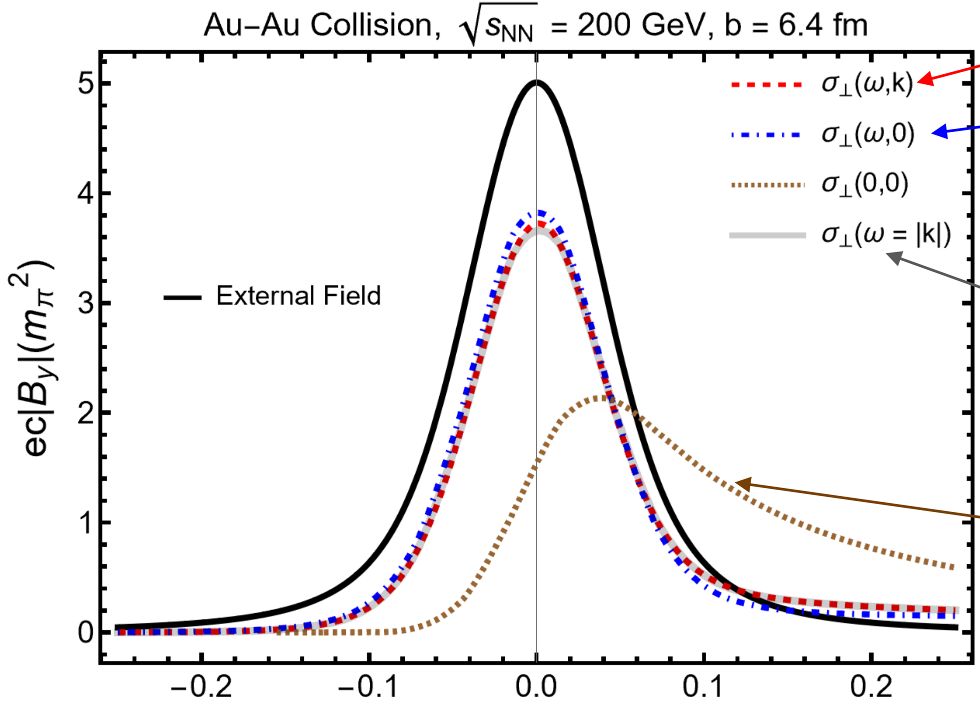
$$B_y(t,0) = \int\frac{d^4k}{(2\pi)^4}e^{-i\omega t}\widetilde{B}_y(\omega,\boldsymbol{k})$$

Magnetic field at the origin

Magnetic Field of Heavy Ion Collisions at the Origin,

Electric field is shown in supplemental slides

$$\boldsymbol{B}(t,0) = \int \frac{d^4k}{(2\pi)^4} e^{-i\omega t} \frac{\mu_0 i\boldsymbol{k} \times \tilde{\boldsymbol{j}}_{\perp f}(\omega,\boldsymbol{k})}{\boldsymbol{k}^2 - \omega^2 - \mu_0 \Pi_\perp(\omega,\boldsymbol{k})}$$



$T = 300\,\mathrm{MeV}$ $\sigma_0 = 5.01\,\mathrm{MeV}$

Full time and space dependent response

Only time dependent response (Drude model)

$$\sigma_\perp(\omega,\boldsymbol{k}) = -i\frac{\Pi_\perp(\omega,\boldsymbol{k})}{\omega} \Rightarrow \sigma_\perp(\omega,0) = \frac{\sigma_{\text{stat}}}{1 - i\omega/\kappa}$$

Light-cone conductivity

$$\sigma_\perp(\omega = |\boldsymbol{k}|) = i\frac{m_D^2}{4\omega}\left(\frac{\kappa^2}{\omega^2}\ln(\xi)\,\xi + \frac{i\kappa}{\omega}(\xi+1)\right)$$

Static Conductivity

K. Tuchin, Phys. Rev. C 88 (2013)

$$\sigma_0 = \frac{m_D^2}{3\kappa}$$

Ohm's Law

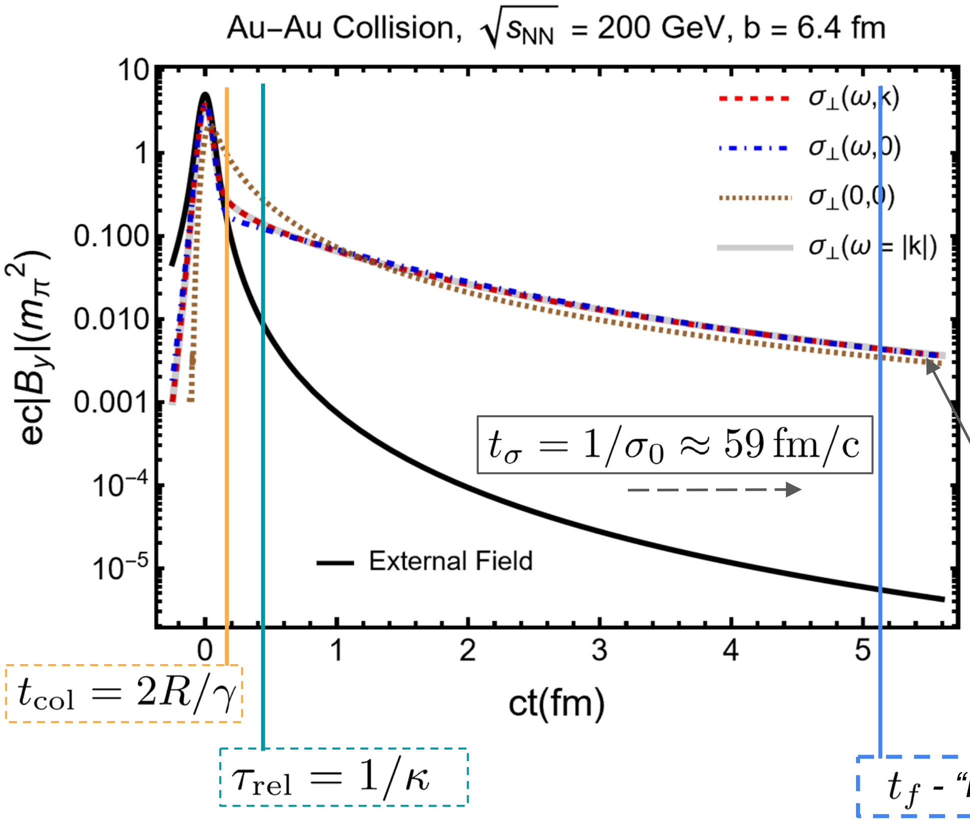
$$\mathbf{J} = \sigma_0 \mathbf{E}$$

Magnetic field persists due to induced currents.

Magnetic Field of Heavy Ion Collisions at the Origin,

Electric field is shown in supplemental slides

$$\boldsymbol{B}(t,0) = \int \frac{d^4k}{(2\pi)^4} e^{-i\omega t} \frac{\mu_0 i\boldsymbol{k} \times \widetilde{\boldsymbol{j}}_{\perp f}(\omega,\boldsymbol{k})}{\boldsymbol{k}^2 - \omega^2 - \mu_0 \Pi_\perp(\omega,\boldsymbol{k})}$$



- Dynamic effects are important for t < 1/κ
- After $t \gg 1/\kappa$ the solutions begin to approach the static solution:

$$B_y(t) = -\mu_0 \frac{Zq\beta}{(2\pi)} \frac{b\sigma_0}{4t^2} e^{\frac{-b^2\sigma_0}{16t}}$$

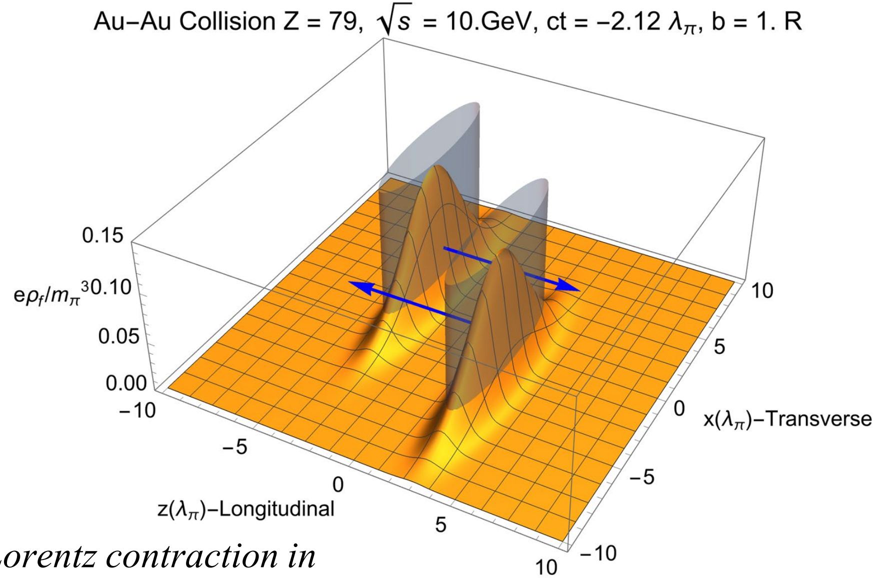
- At the freeze-out time the QGP begins to hadronize.

"Why does the light-cone solution match the full solution?"

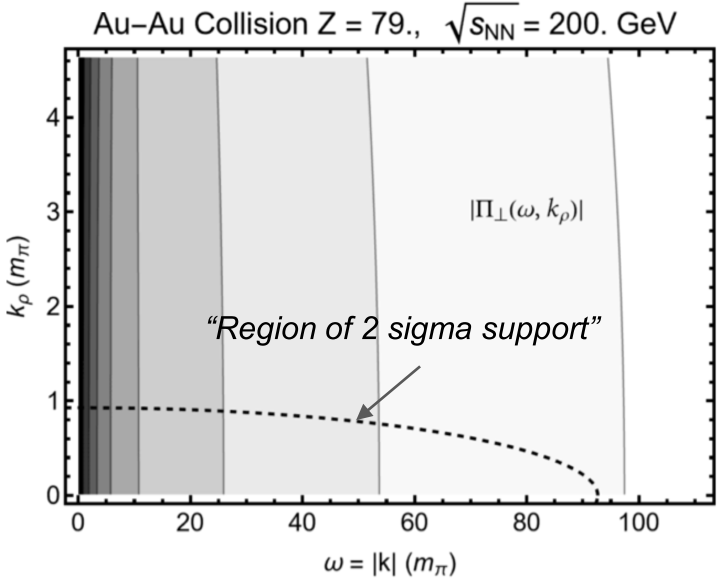
Light-cone Conductivity for Ultrarelativistic Ions

$$\widetilde{\rho}_{f\pm}(\omega, k_\rho) = 2\pi Z q c\, e^{-(k_\rho^2+\omega^2/(\beta\gamma)^2)\frac{R^2}{4}}\; e^{\mp\frac{i k_\rho b\cos\theta}{2}}$$

k_ρ Width $\sqrt{2}/R$ $\quad$ ω Width $\beta\gamma\sqrt{2}/R$



"Lorentz contraction in direction of motion"



The conductivity does not vary with k_ρ and we can use its value at $k_\rho = 0$

$$\xi \equiv 1 - 2i\frac{\omega}{\kappa}$$

$$\sigma_\perp(\omega = |\boldsymbol{k}|) = i\frac{m_D^2}{4\omega}\left(\frac{\kappa^2}{\omega^2}\ln(\xi)\,\xi + \frac{i\kappa}{\omega}(\xi+1)\right)$$

Magnetic Field of Heavy Ion Collisions at the Origin,

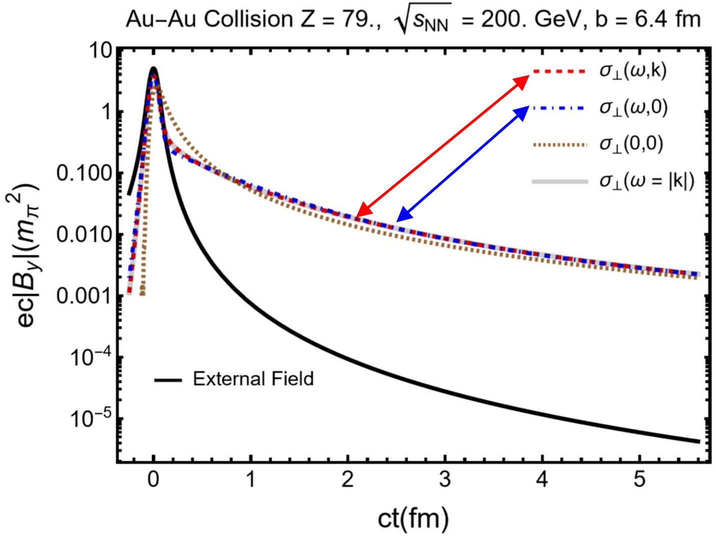
Expansion of the light cone conductivity in ω/κ:

> *"Late time solutions do not depend on γ."*

$$\sigma_\perp(\omega = |\boldsymbol{k}|) = \frac{\sigma_{\rm stat}}{1 - i\omega/\kappa}\left(1 + \frac{\omega^2}{\kappa^2}\right) - \frac{6\sigma_{\rm stat}}{5}\frac{\omega^2}{\kappa^2} + O\left(\frac{\omega^4}{\kappa^4}\right)$$

"Drude conductivity" $\sigma_\perp(\omega, 0)$

$$\boldsymbol{B}(t,0) = \int \frac{d^4k}{(2\pi)^4} e^{-i\omega t} \frac{\mu_0 i\boldsymbol{k} \times \tilde{\boldsymbol{j}}_{\perp f}(\omega, \boldsymbol{k})}{\boldsymbol{k}^2 - \omega^2 - \mu_0 \Pi_\perp(\omega, \boldsymbol{k})}$$



$$\sigma_\perp(\omega, \boldsymbol{k}) = -i\frac{\Pi_\perp(\omega, \boldsymbol{k})}{\omega} \quad \Rightarrow \quad \sigma_\perp(\omega, 0) = \frac{\sigma_{\rm stat}}{1 - i\omega/\kappa}$$

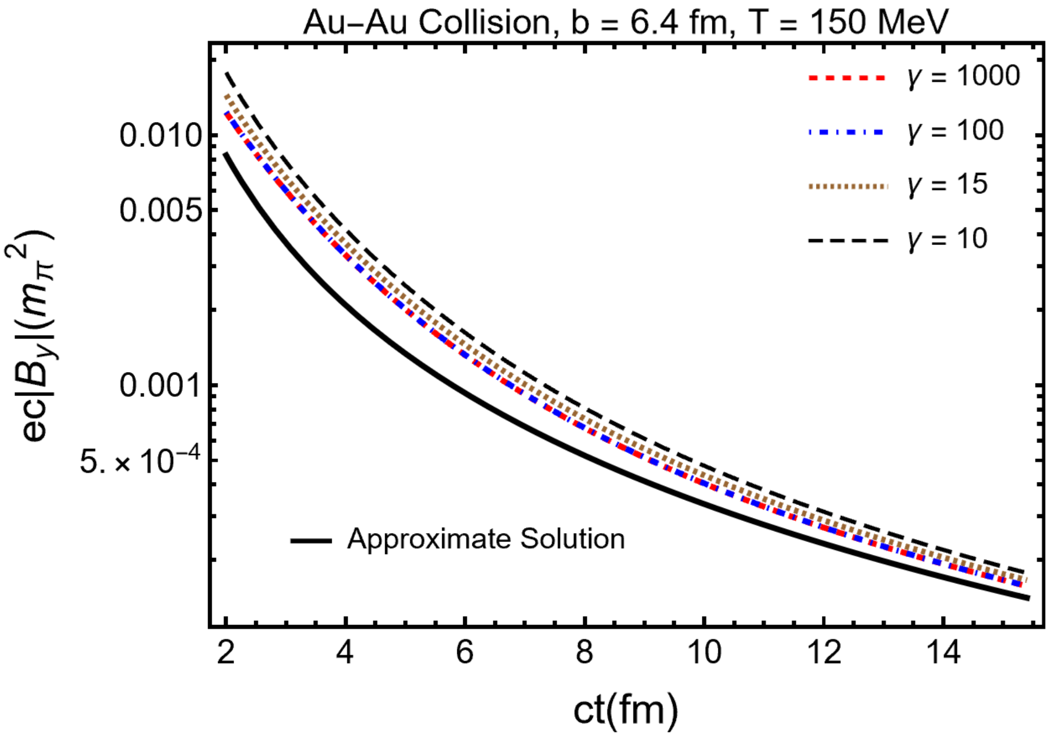
Simplified form of the denominator allows for contour integration:

$$B_y(t) = -\mu_0 \frac{Zq\beta}{(2\pi)} \frac{b\sigma_0}{4t^2} e^{\frac{-b^2\sigma_0}{16t}}$$

K. Tuchin, Phys. Rev. C 88 (2013)

Freeze-out Magnetic Field at the Origin,

Freeze-out magnetic field induces polarization in final hadron spectrum



Numerical solutions for the full polarization tensor:

$$\boldsymbol{B}(t,0) = \int \frac{d^4k}{(2\pi)^4} e^{-i\omega t} \frac{\mu_0 i\boldsymbol{k} \times \widetilde{\boldsymbol{j}}_{\perp f}(\omega, \boldsymbol{k})}{\boldsymbol{k}^2 - \omega^2 - \mu_0 \Pi_\perp(\omega, \boldsymbol{k})}$$

Approximation expected to apply till

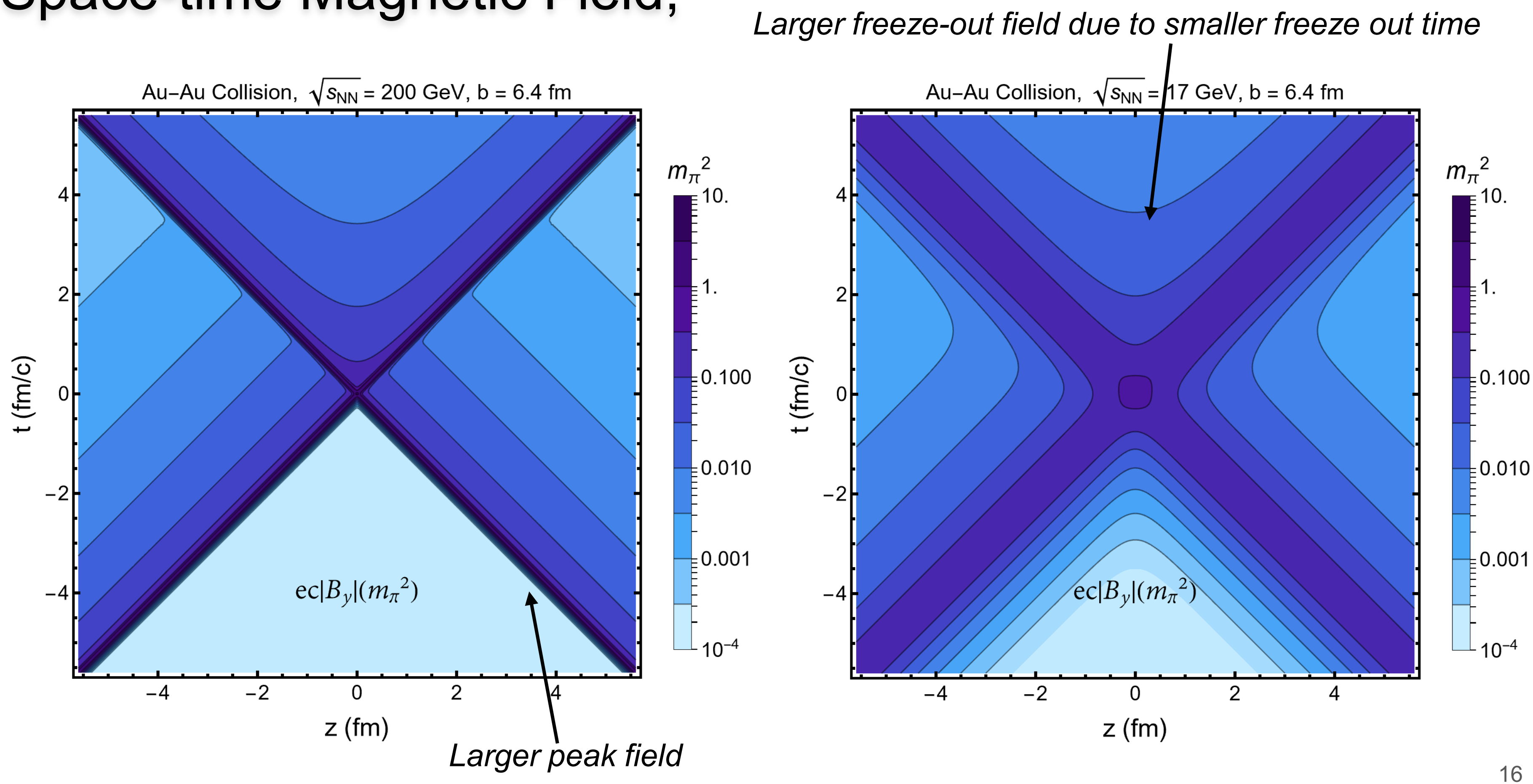
$$\gamma\beta \approx \sqrt{\kappa/\sigma_{\text{stat}}} \approx 12$$

"Static Conductivity solution"

$$B_y(t) = -\mu_0 \frac{Zq\beta}{(2\pi)} \frac{b\sigma_0}{4t^2} e^{\frac{-b^2\sigma_0}{16t}}$$

"Freeze-out magnetic field depends more on t_f than on collision energy"

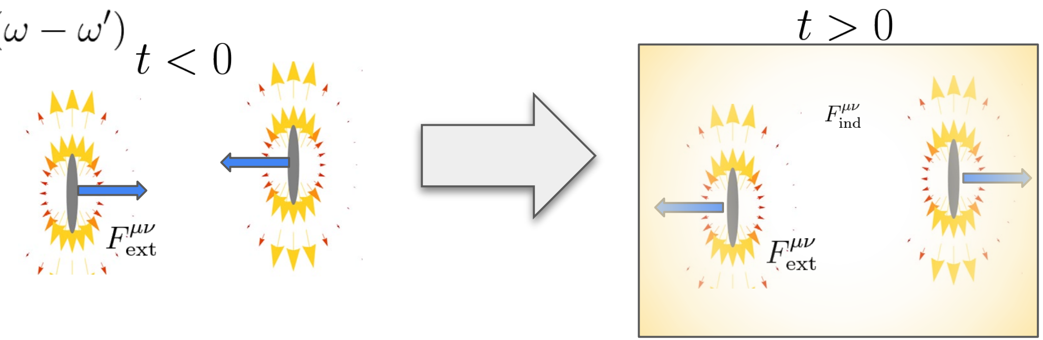
Space-time Magnetic Field,



Outlook: Just at the Beginning

- Early Universe Plasma and BNN (See lecture by Cheng Tao Yang)
  - BBN fusion reactions in damped electron-positron-plasma.
- Electric field: pair production and characterization of electric field in QGP.
- QGP Switch-On and Evolution: create a realistic spacetime picture of the plasma.
  - This can be done by adding a source to the neutral plasma in the boltzmann equation.

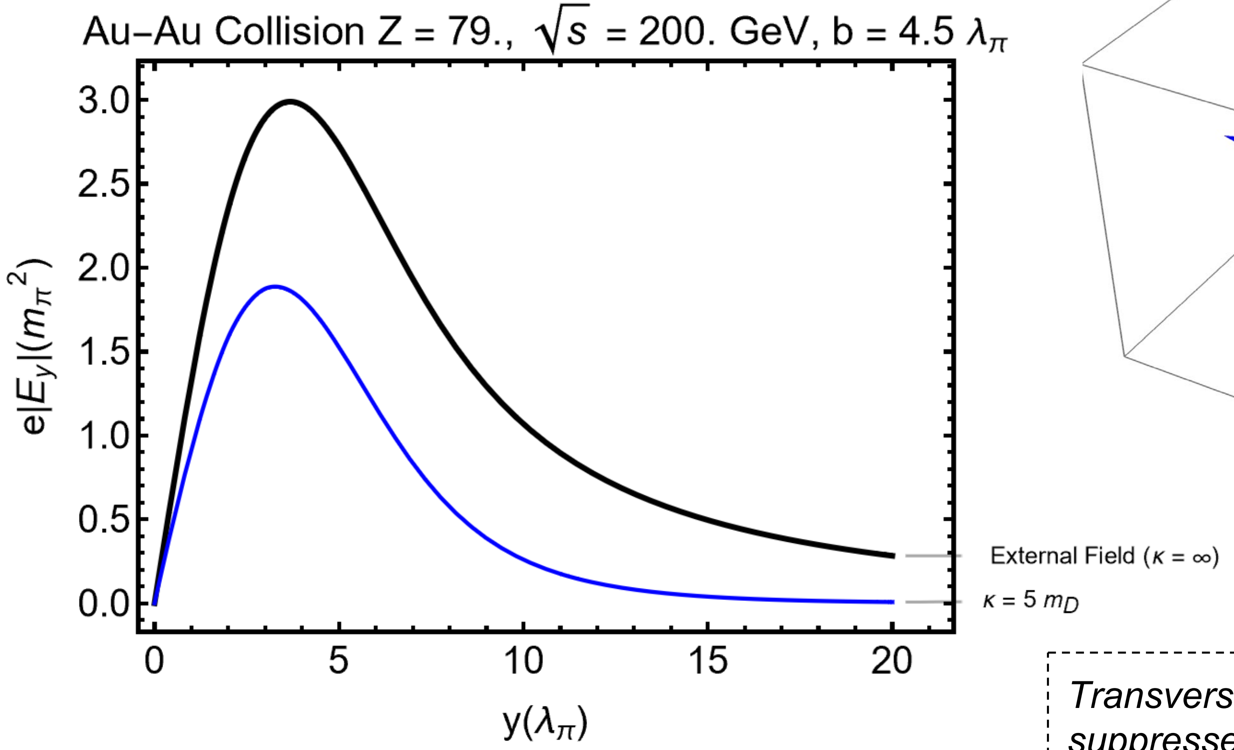
$$\tilde{j}^{\mu}_{\rm ind}(\omega, \boldsymbol{k}) = \int \frac{d\omega'}{2\pi} \Pi^{\mu}_{\nu}(\omega', \omega, \boldsymbol{k}) \widetilde{A}^{\nu}(\omega', \boldsymbol{k}) \tilde{\Theta}(\omega - \omega')$$

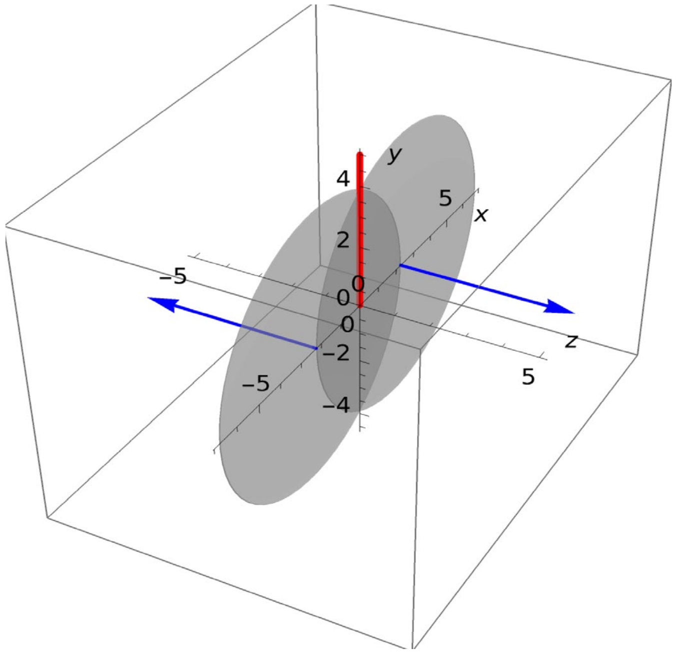


Thank You!

Supplemental Slides

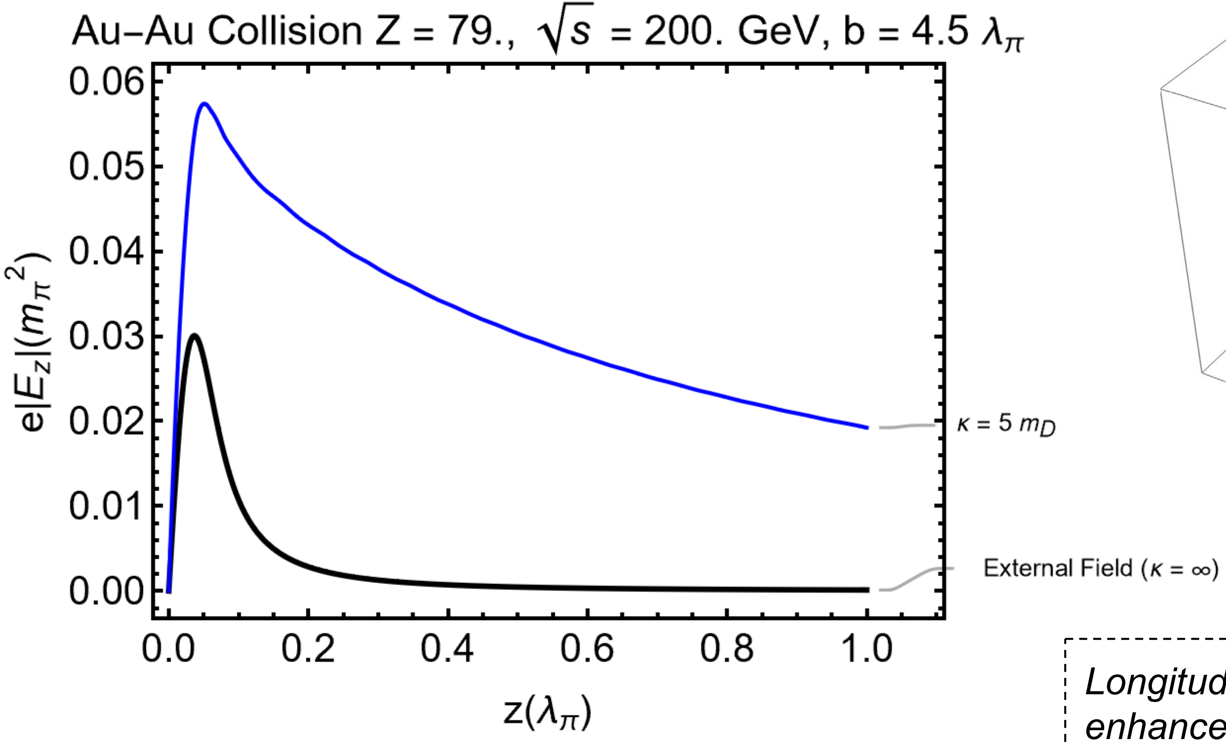
Electric Field

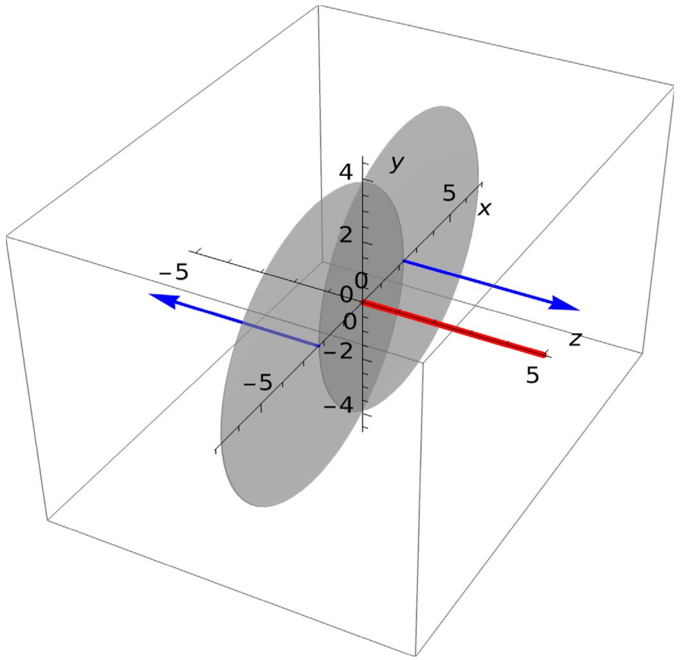




Transverse electric field is suppressed.

Electric Field

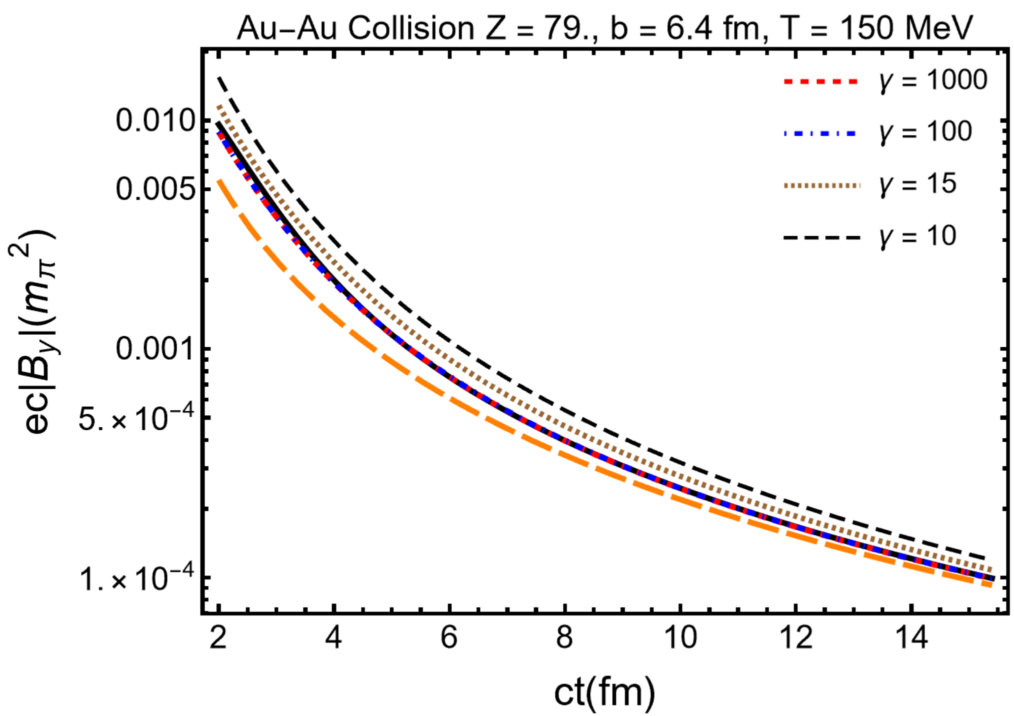




Longitudinal Electric field is enhanced

Freeze-out Magnetic Field at the Origin,

Freeze-out magnetic field induces polarization in final hadron spectrum



Numerical solutions for the full polarization tensor:

$$\boldsymbol{B}(t,0)=\int\frac{d^4k}{(2\pi)^4}e^{-i\omega t}\frac{\mu_0 i\boldsymbol{k}\times\widetilde{\boldsymbol{j}}_{\perp f}(\omega,\boldsymbol{k})}{\boldsymbol{k}^2-\omega^2-\mu_0\Pi_\perp(\omega,\boldsymbol{k})}$$

Approximate result expected to apply till

$$\gamma\beta\approx\sqrt{\kappa/\sigma_{\rm stat}}\approx 12$$

"Dynamic solution"

$$B_y(t)=-\mu_0\frac{Zq\beta}{(2\pi)}\frac{b\kappa\sigma_{\rm stat}\left(1-\kappa t\mathrm{Ei}(\kappa t)e^{-\kappa t}\right)}{4t}$$

"Static solution"

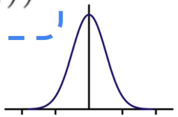
$$B_y(t)=-\mu_0\frac{Zq\beta}{(2\pi)}\frac{b\sigma_{\rm stat}}{4t^2}$$

"Freeze-out magnetic field depends more on t_f than on collision energy"

Outlook: Just at the Beginning

- QGP Switch-On and Evolution: In order to accurately model in space and time QGP we would like to create a realistic spacetime picture of the plasma.
  - This can be done by adding a source to the neutral plasma in the boltzmann equation

$$(p\cdot\partial)f(x,p)+qF^{\mu\nu}p_\nu\frac{\partial f(x,p)}{\partial p^\mu}=(p\cdot u)\kappa\left[f_{\rm eq}(p)\frac{n(x)}{n_{\rm eq}}-f(x,p)\right]+(p\cdot u)f_{\rm eq}(p)\Delta(u_\mu(x^\mu-x_0^\mu))$$

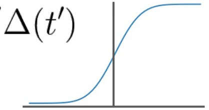


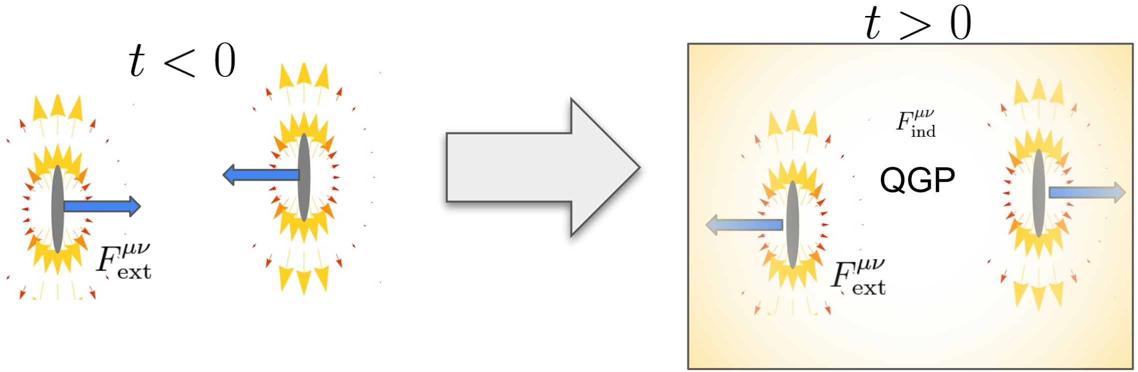
The leads to a convolution integral in Ohm's Law of the form,

$$\tilde{j}^\mu_{\rm ind}(\omega,\boldsymbol{k})=\int\frac{d\omega'}{2\pi}\Pi^\mu_\nu(\omega',\omega,\boldsymbol{k})\tilde{A}^\nu(\omega',\boldsymbol{k})\tilde{\Theta}(\omega-\omega')$$

where,

$$\Theta(t)=\int_{-\infty}^{t}dt'\Delta(t')$$





Free Charge Distribution

For simplicity we prescribe the external fields using a gaussian charge distribution normalized to the radius and charge of the nucleus.

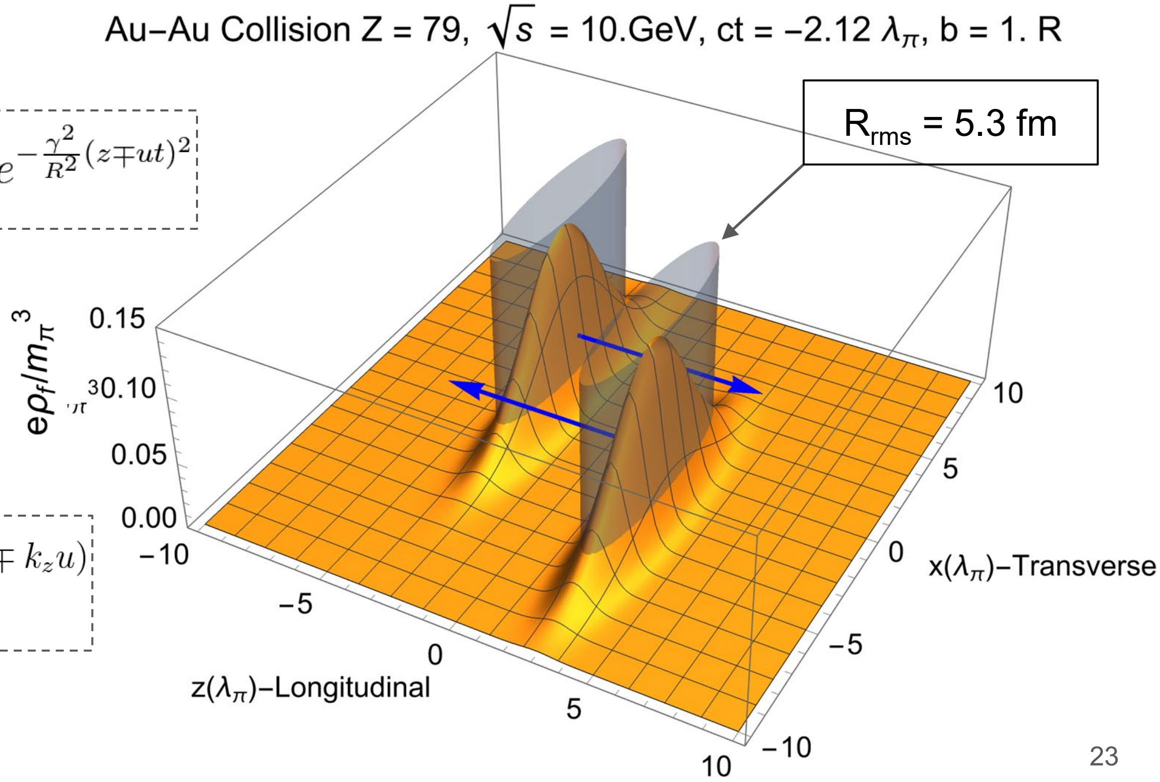
Free Charge in Position Space

$$\rho_{\pm b/2}(t,\boldsymbol{x}) = \frac{Zq\gamma}{\pi^{3/2}R^3} e^{-\frac{1}{R^2}(x\mp b/2)^2} e^{-\frac{1}{R^2}y^2} e^{-\frac{\gamma^2}{R^2}(z\mp ut)^2}$$

Free Charge in Fourier Space

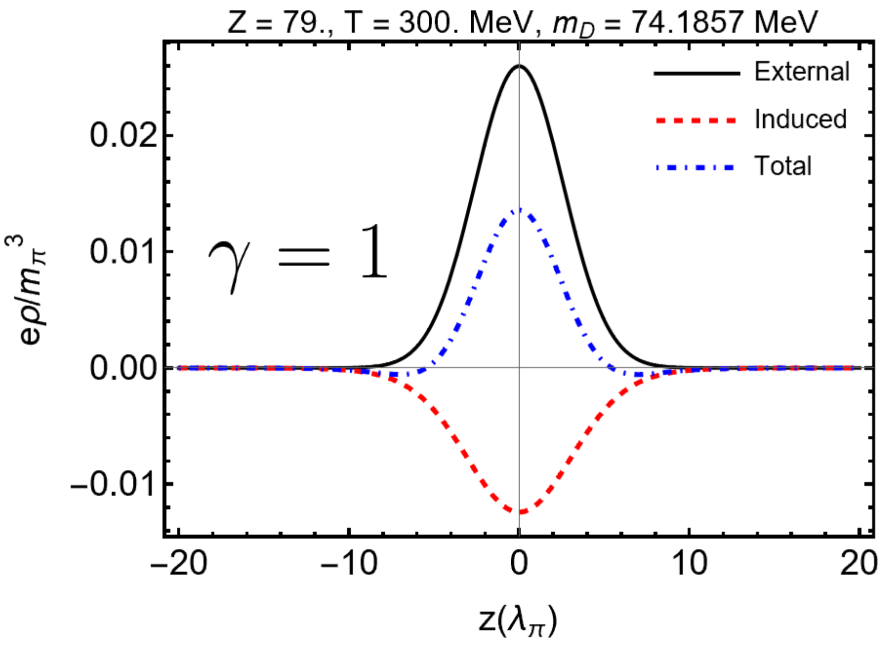
$$\widetilde{\rho}_{\pm b/2}(\omega,\boldsymbol{k}) = 2\pi Zqc\, e^{-(k_x^2+k_y^2+k_z^2/\gamma^2)\frac{R^2}{4}} e^{\mp\frac{ik_x b}{2}} \delta(\omega \mp k_z u)$$

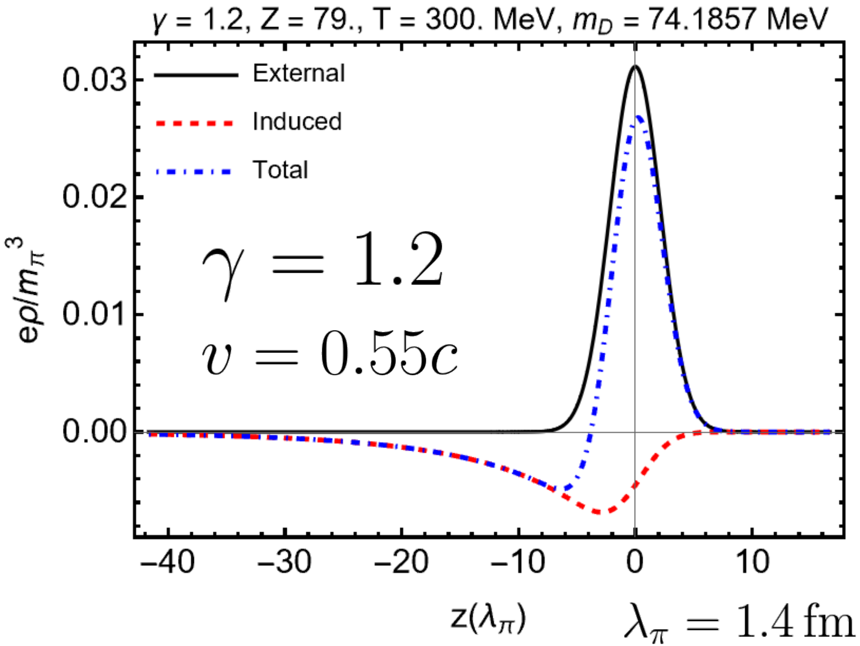
$$\widetilde{\boldsymbol{j}}_{\pm b/2}(\omega,\boldsymbol{k}) = \pm u\hat{\boldsymbol{z}}\widetilde{\rho}_{\pm b/2}(\omega,\boldsymbol{k})$$



Induced Charge

As γ increases the external charge density is contracted (grows large in magnitude but smaller in space). The functional dependence of the tail changes little so appears to be small and more smeared out at high gamma.





$$m_{D(EM)}^{2} = \sum_{u,d,s} q_f^2 T^2 \frac{N}{3} = 2\frac{e^2T^2}{3}$$

$$\kappa_{(\mathrm{QCD})} \approx 5\, m_{D(\mathrm{EM})}$$

$$\lambda_\pi = 1.4\,\mathrm{fm}$$

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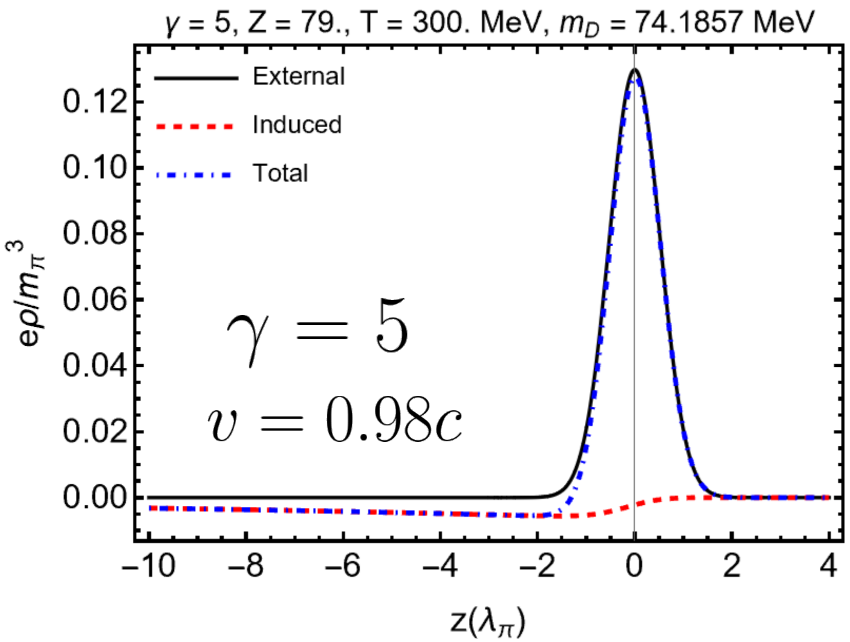
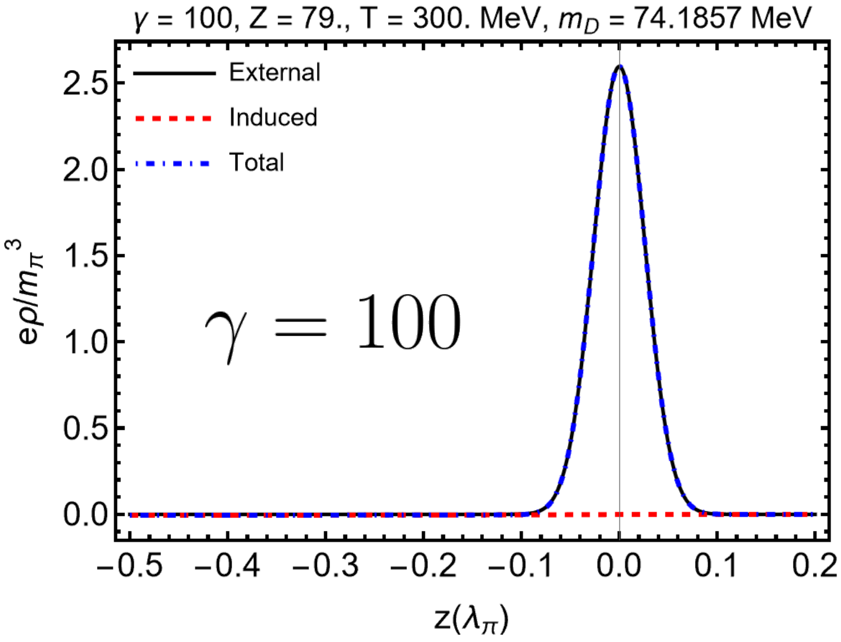
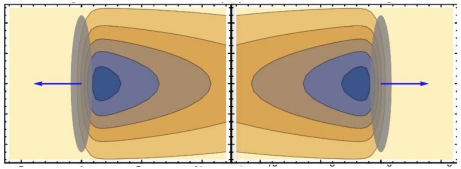




$$m_{D\,(EM)}^{\;2} = \sum_{u,d,s} q_f^2 T^2 \frac{N}{3} = 2\frac{e^2T^2}{3}$$

$$\kappa_{(\mathrm{QCD})} \approx 5\, m_{D(\mathrm{EM})}$$

Induced Charge - (Single Nuclei)



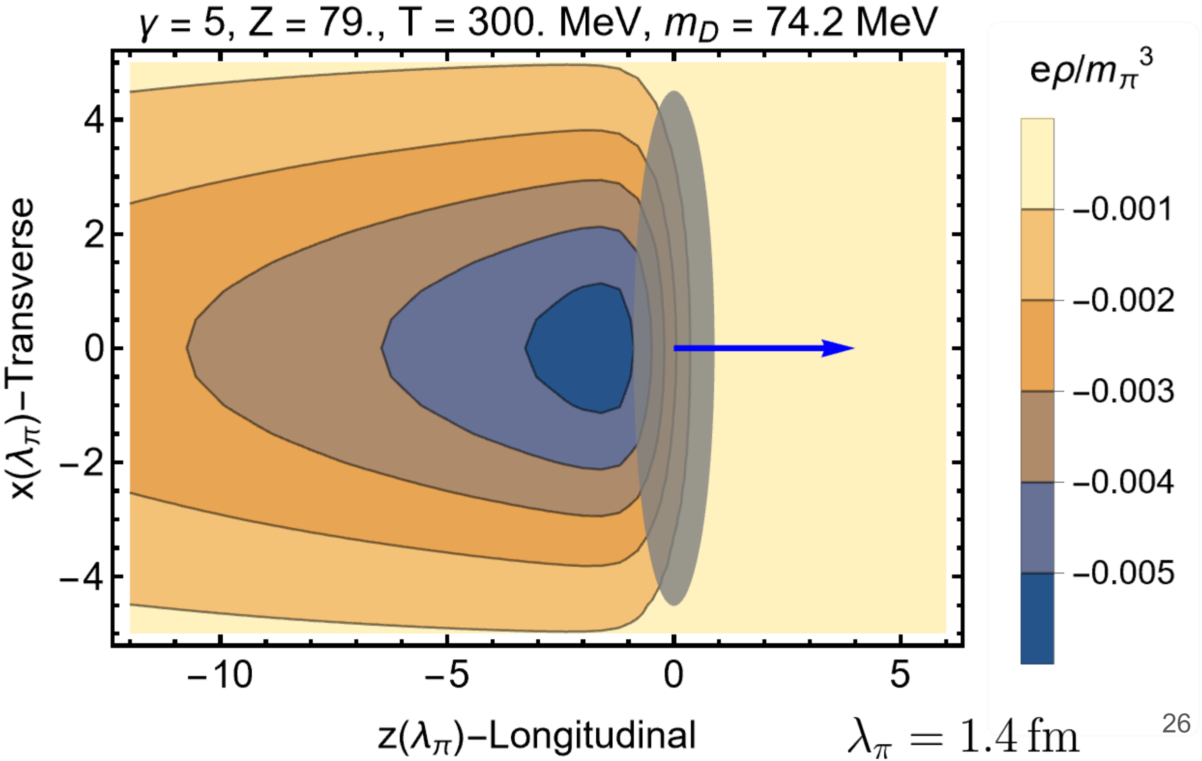
Plot below shows a single ion traveling through the plasma. A trailing negatively charged wake is shown trailing behind the positive Ion.

$$\widetilde{j}^{\mu}_{\rm ind}(k) = \Pi^{\mu}_{\nu}(k)\widetilde{A}^{\nu}(k)$$

$$\widetilde{\rho}_{\rm ind} = \Pi_L \widetilde{\phi}\left(1 - \frac{|\boldsymbol{k}|^2}{\omega^2}\right)$$

$$\widetilde{\boldsymbol{j}}^{T}_{\rm ind} = \Pi_T \widetilde{\boldsymbol{A}}_T \quad \widetilde{j}^{L}_{\rm ind} = \frac{\omega}{|\boldsymbol{k}|}\widetilde{\rho}_{\rm ind}$$

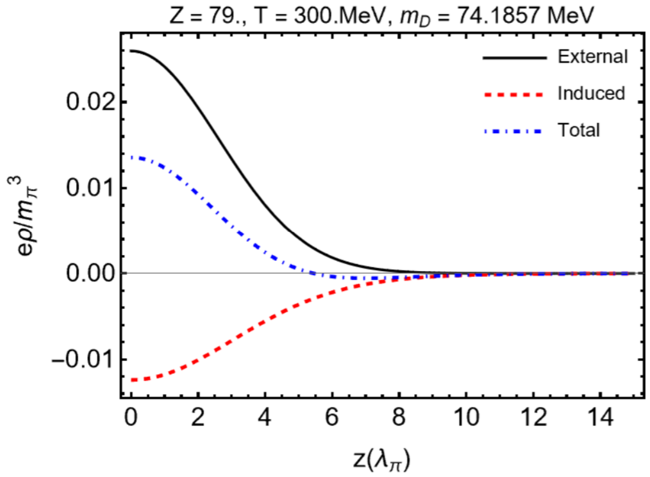
$$\kappa_{\rm (QCD)} \approx 5\, m_{D\rm (EM)}$$

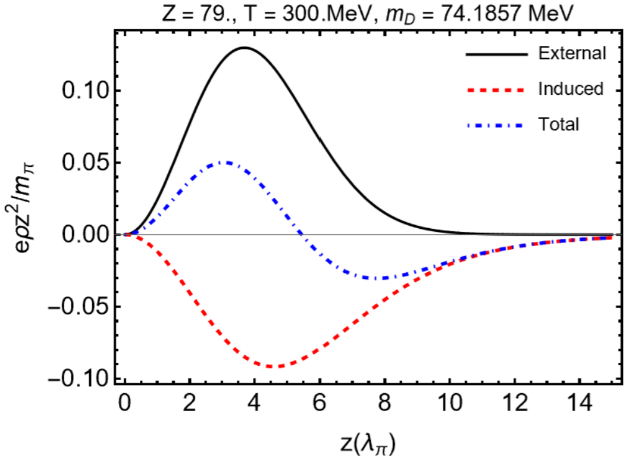


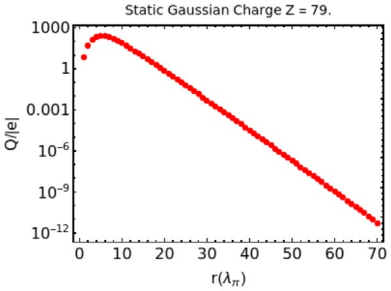
$\lambda_\pi = 1.4\,\mathrm{fm}$

Charge Conservation in the Infinite Plasma

In the case of an infinite plasma in both space and time the induced charge balancing the induced screening charges has moves to infinity.







Plasma Frequency

We can use the dispersion relation to solve for the natural oscillations of the effective particles within the plasma i.e. plasmons, This is done by solving the dispersion relation in the limit k→0,

$$\frac{1}{(k\cdot\tilde{u})}\left((k\cdot\tilde{u})^2+\mu_0\Pi_L(k)\right)\left(k^2+\mu_0\Pi_T(k)\right)^2=0$$

for both modes one finds,

$$\omega=-i\frac{\kappa}{2}\pm\sqrt{\omega_p^2-\frac{\kappa^2}{4}}$$ Where $$\omega_p^2=\frac{m_D^2}{3}$$

For an oscillatory electric field, these modes are damped by the relaxation parameter

$$\mathbf{E}=E_0e^{-i\omega t}\longrightarrow\mathbf{E}=E_0e^{-\frac{\kappa}{2}t\pm it\sqrt{\omega_p^2-\frac{\kappa^2}{4}}}$$

Susceptibility

$$\Pi^i_j = \begin{bmatrix} \Pi^x_x & 0 & 0 \\ 0 & \Pi^x_x & 0 \\ 0 & 0 & -\frac{\omega^2}{|\mathbf{k}|^2}\Pi^0_0 \end{bmatrix}$$

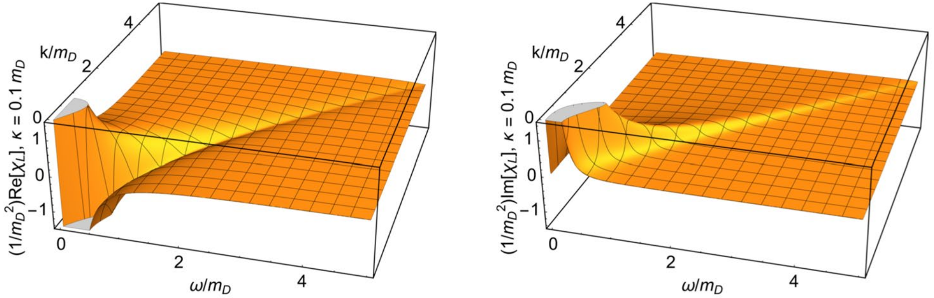


FIG. 3. Longitudinal susceptibility $\chi_L(\omega, k)$ in units of m_D^2 for $\kappa = 0.1\, m_D$. Left panel: real part of $\chi_L(\omega, k)$; right panel: imaginary part of $\chi_L(\omega, k)$.

$$K^i_j(\omega, \mathbf{k}) = 1 + \frac{\Pi^i_j(\omega, \mathbf{k})}{\omega^2 \varepsilon_0} = 1 + \chi^i_j(\omega, \mathbf{k})$$

(Rukhadze and Silin, 1961)

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$$

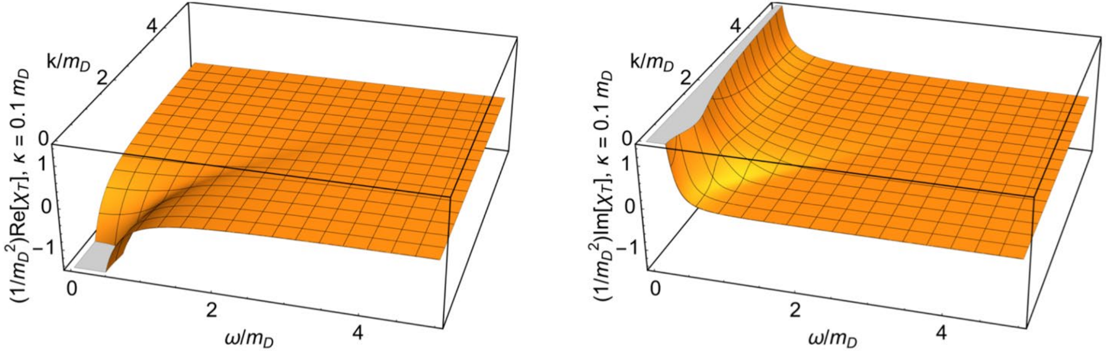


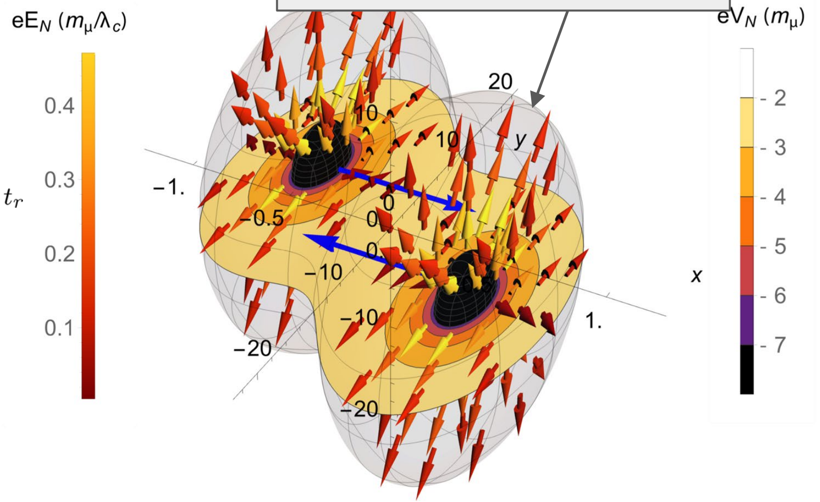
FIG. 4. Transverse susceptibility $\chi_T(\omega, k)$ in units of m_D^2 for $\kappa = 0.1\, m_D$. Left panel: real part of $\chi_T(\omega, k)$; right panel: imaginary part of $\chi_T(\omega, k)$.

Fields of Colliding Ions

$$eE(\mathbf{r},t) = Z\alpha\hbar c \left(\frac{(\mathbf{n}-\beta)}{\gamma^2(1-\mathbf{n}\cdot\beta)^3|\mathbf{r}-\mathbf{r}_s|^2} + \frac{\mathbf{n}\times((\mathbf{n}-\beta)\times\dot{\beta})}{c(1-\mathbf{n}\cdot\beta)^3|\mathbf{r}-\mathbf{r}_s|} \right)_{t_r}$$

Velocity Field   Radiation Field

Surface indicates where potential reaches $2m_\mu$



Blue arrows indicate direction of motion

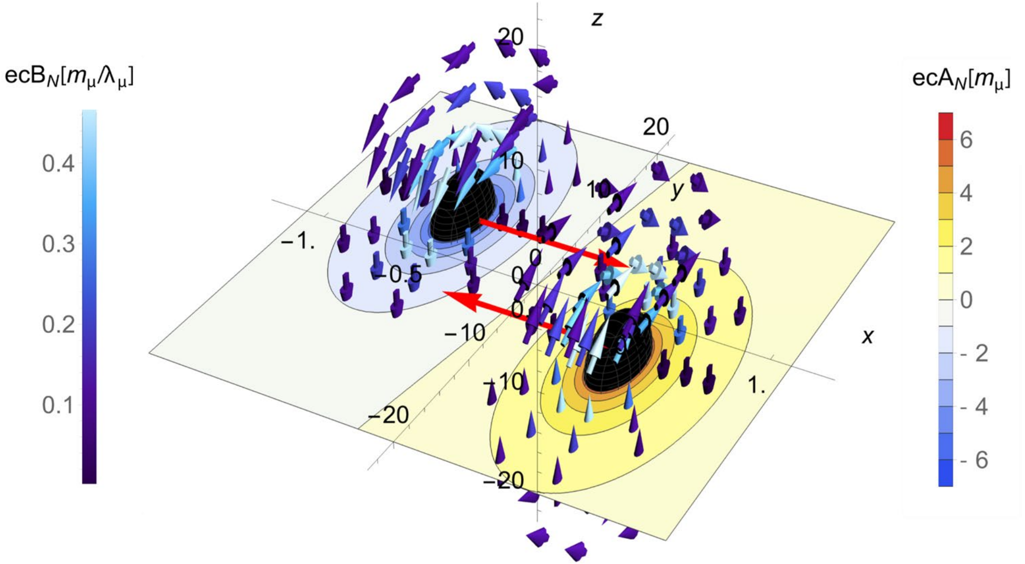


Figure 19: Here we see the fields and gauge potential of two colliding Pb ions at $\gamma = 37$ (about 74 GeV per nucleon pair in the CM frame) and impact parameter $b = 3R$ where the ions are separated by $\Delta x = 2\beta ct = 1\lambda_\mu$, where λ_μ is the muon Compton wavelength, approximately 1.87 fm. At this collision energy the potential of the ions surpasses $2m_\mu$ over a distance larger than the muon compton wavelength λ_μ The lorentz contracted nuclei are indicated by black surfaces traveling in the x direction (collision axis). (a) the potential is plotted in units of m_μ in the xy-plane (collision plane) and the electric field vectors are shown in units $\frac{m_\mu}{\lambda_\mu}$ where in these units the Schwinger field $eE_s = 1\frac{m_\mu}{\lambda_\mu}$. The $2m_\mu$ barrier, at which the Dirac equation predicts boundstates to dive in to the negative continuum, [110] is indicated by a gray surface.

Conductivity

$$\sigma^i_j(\omega, \mathbf{k}) = -i\frac{\Pi^i_j(\omega, \mathbf{k})}{\omega}$$

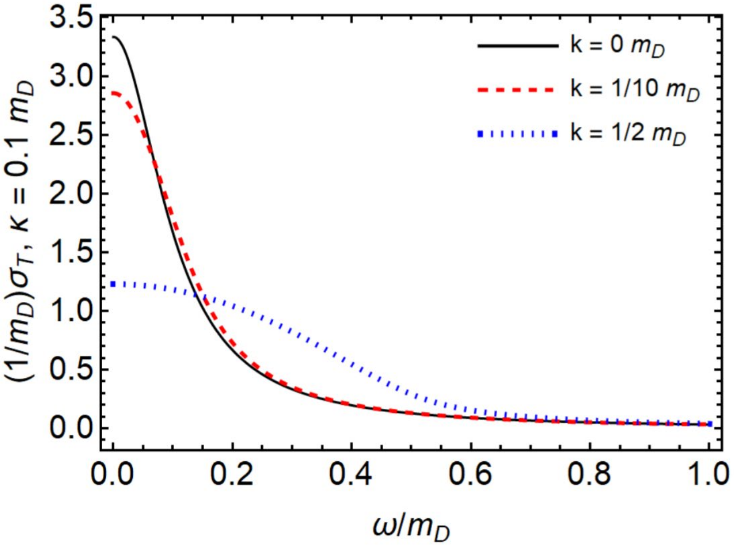


FIG. 8. Real part of σ_T for different values of k.

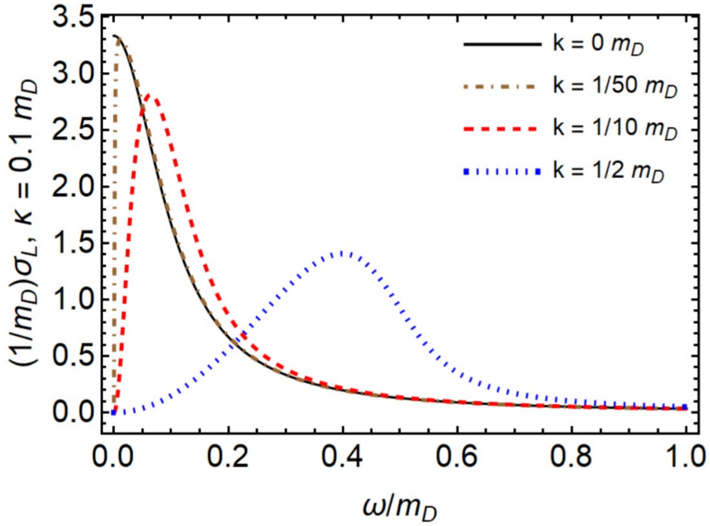


FIG. 7. Real part of σ_L for different values of k.

Discontinuity at k = 0 comes from infinite extent of plasma (Baranger 1989)

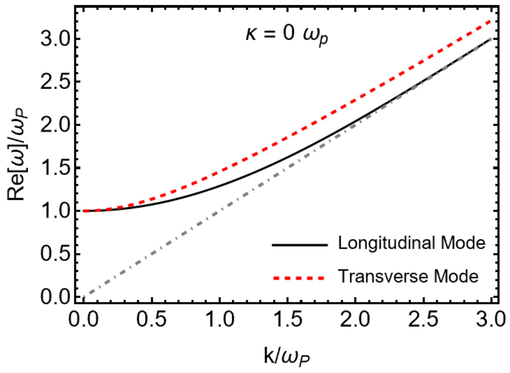
Modes - Dispersion Relation

$$((k \cdot u)^2 + \mu_0 \Pi_L(\omega, k))(k^2 + \mu_0 \Pi_T(\omega, k))^2 = 0$$

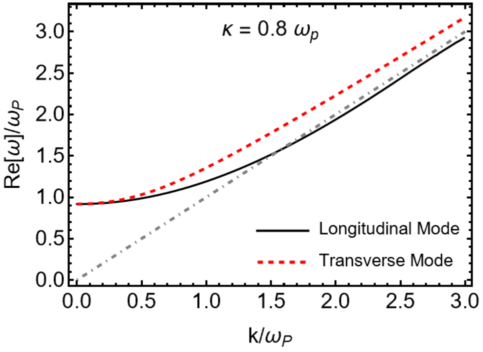
Negative imaginary part means modes are damped

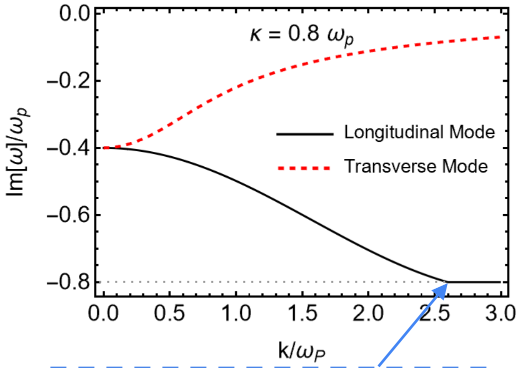
Longitudinal Modes: Electric Charge Oscillations → Langmuir waves (density fluctuations), Debye screening (charge screening)

Transverse Modes: Current Oscillations → Electromagnetic waves in vacuum



Finte κ





Longitudinal solution runs into the branch cut at *-iκ*

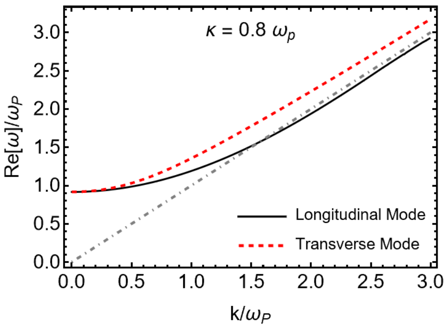
Modes - Dispersion Relation

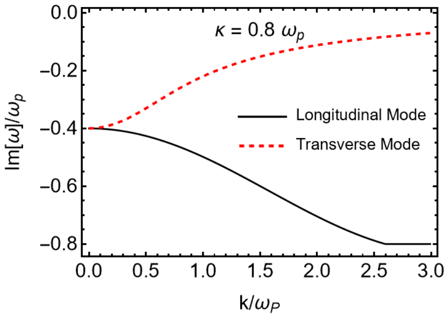
$$\ln \frac{\omega + i\kappa + |\mathbf{k}|}{\omega + i\kappa - |\mathbf{k}|}$$

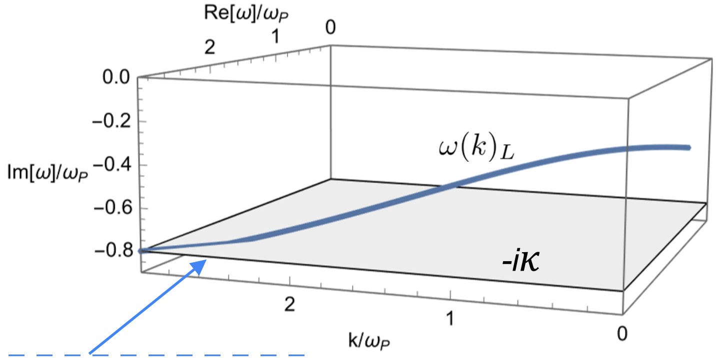
$$((k \cdot u)^2 + \mu_0 \Pi_L(\omega, k))(k^2 + \mu_0 \Pi_T(\omega, k))^2 = 0$$

<u>Longitudinal Modes</u>: Electric Charge Oscillations → Langmuir waves (density fluctuations), Debye screening (charge screening)

<u>Transverse Modes:</u> Current Oscillations → Electromagnetic waves in vacuum







Longitudinal solution runs into the branch cut at *-iκ*

(Schenke et al. 2006)

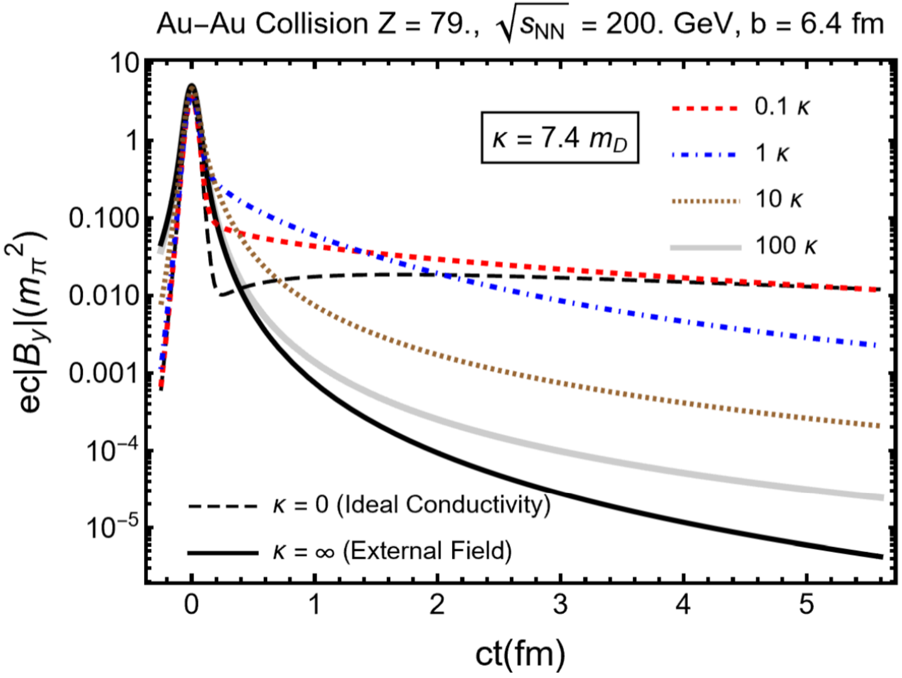


FIG. 6. Comparison of the magnetic field for different values of QCD dampening or equivalently conductivity. Larger values of dampening κ represent smaller conductivities and vice versa as indicated by Eq. (54). The black dashed line and the black line represent the limits of zero and infinite or ideal conductivity respectively. One can see that as κ increases the asymptotic value of the magnetic field decreases.