Vacuum structure and particle production: Role of magnetic moment in nonperturbative QED

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Based on:

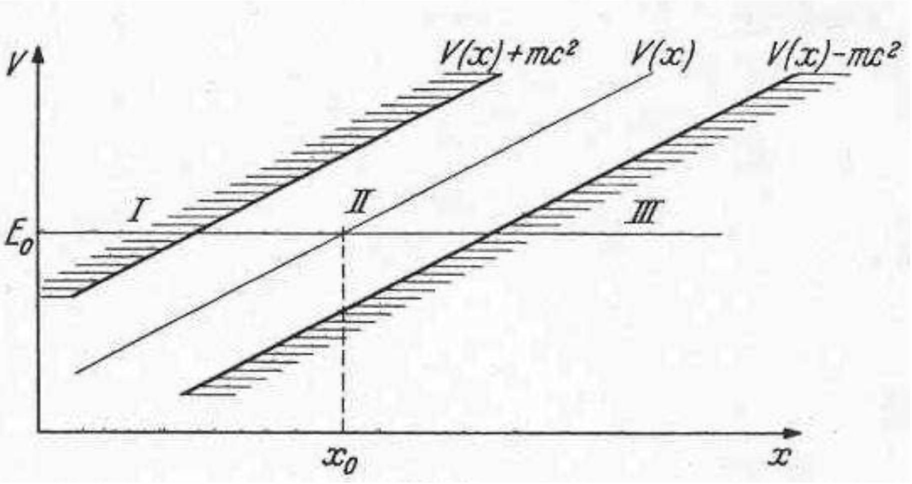
S. Evans and J. Rafelski. Eur. Phys. J. A 57 (2021) no.12, 341

S. Evans and J. Rafelski, (2022) In press – Phys. Lett. B. arXiv:2203.13145

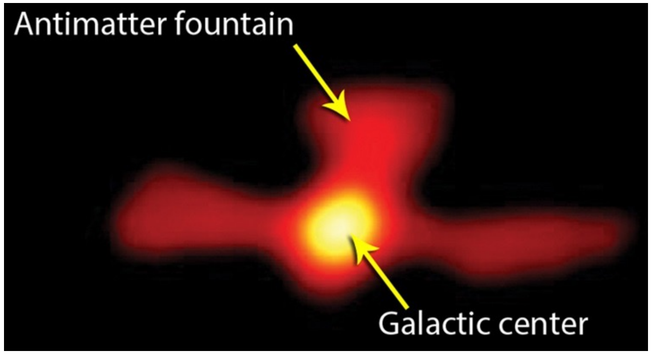
S. Evans et al., "Singular properties of Euler-Heisenberg action", in preparation

Physics

QED in strong fields



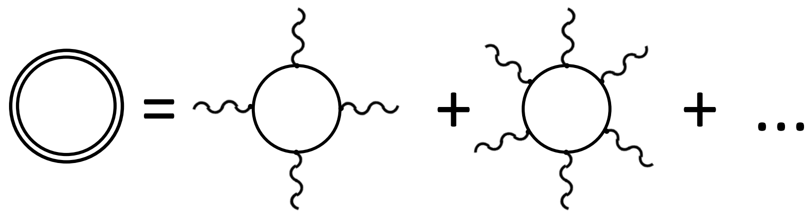
W. Heisenberg and H. Euler, Zeitschrift für Physik 98, 714 (1936).



NASA/W. Purcell, et al. APOD May 1st, 1997.

Euler-Heisenberg-Schwinger action

The vacuum response to spin 1/2 particles is described by the well-known Euler-Heisenberg-Schwinger (EHS) effective action



$$\mathcal{L}_{\text{EHS}} = \frac{1}{8\pi^2} \int_0^\infty \frac{du}{u^3} e^{-i(m^2 - i\epsilon)u} \left(e^2 u^2 ab \frac{\cosh(eau)\cos(ebu)}{\sinh(eau)\sin(ebu)} - 1 \right) ,$$

Leads to an imaginary part of effective action

$$\mathcal{E}_{\text{EHS}} = \frac{m^2 c^2}{e\hbar} = 1.32 \cdot 10^{18} \frac{\text{V}}{\text{m}} = 4.41 \cdot 10^9 c\text{T} .$$

$$a^2 - b^2 = \mathcal{E}^2 - \mathcal{B}^2 = 2S ,$$

$$a^2 b^2 = (\mathcal{E} \cdot \mathcal{B})^2 = P^2 .$$

W. Heisenberg and H. Euler, Zeitschrift für Physik 98, 714 (1936).

V. Weisskopf, Kong. Dan. Vid. Sel. Mat. Fys. Med. 14, N6, 1 (1936)

J. Schwinger, Phys. Rev. 82, 664 (1951).

Euler-Heisenberg-Schwinger action

Euler and Heisenberg recognized the imaginary part of effective action, describing the rate at which the vacuum decays into particle-antiparticle pairs

$$|\langle 0_{t=-\infty}|0_{t=+\infty}\rangle|^2 = e^{-2L^3 T \mathrm{Im}[\mathcal{L}_{\mathrm{EHS}}]} \,,$$

$$\mathrm{Im}[\mathcal{L}_{\mathrm{EHS}}] = \frac{e^2 ab}{8\pi^3}\sum_{n=1}^{\infty}\frac{\coth(n\pi b/a)}{n} e^{-n\pi m^2/ea} \,,$$

$$a^2 - b^2 = \mathcal{E}^2 - \mathcal{B}^2 = 2S \,, \qquad a^2 b^2 = (\mathcal{E}\cdot\mathcal{B})^2 = P^2 \,.$$

$$\mathcal{E}_{EH} = \frac{m^2c^2}{e\hbar} = 1.32\cdot 10^{18}\frac{\mathrm{V}}{\mathrm{m}} = 4.41\cdot 10^{9} c\mathrm{T} \,.$$

W. Heisenberg and H. Euler, Zeitschrift für Physik 98, 714 (1936).

V. Weisskopf, Kong. Dan. Vid. Sel. Mat. Fys. Med. 14, N6, 1 (1936)

J. Schwinger, Phys. Rev. 82, 664 (1951).

Referee: Sauter?

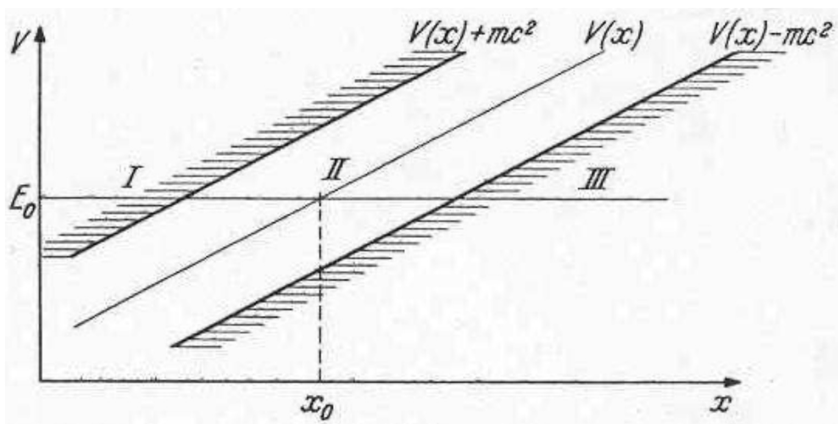
Constant vs localized fields

Conditions for EHS effective action: quasi-constant fields

$$\lambda_C \frac{|\nabla\cdot\mathcal{E}|}{|\mathcal{E}|} \ll 1\,,\ \lambda_C \frac{|\nabla\cdot\mathcal{B}|}{|\mathcal{B}|} \ll 1\,, \qquad \lambda_C = \hbar/mc = 386\ \mathrm{fm}$$

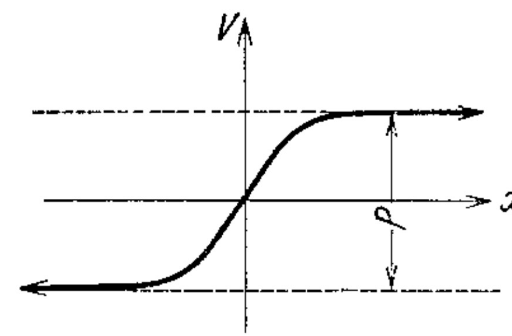
	Constant fields	Static Sauter potential edge
Solution to Dirac equation	F. Sauter, Z. Phys. 69 (1931), 742-764	F. Sauter, Z. Phys. 73 (1932), 547-552
Evaluation of effective action	W. Heisenberg and H. Euler, Zeitschrift für Physik 98, 714 (1936).	A. I. Nikishov. Nucl. Phys. B 21 (1970)

Constant fields:



Static Sauter potential edge:

$$A_0 = \frac{\mathcal{E}_0 L}{2}(\tanh[2z/L]+1)$$



← Both solutions for g=2 →

Role of anomalous magnetic moment

Magnetic moment of the electron:

$$\mu = g\mu_B = \frac{ge\hbar}{2m} \, ,$$

No fermion has exactly *g=2*:

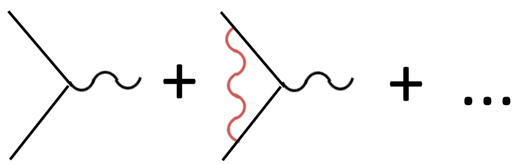
Measured electron *g*-factor: $g = 2.00231930436146(56)$

Hanneke, D.; Fogwell Hoogerheide, S.; Gabrielse, G. Physical Review A. 83 (5) 052122 (2011)

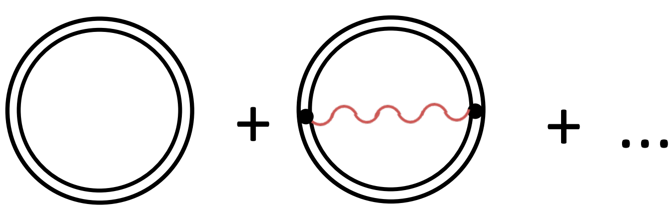
g in QED action

Perturbative QED around *g*=2

$$g \; = \; 2 \; + \; \alpha/\pi + \mathcal{O}(\alpha^2)$$



QED action



V. I. Ritus, Sov. Phys. JETP 42, 774 (1975)

Implementing *g*≠2 from the start

The two measured quantities are *magnetic moment* **and** mass

We insert measured *g*≠2 directly into the action (*the same way we treat mass*)

Re-derive EHS action with *g*≠2:

R. F. O'Connell, Phys. Rev. 1761, 1433 (1968)

W. Dittrich, J. Phys. A 11 (1978), 1191

S. I. Kruglov . Eur. Phys. J. C 22 (2001), 89-98

Euler-Heisenberg-Schwinger with $g \neq 2$

EHS action extended to $|g| \leq 2$: Klein-Gordon-Pauli (KGP) formulation

$$\Big[\Pi^2 - m^2 - \frac{g}{2}\frac{e\sigma_{\alpha\beta}F^{\alpha\beta}}{2}\Big]\psi = 0\ ,$$

$$\sigma_{\alpha\beta}F^{\alpha\beta}/2 = \vec{\sigma}\cdot\vec{\mathcal{B}} + i\vec{\alpha}\cdot\vec{\mathcal{E}}\ ,$$

$$\Pi_\alpha = i\partial_\alpha + eA_\alpha$$

The KGP expression enters the Schwinger proper time evolution operator to produce the g-dependent "EHSg" action:

$$\mathcal{L}_{\mathrm{EHSg}}(a,b,g) = \frac{1}{8\pi^2}\int_0^\infty \frac{du}{u^3}\, e^{-i(m^2-i\epsilon)u} F(a,b,\frac{g}{2})$$

$$F(a,b,\frac{g}{2}) = \frac{(eau)\cosh(\frac{g}{2}eau)}{\sinh(eau)}\frac{(ebu)\cos(\frac{g}{2}ebu)}{\sin(ebu)} - 1\ ,\ \left|\frac{g}{2}\right| \leq 1\ ,$$

Convergence condition for expression

S. I. Kruglov . Eur. Phys. J. C 22 (2001), 89-98

A. Steinmetz, M. Formanek, and J. Rafelski. EPJA 55.3 (2019): 1-17.

Euler-Heisenberg-Schwinger with $g \neq 2$

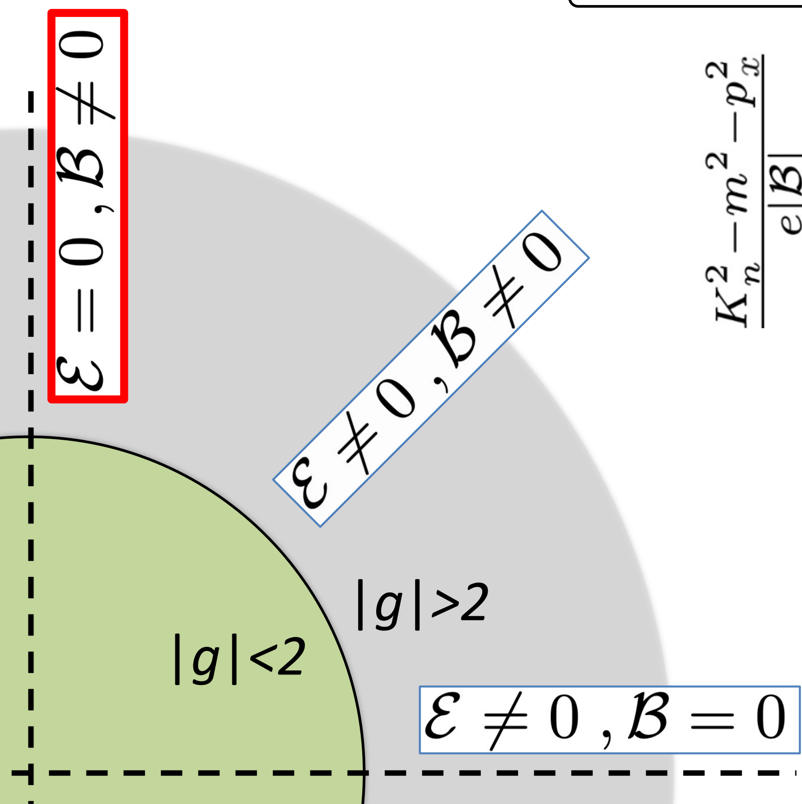
EHS action extended to $|g| \geq 2$: (Magnetic) Landau eigen energy summation:

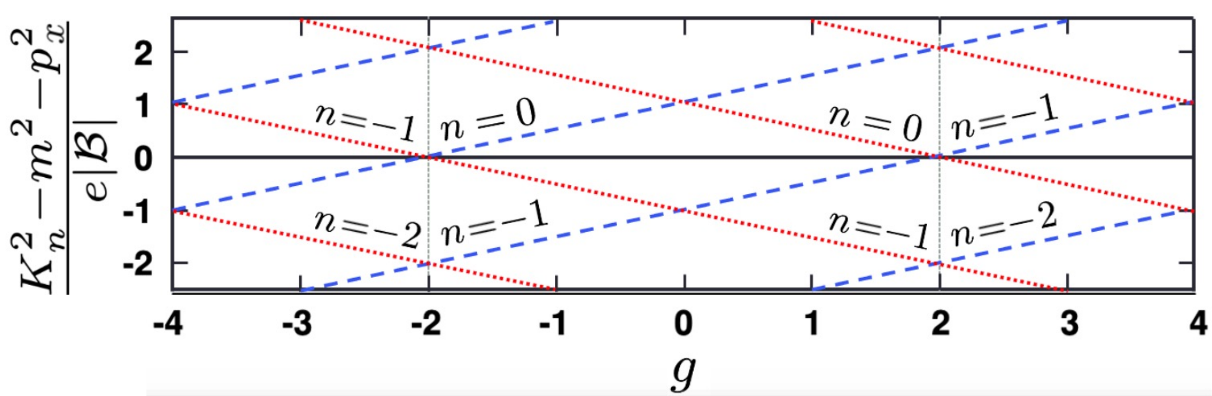
Weisskopf method: V. Weisskopf, Kong. Dan. Vid. Sel. Mat. Fys. Med. 14, N6, 1 (1936)

Landau eigen energies with $|g| \geq 2$:

$$K_n(g,\sigma) = \pm\sqrt{m^2 + p_x^2 + e|\vec{\mathcal{B}}|(2n+1-g\sigma)}\ ,$$

J. Rafelski and L. Labun, (2012) arXiv:1205.1835 [hep-ph]





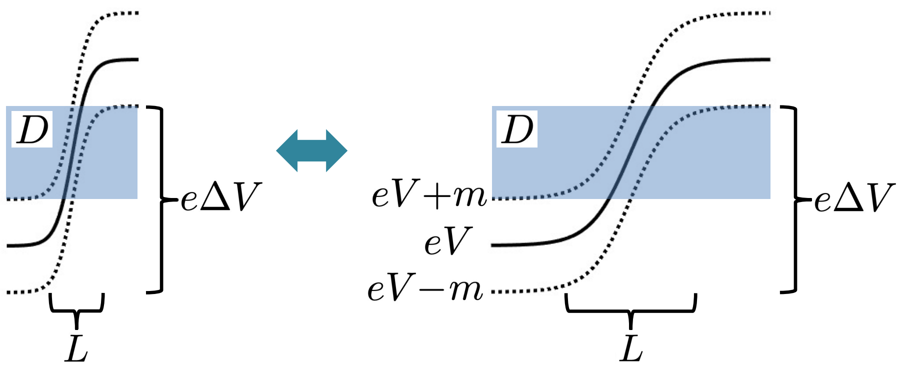
$$\sum_{\sigma,n} K_n(g+4k,\sigma) = \sum_{\sigma=\pm\frac{1}{2}} \sum_{n=0}^{\infty} K_n(g,\sigma)$$

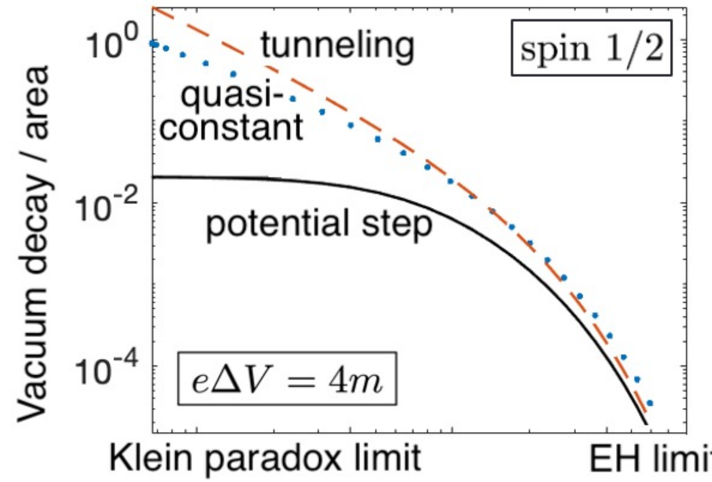
$$\mathcal{L}_{\mathrm{EHSg}}(\mathcal{B}^2, g) = \mathcal{L}_{\mathrm{EHSg}}(\mathcal{B}^2, g+4k)$$

$$-2 \leq g_k = g + 4k \leq 2\ , \qquad k = 0, \pm 1, \pm 2, \ldots$$

Euler-Heisenberg-Schwinger with $g \neq 2$

EHS action extended to $|g| \geq 2$: Electric Sauter action

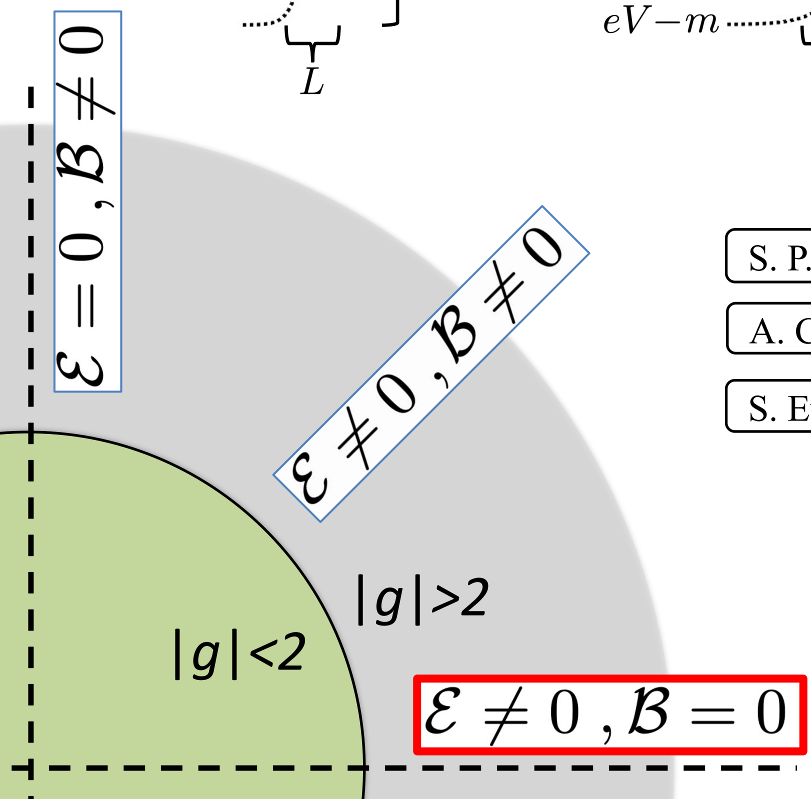




S. P. Kim, H. K. Lee and Y. Yoon. Phys. Rev. D 82, 025015 (2010)

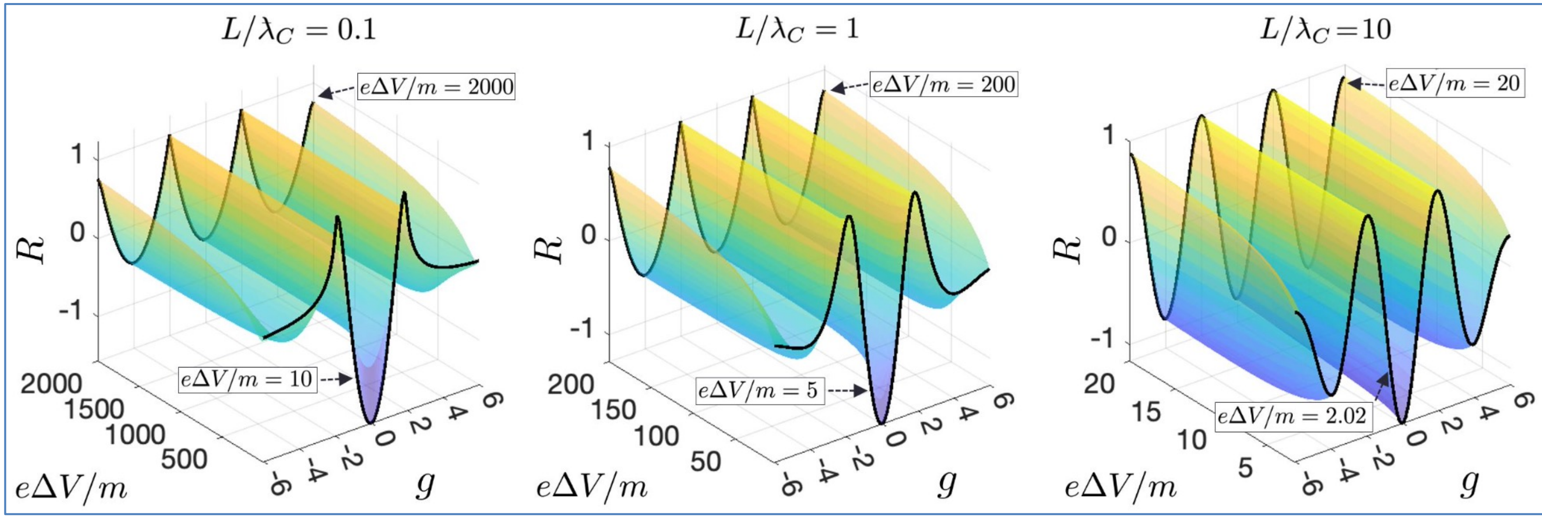
A. Chervyakov and H. Kleinert, Phys. Part. Nucl. 49 no.3, 374-396 (2018)

S. Evans and J. Rafelski. Eur. Phys. J. A 57 (2021) no.12, 341

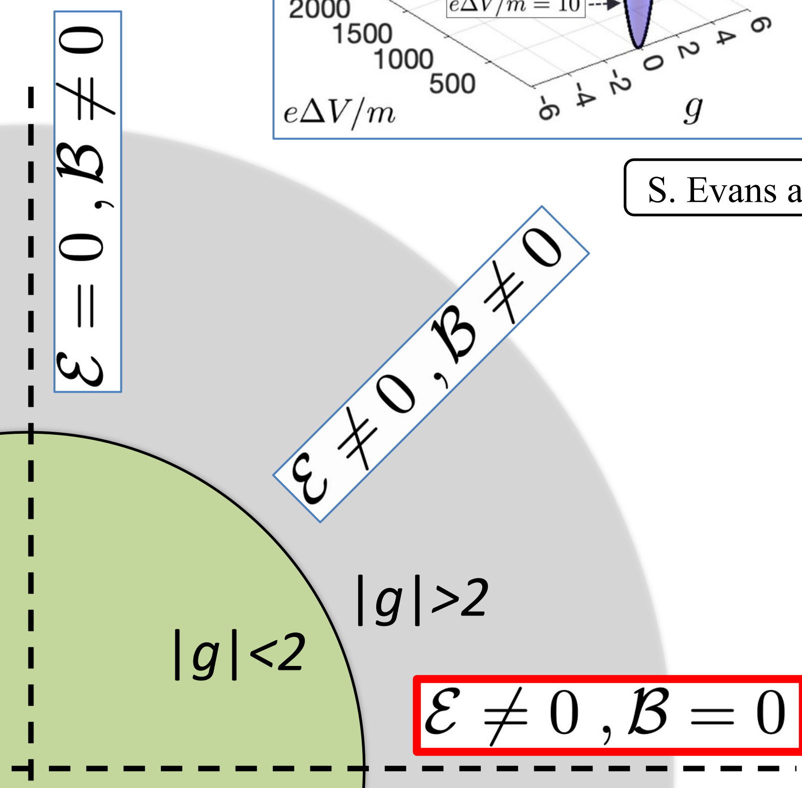


Euler-Heisenberg-Schwinger with $g \neq 2$

EHS action extended to $|g| \geq 2$: Electric Sauter action



S. Evans and J. Rafelski, (2022) In press – Phys. Lett. B. arXiv:2203.13145



Comparing the g-dependent (SSg) Sauter Step action to the known g=2 (SS) result:

$$R = \frac{\mathrm{Im}[\mathcal{L}^{1/2}_{\mathrm{SSg}}]}{\mathrm{Im}[\mathcal{L}^{1/2}_{\mathrm{SS}}]}$$

The same periodicity arises in the pure electric case as in the pure magnetic case.

Euler-Heisenberg-Schwinger with $g \neq 2$

Beta-function and the strong field limit: V. I. Ritus, Sov. Phys. JETP 42, 774 (1975)

Electric SS: eΔV=2000m, L=0.1 λ_C

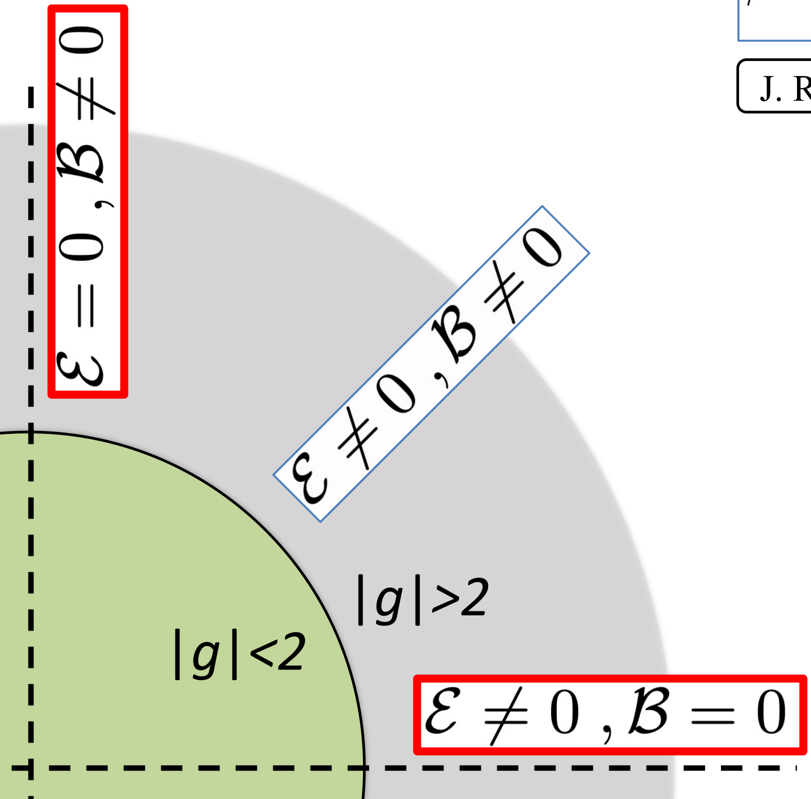
$$R = \frac{\mathrm{Im}[\mathcal{L}^{1/2}_{\mathrm{SSg}}]}{\mathrm{Im}[\mathcal{L}^{1/2}_{\mathrm{SS}}]}$$

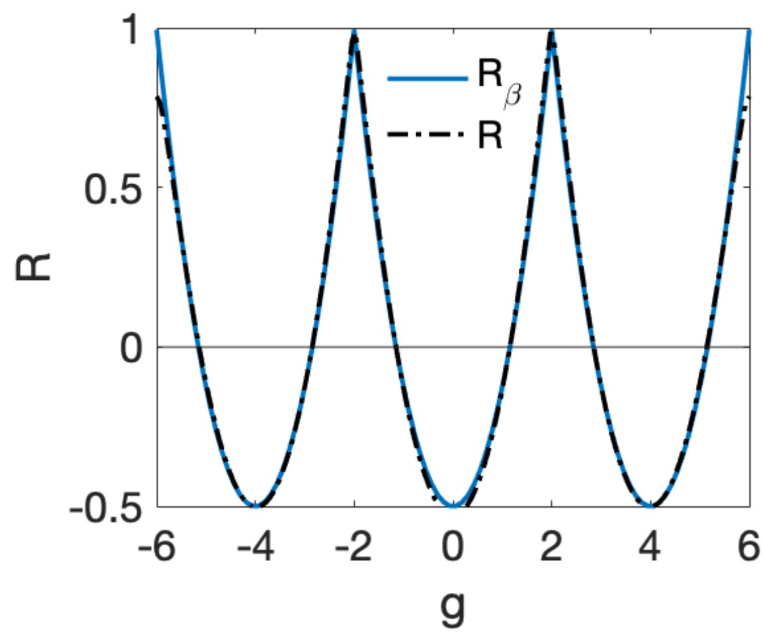
S. Evans and J. Rafelski, (2022) In press – Phys. Lett. B. arXiv:2203.13145

Magnetic action beta-function

$$\beta = \frac{e^3}{12\pi^2} R_\beta \,, \qquad R_\beta = \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos[n\pi g/2]$$

J. Rafelski and L. Labun, (2012) arXiv:1205.1835 [hep-ph]

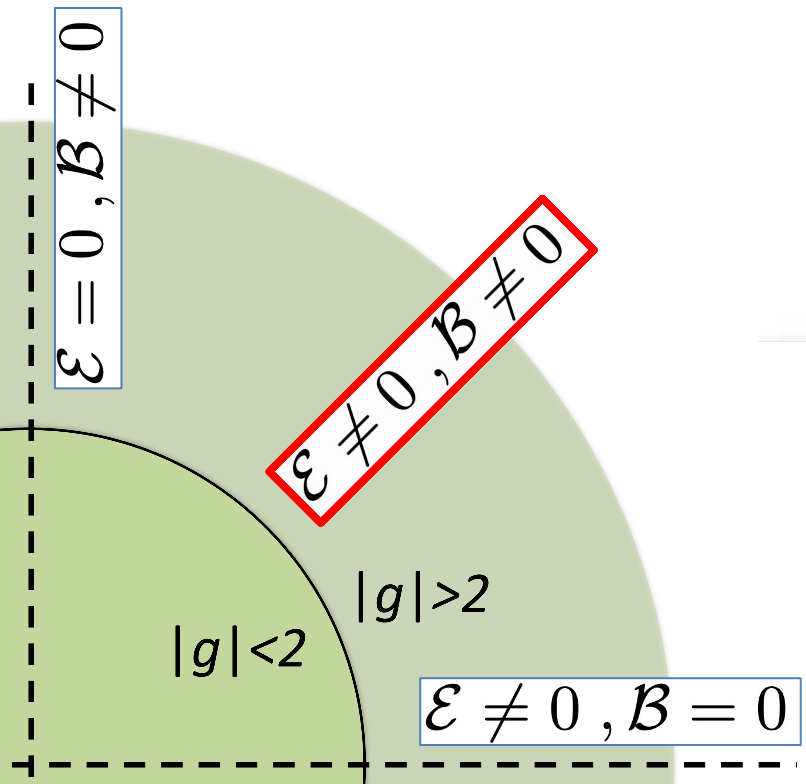


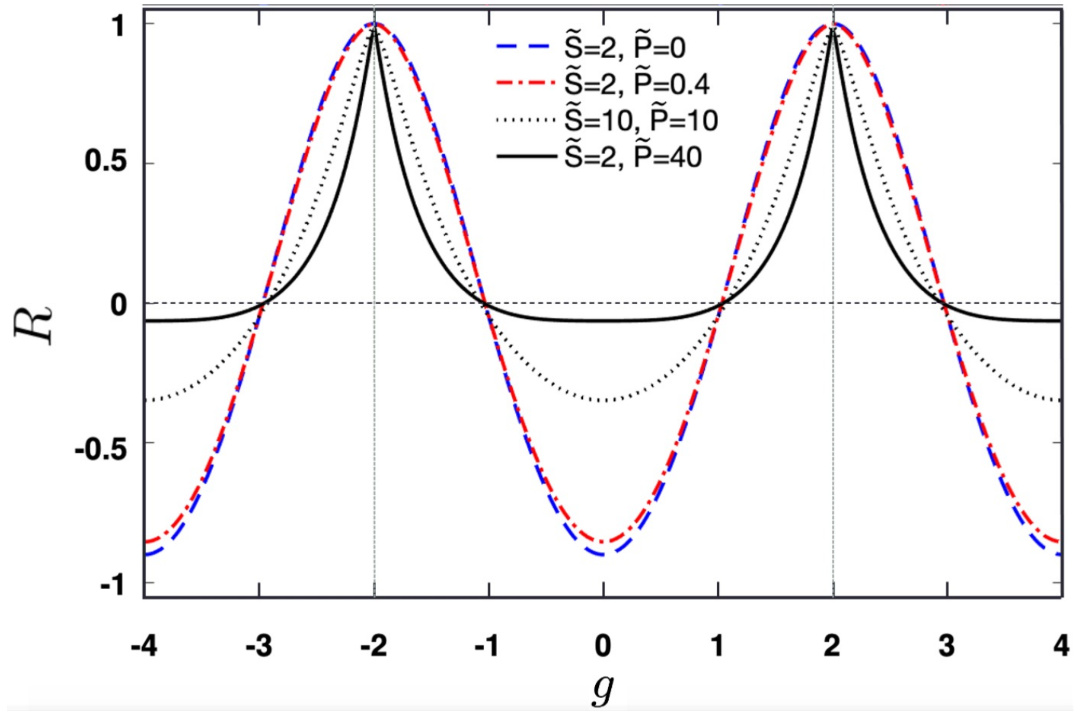


Euler-Heisenberg-Schwinger with $g \neq 2$

EHS action extended to $|g| \geq 2$: Both electric and magnetic fields

$$a^2 - b^2 = \mathcal{E}^2 - \mathcal{B}^2 = 2S \,,$$
$$a^2 b^2 = (\mathcal{E} \cdot \mathcal{B})^2 = P^2 \,.$$
$$\tilde{S} = e^2 S/m^4 \,,\; \tilde{P} = e^2 P/m^4$$





S. Evans and J. Rafelski, in preparation

$$R = \frac{\mathrm{Im}[\mathcal{L}_{\mathrm{EHSg}}]}{\mathrm{Im}[\mathcal{L}_{\mathrm{EHSg}}]_{g=2}}$$

The same periodicity arises, with a new cusp at g=2 for finite pseudoscalar $E \cdot B$

Applications: Conserved quantities

The electron's three physical quantities are: magnetic moment μ, gyromagnetic ratio g, and mass m

-- Mass is not protected by conservation laws, and is modified by presence of external fields:

V. I. Ritus, Annals Phys. 69 (1972) 555

S. Evans and J. Rafelski. Phys. Rev. D 102, 036014 (2020)

Like the Dirac current, the KGP current can be split (Gordon decomposition) into independently conserved convection and magnetic current:

-- Convection current conserves charge e

-- Magnetic current conserves either magnetic moment μ or gyromagnetic ratio g

Applications: Conserved quantities

So conserved quantities are either:

μ and e	or g and e
$\mu = \frac{g(\mathcal{E},\mathcal{B})e\hbar}{2m(\mathcal{E},\mathcal{B})}$,	$\mu(\mathcal{E},\mathcal{B}) = \frac{ge\hbar}{2m(\mathcal{E},\mathcal{B})}$,
↓	↓
$\frac{g(\mathcal{E},\mathcal{B})}{g(0,0)} = \frac{m(\mathcal{E},\mathcal{B})}{m(0,0)}$	$g = g(\mathcal{E},\mathcal{B}) = g(0,0)$

Applications: Conserved quantities

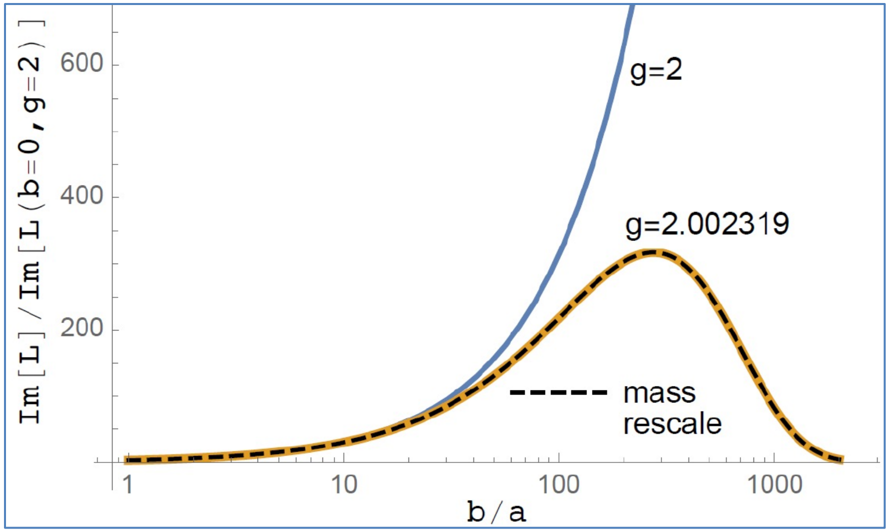
Probing the *g*-factor is possible in strong fields:

$$\mathrm{Im}[\mathcal{L}_{\mathrm{EHSg}}] = \frac{e^2ab}{8\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^n\cos(\frac{g}{2}n\pi)}{n}\frac{\cosh(\frac{g}{2}n\pi b/a)}{\sinh(n\pi b/a)}e^{-n\pi m^2/ea}\,,$$

$$\Im[\mathcal{L}_{\mathrm{EH}}] \sim \frac{(ea)(eb)}{8\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\cos[\frac{g}{2}n\pi]e^{-n\pi\tilde{m}^2/ea}\,,$$

$$\tilde{m}^2 = m^2 + \left|\frac{|g_k-4k|}{2}-1\right|eb$$

$$-2 \le g_k = g+4k \le 2\,, \qquad k = 0, \pm1, \pm2, \ldots$$



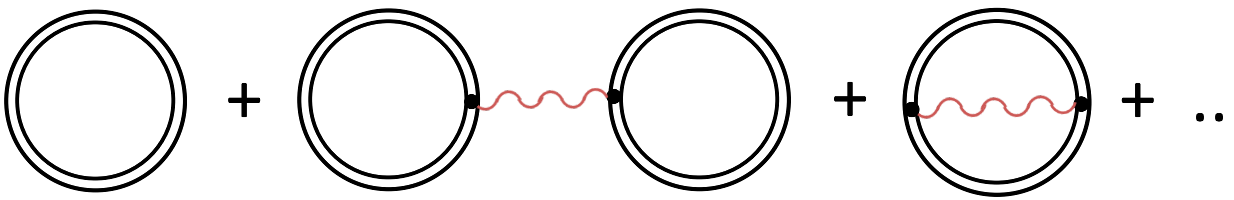
S. Evans, J. Rafelski, PRD 98 (2018) no. 1, 016006

Future work

- Periodic Beta function in the strong field limit: opportunity to study asymptotically free effective action within an Abelian theory

G. K. Savvidy, Phys. Lett. B 71 (1977), 133-134

- Comparison with higher order reducible corrections and exploration of asymptotic strong field limits



H. Gies and F. Karbstein, JHEP 1703, 108 (2017)

- Temperature representation for arbitrary *g* and EM fields

B. Müller, W. Greiner, and J. Rafelski. “Interpretation of external fields as temperature.’ Physics Letters A 63.3 (1977)

W. G. Unruh, “Notes on black-hole evaporation.” Physical Review D 14.4 (1976)

L. Labun and J. Rafelski, “Acceleration and vacuum temperature.” Phys. Rev. D 86, 041701(R) (2012)

Thank you!