NEW INSIGHTS INTO THE MASS GENERATION AND STRUCTURE OF THE QCD GROUND STATE

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Lagrangian of QCD

H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. B **47** (1973) 365

$$L_{QCD} = L_{YM} + L_{qg}$$

$$L_{YM} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + L_{g.f.} + L_{gh.}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \quad a = N_c^2 - 1, \quad N_c = 3$$

$$L_{qg} = i\bar{q}^j_\alpha D_{\alpha\beta} q^j_\beta + \bar{q}^j_\alpha m^j_0 q^j_\beta, \quad \alpha, \beta = 1, 2, 3, \quad j = 1, 2, 3, \ldots N_f$$

$$D_{\alpha\beta} q^j_\beta = (\delta_{\alpha\beta}\partial_\mu - ig(1/2)\lambda^a_{\alpha\beta} A^a_\mu)\gamma_\mu q^j_\beta \qquad (cov.\ der.)$$

The λ^as are generators of $SU(3)$ color gauge group

$$[\lambda^a, \lambda^b] = 2if^{abc}\lambda^c$$

$$q^A \;\rightarrow\; U(x)q^A(x), \quad U(x) = \exp(i\Theta^a(x)\lambda^a/2)$$

$$A_\mu(x) = A^a_\mu(x)\lambda^a/2 \;\rightarrow\; U(x)A_\mu(x)U^{-1}(x) + \frac{i}{g}U(x)\partial_\mu U^{-1}(x)$$

$$SU(N_f) \times SU(N_f) \times U_B(1) \times U_A(1), \quad q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$$

$$L_q = \bar{q}^A_\alpha \delta^{AB} m_0 q^B_\alpha$$

$$SU(N_f) \times SU(N_f) \rightarrow SU(N_f)$$

Properties of the QCD Lagrangian

QCD without quarks is Yang-Mills (YM)

1). Universal coupling constant is strong, $g \sim 1$.

2). In general, in QCD the PT does not work.

3). $g \ll 1$ only in the AF regime, where the PT works.

4). But there is a scale violation there, appears the finite constant Λ^2_{QCD}.

5). However, any scale parameter, having the dimension of mass squared, for example such as $M^2_g A_\mu A_\mu$, explicitly violates $SU(3)$ color gauge symmetry/invariance of the QCD Lagrangian.

Conceptual problems of QCD

A. The dynamical generation of a mass squared at the fundamental quark-gluon level, since the QCD Lagrangian forbids such kind of terms apart from the current quark mass.

From the beginning of QCD it has been asked the question

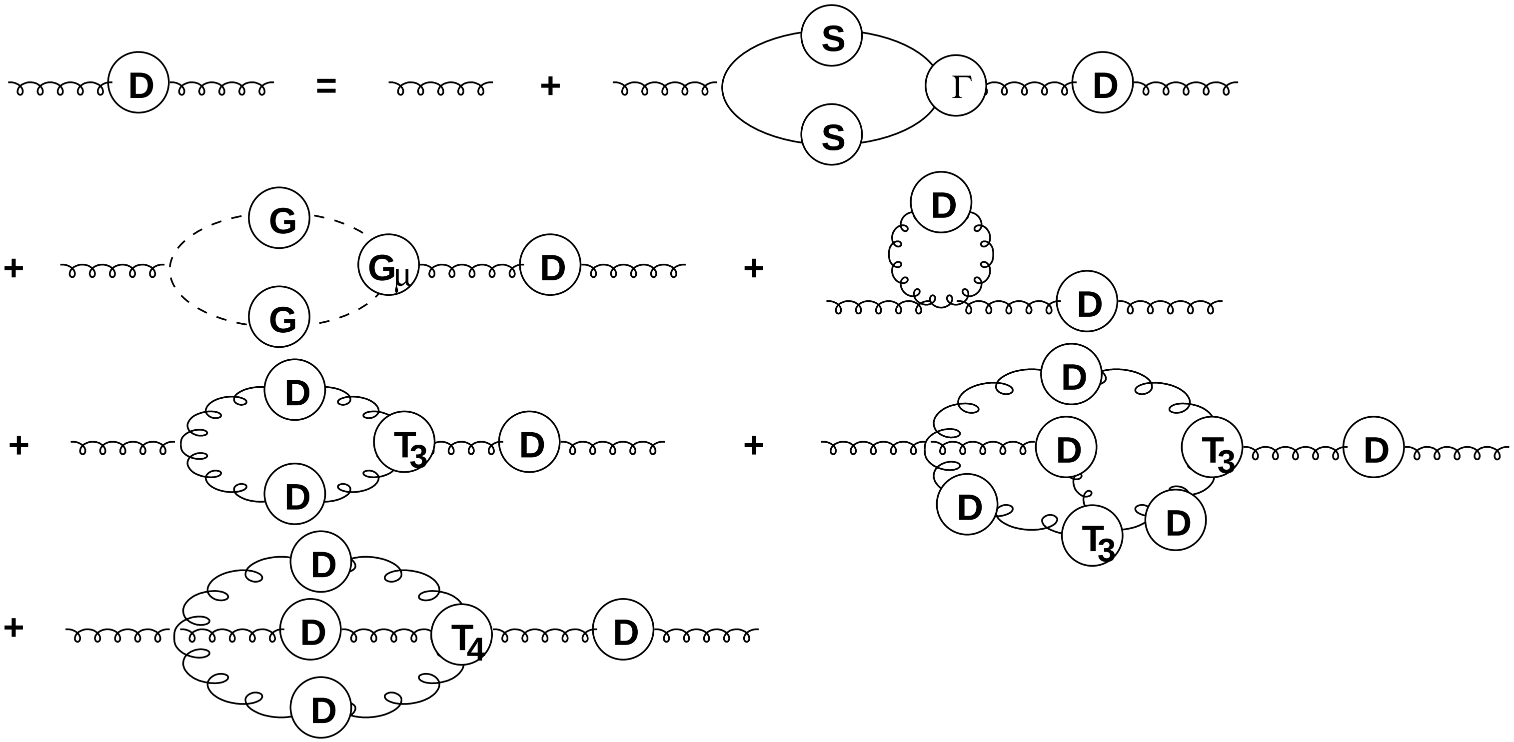
How does one get a mass out of massless theory?

’Mass without mass’

Such a mass has been called ’mass gap’

B. Whether the symmetries of the QCD Lagrangian and its ground state coincide or not?

Gluon SD equation



$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i\Pi_{\rho\sigma}(q; D) D_{\sigma\nu}(q)$$

$$\Pi_{\rho\sigma}(q; D) = \Pi^q_{\rho\sigma}(q) + \Pi^g_{\rho\sigma}(q; D) + \Pi^t_{\rho\sigma}(D)$$

$$\Pi^g_{\rho\sigma}(q; D) = \Pi^{gh}_{\rho\sigma}(q) + \Pi^{(1)}_{\rho\sigma}(q; D^2) + \Pi^{(2)}_{\rho\sigma}(q; D^4) + \Pi^{(2')}_{\rho\sigma}(q; D^3)$$

$$\Pi^t_{\rho\sigma}(D) \sim \int d^4k D_{\alpha\beta}(k) T^0_{\rho\sigma\alpha\beta} = g_{\rho\sigma}\Delta^2_t(D) = [T_{\rho\sigma}(q) + L_{\rho\sigma}(q)]\Delta^2_t(D)$$

$$\Pi_{\rho\sigma}(q; D) \equiv \Pi_{\rho\sigma}(q; \tilde{\lambda}, \alpha, D), \quad L_{\rho\sigma}(q) = \frac{q_\rho q_\sigma}{q^2}$$

$$(Eucl.\ sign.)\ q^2 \to 0,\ q_i \to 0,\ q^2 \to -q^2,\ g_{\rho\sigma} \to \delta_{\rho\sigma}$$

Transversity of the full gluon self-energy

ST identities

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi, \quad q_\mu q_\nu D^0_{\mu\nu}(q) = i\xi_0$$

$$D_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)d(q^2) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

$$D^0_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q) + \xi_0 L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

$$T_{\mu\nu}(q) = \delta_{\mu\nu} - (q_\mu q_\nu/q^2) = \delta_{\mu\nu} - L_{\mu\nu}(q)$$

Subtraction scheme and the transverse relations

$$\Pi^{(s)}_{\rho\sigma}(q;D) = \Pi_{\rho\sigma}(q;D) - \Pi_{\rho\sigma}(0;D) = \Pi_{\rho\sigma}(q;D) - \delta_{\rho\sigma}\Delta^2(D),$$

and thus $\Pi^{(s)}_{\rho\sigma}(0;D) = 0$, the subtraction is assumed at $q^2 = -\mu^2 \to 0$

$$\Pi_{\rho\sigma}(0) = \Pi^{q}_{\rho\sigma}(0) + \Pi^{g}_{\rho\sigma}(0;D) + \Pi^{t}_{\rho\sigma}(D)$$

$$= \delta_{\rho\sigma}\Delta^2(D) = \delta_{\rho\sigma}[\Delta^2_q + \Delta^2_g(D) + \Delta^2_t(D)]$$

$$q_\rho q_\sigma \Pi_{\rho\sigma}(q;D) = \frac{(\xi_0 - \xi)}{\xi\xi_0}(q^2)^2$$

$$q_\rho q_\sigma \Pi^{(s)}_{\rho\sigma}(q;D) = \frac{(\xi_0 - \xi)}{\xi\xi_0}(q^2)^2 - q^2\Delta^2(D)$$

$$\Pi_{\rho\sigma}(q) = T_{\rho\sigma}(q) q^2 \Pi_t(q^2) - q_\rho q_\sigma \Pi_l(q^2),$$

$$\Pi^{(s)}_{\rho\sigma}(q) = T_{\rho\sigma}(q) q^2 \Pi^{(s)}_t(q^2) - q_\rho q_\sigma \Pi^{(s)}_l(q^2)$$

$$\Pi^{(s)}_t(q^2) = \Pi_t(q^2) - \frac{\Delta^2(D)}{q^2},$$

$$\Pi^{(s)}_l(q^2) = \Pi_l(q^2) + \frac{\Delta^2(D)}{q^2} = -\frac{(\xi_0 - \xi)}{\xi\xi_0} + \frac{\Delta^2(D)}{q^2}$$

$$\Pi_{\rho\sigma}(q) = T_{\rho\sigma}(q)\left[q^2\Pi_t^{(s)}(q^2) + \Delta^2(D)\right] + L_{\rho\sigma}\frac{(\xi_0 - \xi)}{\xi\xi_0}q^2$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q)\left[q^2\Pi_t^{(s)}(q^2) + \Delta^2(D)\right] D_{\sigma\nu}(q)$$

$$+ D^0_{\mu\rho}(q) i L_{\rho\sigma}(q)\frac{(\xi_0 - \xi)}{\xi\xi_0}q^2 D_{\sigma\nu}(q)$$

However, let us underline that contracting it with q_μ and q_ν, one obtains identities $\xi = \xi$ and $\xi_0 = \xi_0$, and not ξ as a function of ξ_0, i.e., $\xi = f(\xi_0)$. In fact, this expression is not an equation, but it is an identity! The only way to get out of these troubles is to satisfy (i.e., put zero) at least one of the transverse relations. Only this will make from the above expression an equation for the full gluon propagator, and thus to fix $\xi = f(\xi_0)$.

Preservation of the exact gauge symmetry

$$q_\rho q_\sigma \Pi_{\rho\sigma}(q; D^{PT}) = q_\rho q_\sigma \Pi^{(s)}_{\rho\sigma}(q; D^{PT}) = 0, \quad \xi = \xi_0, \quad \Delta^2(D) = 0$$

$$D^{PT}_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q) q^2 \Pi^{(s)}_t(q^2) D^{PT}_{\sigma\nu}(q)$$

$$q_\mu q_\nu D^{PT}_{\mu\nu}(q) = q_\mu q_\nu D^0_{\mu\nu}(q) = i\xi = i\xi_0$$

$$D^{PT}_{\mu\nu}(q) = i \left[\frac{1}{1 + \Pi^{(s)}_t(q^2)} T_{\mu\nu}(q) + \xi_0 L_{\mu\nu}(q) \right] \frac{1}{q^2}$$

This system of eqs. is free of all the types of the scale parameters having the dimensions of mass squared, forbidden by the exact gauge symmetry of the QCD Lagrangian. Therefore, the gauge symmetry of the QCD ground state coincides with the symmetry of its Lagrangian.

Violation of the exact gauge symmetry

$$q_\mu q_\nu \Pi_{\rho\sigma}(q;D) = q^2 \Delta^2(D) \neq 0$$

$$q_\mu q_\nu \Pi^{(s)}_{\rho\sigma}(q;D) = 0$$

$$\frac{(\xi_0 - \xi)}{\xi \xi_0} = \frac{\Delta^2(D)}{q^2}$$

$$\Delta^2(D) = \Delta_q^2 + \Delta_g^2(D) + \Delta_t^2(D)$$

$$\Delta_q^2 = \Delta_g^2(D) = 0, \quad \Delta^2(D) \to \Delta_t^2(D)$$

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\rho}(q) i T_{\rho\sigma}(q) \left[q^2 \Pi_t^{(s)}(q^2) + \Delta_t^2(D) \right] D_{\sigma\nu}(q)$$

$$D^0_{\mu\rho}(q) i L_{\rho\sigma}(q) \Delta_t^2(D) D_{\sigma\nu}(q)$$

$$d(q^2) = \frac{1}{1 + \Pi(q^2; D) + (\Delta_t^2(D)/q^2)}$$

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi_0 \left(1 - \xi \frac{\Delta_t^2(D)}{q^2} \right) = i\xi$$

$$\xi \equiv \xi(q^2) = \frac{\xi_0 q^2}{q^2 + \xi_0 \Delta_t^2(D)}$$

The existence of the mass gap

$$D_{\mu\nu}(q) = \frac{iT_{\mu\nu}(q)}{q^2 + q^2\Pi(q^2; D) + \Delta_t^2(D)} + iL_{\mu\nu}(q)\frac{\lambda^{-1}}{q^2 + \lambda^{-1}\Delta_t^2(D)}$$

where we have introduced the useful notation, namely $\xi_0 = \lambda^{-1}$

$$q_\mu q_\nu D_{\mu\nu}(q) = i\xi(q^2) = i\frac{\lambda^{-1}q^2}{(q^2 + \lambda^{-1}\Delta_t^2(D))}$$

$\lambda^{-1} = 0$, $\lambda^{-1} = 1$. The formal $\lambda^{-1} = \infty$.

This system of the regularized equations constitutes that the $SU(3)$ color gauge symmetry of the QCD Lagrangian is not a symmetry of its ground state.

$$\Delta_t^2(D) = \Delta^2 \times f(\tilde{\lambda}), \quad \Delta^2 > 0, \quad f(\tilde{\lambda}) \sim \tilde{\lambda}^2, \quad \tilde{\lambda} \to \infty$$

Why do we need the mass gap at the fundamental level?

$$D_{\mu\nu}(q) \sim g_{\mu\nu}\Big[\frac{g^2}{1 + g^2 b_0 \ln(q^2/\Lambda^2_{QCD})}\Big](1/q^2), \quad q^2 \to \infty$$

Any mass to which can be assigned some physical meaning

$$M \sim \mu \exp(-1/b_0 g^2), \qquad g^2 \to 0$$

It vanishes to every order of the PT. None a finite mass can survive in the PT weak coupling limit or, equivalently, in the PT $q^2 \to \infty$ regime. So the question where the finite mass comes from? cannot be answered by the PT! The renormalization group equations predicts the existence of a such finite mass, but can not explain how it appears under the PT logarithms!

The mass gap and asymptotic freedom

If the tadpole term survives the renormalization beyond the PT, then the AF behavior of the full gluon propagator will be explained.

$$d(q^2) = \frac{1}{1 + \Pi(q^2; d) + (\Delta_t^2(d)/q^2)}$$

$d(q^2) = \alpha_s(q^2; \tilde{\lambda})/\alpha_s(\tilde{\lambda})$ and in the PT $q^2 \to \infty$ limit

$$\alpha_s(q^2; \tilde{\lambda}) = \frac{\alpha_s(\tilde{\lambda})}{1 + b_0 \alpha_s(\tilde{\lambda}) \ln(q^2/\Delta_t^2(\tilde{\lambda}))}$$

$$\Delta_t^2(\tilde{\lambda}) = f(\tilde{\lambda})\Delta^2 = f(\tilde{\lambda}) A^{-1} A \Delta^2 = f(\tilde{\lambda}) \Lambda_{QCD}^2$$

$$\alpha_s(q^2) = \frac{\alpha_s}{1 + b_0\alpha_s \ln(q^2/\Lambda^2_{QCD})},$$

$$\alpha_s = \frac{\alpha_s(\tilde{\lambda})}{1 - b_0\alpha_s(\tilde{\lambda}) \ln f(\tilde{\lambda})}, \quad \alpha_s \equiv \alpha_s(M_Z) = 0.1184$$

$$\ln f(\tilde{\lambda}) = \frac{\alpha_s - \alpha_s(\tilde{\lambda})}{\alpha_s b_0 \alpha_s(\tilde{\lambda})} \to \frac{1}{b_0\alpha_s(\tilde{\lambda})}, \quad \alpha(\tilde{\lambda}) \to 0, \quad \tilde{\lambda} \to \infty$$

$$f(\tilde{\lambda}) = \exp(1/b_0\alpha_s(\tilde{\lambda}))$$

$$\lim_{\tilde{\lambda}\to\infty} \Delta_t^2(\tilde{\lambda}) \exp\left[-\frac{1}{b_0 \alpha_s(\tilde{\lambda})}\right] = \Lambda_{QCD}^2, \quad \alpha(\tilde{\lambda}) \to 0$$

$$\alpha_s(q^2) = \frac{\alpha_s}{1 + b_0 \alpha_s \ln(q^2/\Lambda_{QCD}^2)}$$

At very large gluon momentum q^2 one recovers

$$\alpha_s(q^2) = \frac{1}{b_0 \ln(q^2/\Lambda_{QCD}^2)} \to 0, \quad q^2 \to \infty$$

The summation of the severe IR singularities

Starting the NL iteration procedure of the initial gluon SD equation, one arrives at

$$D(q) = 1/q^2 + (1/q^2)\Delta^2 f(\tilde{\lambda}, ...)(1/q^2) + ...$$

$$D_{\mu\nu}(q) \sim T_{\mu\nu} \frac{\Delta^2}{(q^2)^2} \sum_{k=0}^{\infty} \Big(\frac{\Delta^2}{q^2} \Big)^k \Phi_k(\tilde{\lambda}, ...) + ..., \quad q^2 \to 0$$

The free gluon states can not appear in the physical spectrum at large distances ($q^2 \to 0$). If this statement will survive the INP renormalization program beyond the PT then confinement of the free gluons will be proven.

The massive gluons (Minkowski signature)

$$D_{\mu\nu}(q) = \frac{-iT_{\mu\nu}(q)}{q^2 + q^2\Pi(q^2; D) - M^2} - iL_{\mu\nu}(q)\frac{\lambda^{-1}}{q^2 - \lambda^{-1}M^2}$$

If the denominator may have a pole at $q^2 = M_g^2$, then one obtains

$$M^2 = [1 + \Pi(M_g^2)]M_g^2 = \tilde{Z}_3^{-1}M_g^2, \quad \tilde{Z}_3^{-1} = [1 + \Pi(M_g^2)]$$

$$D^R_{\mu\nu}(q) = \frac{-iZ_3\tilde{Z}_3^{-1}}{(q^2 - M_g^2)[1 + Z_3\tilde{\Pi}(q^2)]}T_{\mu\nu}(q) - iL_{\mu\nu}(q)\frac{\tilde{\lambda}^{-1}}{[q^2 - \tilde{\lambda}^{-1}M_g^2]}$$

$$D^R_{\mu\nu}(q) = \tilde{Z}_3^{-1}D_{\mu\nu}(q), \quad \tilde{\lambda} = \lambda\tilde{Z}_3, \quad Z_3\tilde{Z}_3^{-1} = 1 - Z_3M_g^2\Pi'(M_g^2)$$

$$\tilde{\Pi}(M_g^2) = 0, \qquad \tilde{\Pi}(0) = -M_g^2 \Pi'(M_g^2)$$

Neglecting the contribution from the full gluon-self energy,

$$\Pi(q^2) = \tilde{\Pi}(q^2) = 0,\ Z_3 = \tilde{Z}_3 = 1,\ \tilde{\lambda} = \lambda,\ M_g^2 = M^2$$

$$D^0_{\mu\nu}(q; M_g^2) = \frac{-i}{(q^2 - M_g^2)} \left[g_{\mu\nu} - (1 - \lambda^{-1}) \frac{q_\mu q_\nu}{(q^2 - \lambda^{-1} M_g^2)} \right]$$

On the mass-shell $q^2 = M_g^2$

$$D^0_{\mu\nu}(q; M_g^2) = \frac{-i}{(q^2 - M_g^2)} \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{M_g^2} \right] \sim_{q^2 \to \infty} const$$

The PT full gluon propagator

$$D_{\mu\nu}(q) = \frac{-iT_{\mu\nu}(q)}{q^2[1+\Pi(q^2)]} - i\lambda^{-1}L_{\mu\nu}(q)\frac{1}{q^2}$$

$M_g^2[1+\Pi(M_g^2)] = 0$, when $q^2 = M_g^2$ then $[1+\Pi(M_g^2)] = 0$

$$\Pi(q^2) = \Pi(M_g^2) + (q^2 - M_g^2)\Pi'(M_g^2) + (q^2 - M_g^2)^2\Pi''(M_g^2) + \ldots$$

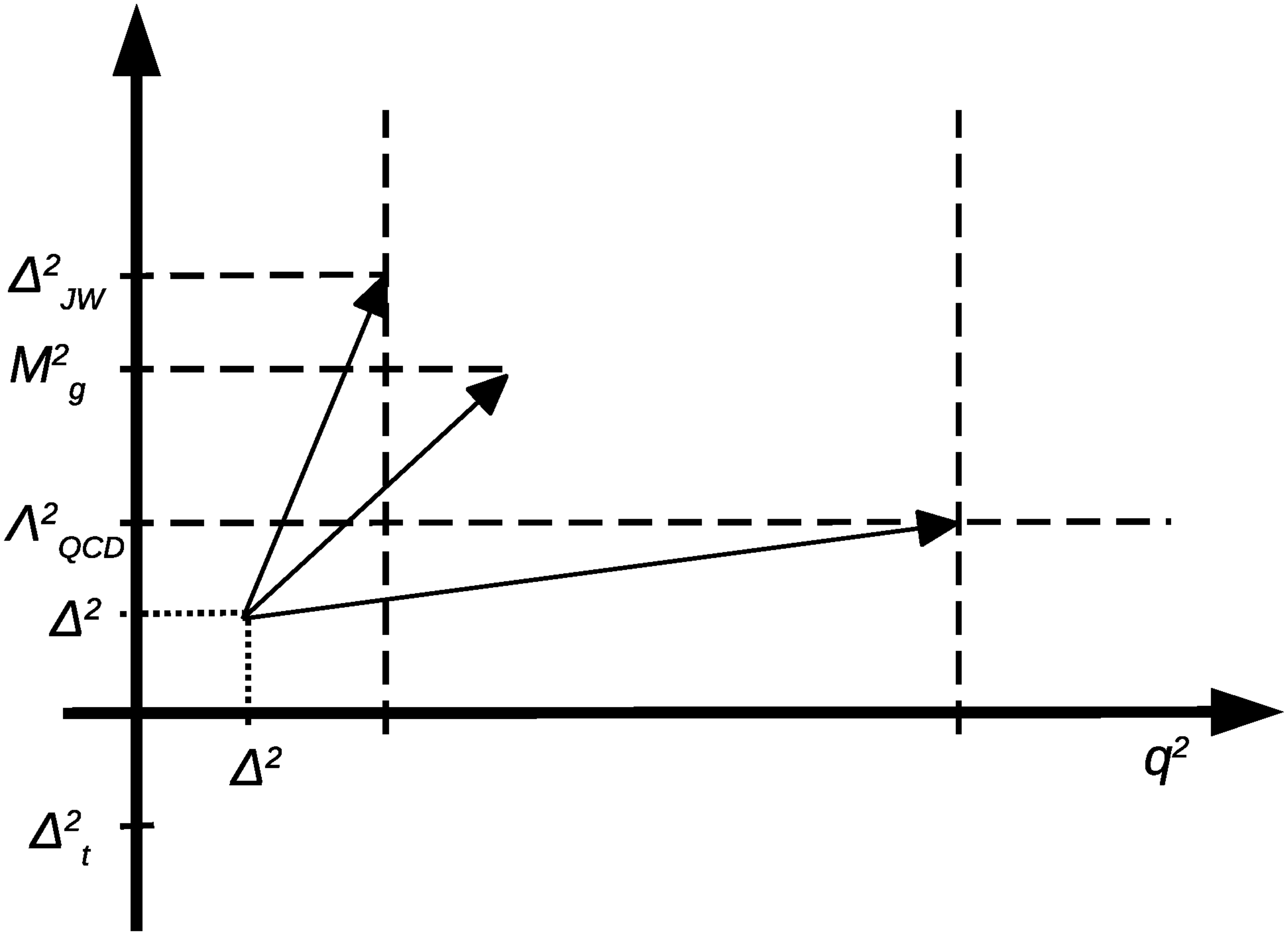
$$q^2[1+\Pi(q^2)] = (q^2 - M_g^2)q^2 R(q^2),\ R(q^2) = \Pi'(M_g^2) + (q^2 - M_g^2)\Pi''(M_g^2) + \ldots$$

$$(q^2 - M_g^2)R(q^2) = \Pi(q^2) - \Pi(M_g^2) = [1+\Pi(q^2)]$$

Conclusions

Mass Gap existence and the Yang-Mills theory: If a non-trivial quantum Yang-Mills theory with compact simple gauge group $SU(3)$ exists on R^4 then it has a mass gap $\Delta^2 > 0$. Existence of the mass gap includes establishing that the exact gauge symmetry of the theory's Lagrangian is not a symmetry of its ground state.

At the fundamental quark-gluon level the scale parameter having the dimension of mass squared is dynamically generated by the self-interaction of the multiple massless gluon modes involving only the point-like 4-gluon vertex. We identify such mass scale parameter with the tadpole term, which renormalized version has been called the mass gap. To understand this phenomenon is a necessary first step to understand the existence of the physical mass spectrum at the hadronic level in QCD as a theory of strong interactions.



Δ²JW
M²g
Λ²QCD
Δ²
Δ²
q²
Δ²t