Cooling of quantum particles by external fields

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On the Schrödinger–Langevin Equation*

M. D. Kostin
Department of Chemical Engineering, Princeton University, Princeton, New Jersey 08540
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It is shown that the Heisenberg–Langevin equation can be used to derive a Schrödinger equation for a Brownian particle interacting with a thermal environment. The equation derived is

$$i\hbar(\partial\psi/\partial t)=-(\hbar^2/2m)\nabla^2\psi+V\psi+V_R\psi+[(\hbar f/2im)\ \ln(\psi/\psi^*)+W(t)]\psi(\mathbf{r},t),$$

$$W(t)=-(\hbar f/2im)\int\psi^*\ \ln(\psi/\psi^*)\psi d\mathbf{r},$$

where f is the friction constant and V_R is a random potential.

The Kostin equation, the deceleration of a quantum particle and coherent control

Harald Losert[1*], Freyja Ullinger[1,2], Matthias Zimmermann[2], Maxim A. Efremov[1,2], Ernst M. Rasel[3] and Wolfgang P. Schleich[1,4]






Overview

- Deceleration of a classical particle
- Deceleration of a quantum particle
- Nonlinear Schrödinger equation
- Deceleration à la Kostin

Dissipative Force

$$m \frac{\mathrm{d}}{\mathrm{d}t} v = -\gamma m v$$

$$\bar{v}(t) = v_0 \, \mathrm{e}^{-\gamma t}$$

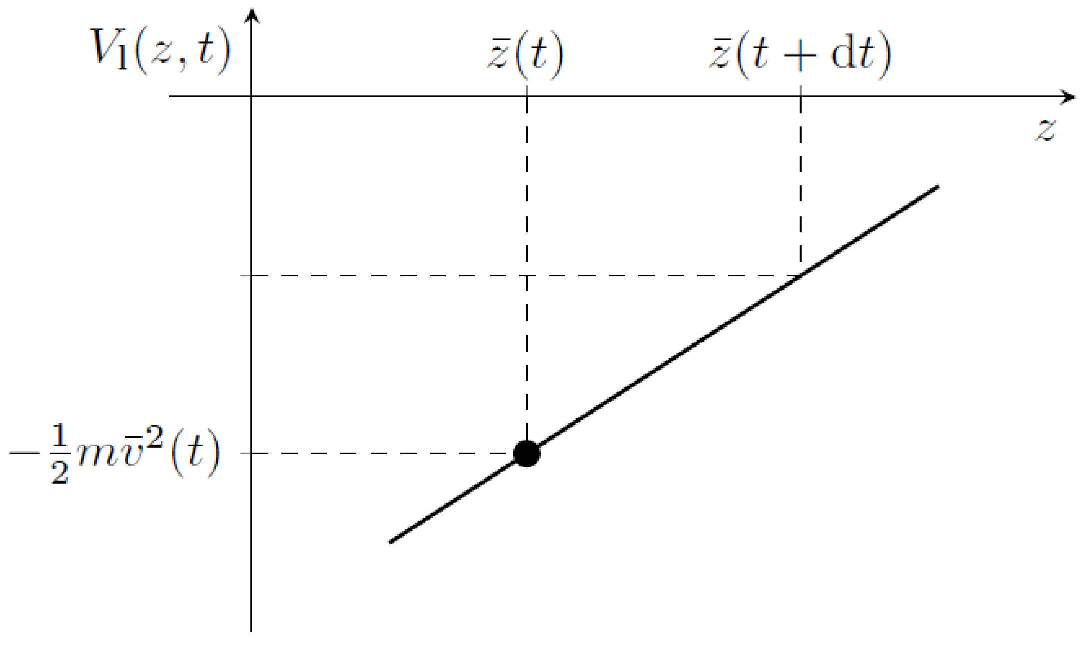
$$\bar{z}(t) = z_0 + v_0 \frac{1}{\gamma} \left(1 - \mathrm{e}^{-\gamma t}\right)$$

$$\bar{z}(t) = z_\infty - \bar{v}(t) \, \frac{1}{\gamma}$$

Time-dependent homogenous force

$$F_{\mathrm{h}}(t) = -m\gamma v_0 \mathrm{e}^{-\gamma t}$$

$$V_{\mathrm{l}}(z,t) = m\gamma\bar{v}(t)\left[z - \bar{z}(t)\right] - \frac{m}{2}\bar{v}^2(t)$$



Inverted harmonic oscillator

$$\bar{z}(t) = z_\infty - \bar{v}(t)\,\frac{1}{\gamma}$$

$$E_{\mathrm{kin}}(t) = \frac{m}{2}\gamma^2\left[\bar{z}(t) - z_\infty\right]^2$$

$$V_{\mathrm{io}}(z) \equiv -\frac{m}{2}\gamma^2\left(z - z_\infty\right)^2$$

Deceleration of a quantum particle

$$\psi(z,t=0) \equiv \frac{1}{\sqrt[4]{\pi}}\sqrt{\frac{1}{\Delta z(0)}}\exp\left\{-\frac{1}{2}\left[\frac{z-\bar{z}(0)}{\Delta z(0)}\right]^2\right\}\mathrm{e}^{\mathrm{i}m\bar{v}(0)[z-\bar{z}(0)]/\hbar}$$

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+V\right)\psi$$

$$W(z,t) = |\psi(z,t)|^2 = \frac{1}{\sqrt{\pi}}\frac{1}{\Delta z(t)}\exp\left\{-\left[\frac{z-\bar{z}(t)}{\Delta z(t)}\right]^2\right\}$$

Schrödinger equation revisited

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$$Z = Z(\boldsymbol{r},t) \equiv A(\boldsymbol{r},t)\,\mathrm{e}^{\mathrm{i}\theta(\boldsymbol{r},t)}$$

$$\mathrm{i}\frac{\partial Z}{\partial t} + \beta\boldsymbol{\nabla}^2 Z = \left\{\mathrm{i}\frac{1}{2A^2}\left[\frac{\partial}{\partial t}A^2 + 2\beta\boldsymbol{\nabla}\cdot\left(A^2\boldsymbol{\nabla}\theta\right)\right] + \left[-\frac{\partial\theta}{\partial t} - \beta(\boldsymbol{\nabla}\theta)^2 + \beta\frac{\boldsymbol{\nabla}^2 A}{A}\right]\right\} Z$$

Potential

$$V(z,t) \equiv V_2(z,t) + V_1(z,t) + V_0(t)$$

$$V_2(z,t) \equiv -\frac{m}{2}\frac{1}{\Delta z(t)}\left[\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\Delta z\right) - \frac{\hbar^2}{m^2}\frac{1}{\Delta z^3(t)}\right][z - \bar{z}(t)]^2$$

$$V_1(z,t) = m\gamma\bar{v}(t)\,[z - \bar{z}(t)] - \frac{m}{2}\bar{v}^2(t)$$

$$V_0(t) \equiv m\bar{v}^2 - \frac{\mathrm{d}}{\mathrm{d}t}s - \frac{\hbar^2}{2m}\frac{1}{\Delta z^3(t)}$$

Solution

$$\psi(z,t) \equiv \sqrt{W(z,t)}\,\mathrm{e}^{\mathrm{i}S(z,t)/\hbar}$$

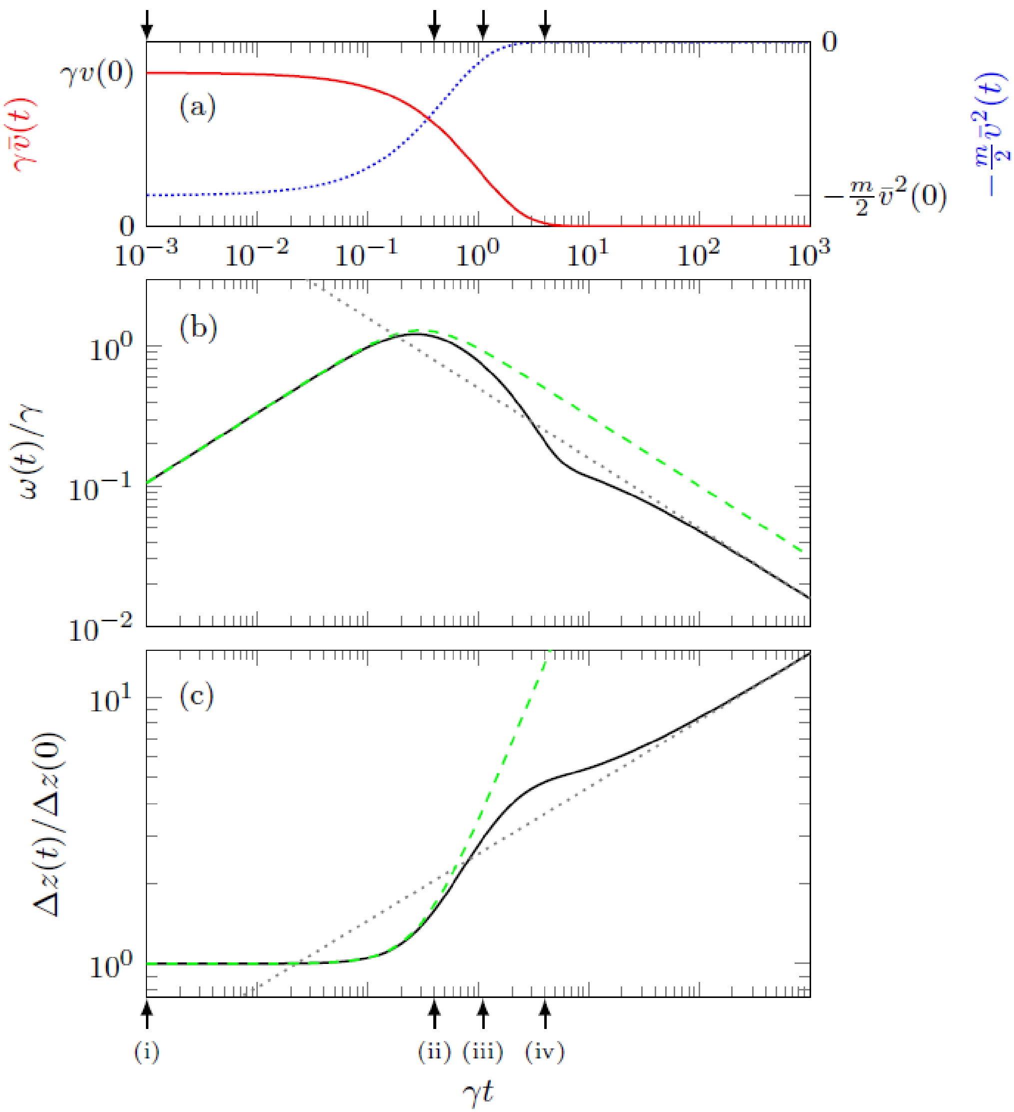
$$S(z,t) \equiv \frac{m}{2}\frac{1}{\Delta z(t)}\left(\frac{\mathrm{d}}{\mathrm{d}t}\Delta z(t)\right)[z-\bar{z}(t)]^2 + m\bar{v}(t)\,[z-\bar{z}(t)] + s(t)$$

Nonlinear Schrödinger equation

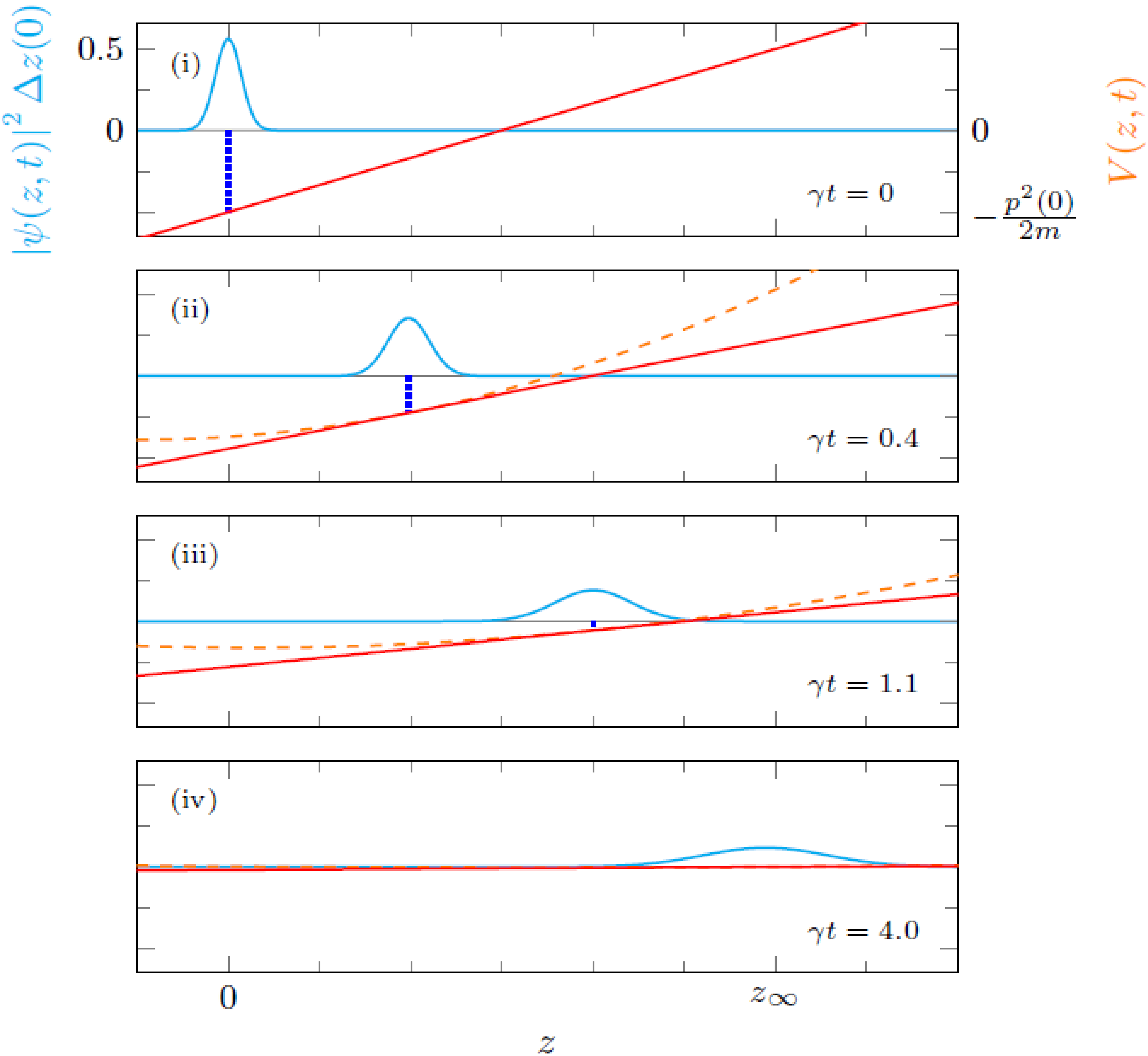
$$V = -\frac{m}{2}\frac{1}{\Delta z}\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\Delta z - \frac{\hbar^2}{m^2}\frac{1}{\Delta z^3}\right)(z-\bar{z})^2 + m\gamma\bar{v}\,(z-\bar{z}) + \frac{1}{2}m\bar{v}^2 - \frac{\mathrm{d}}{\mathrm{d}t}s - \frac{\hbar^2}{2m}\frac{1}{\Delta z^3}$$

$$S = \frac{m}{2}\frac{1}{\Delta z}\left(\frac{\mathrm{d}}{\mathrm{d}t}\Delta z\right)(z-\bar{z})^2 + m\bar{v}\,(z-\bar{z}) + s$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\Delta z^{(\mathrm{P})} - \frac{\hbar^2}{m^2\left(\Delta z^{(\mathrm{P})}\right)^3} = -\gamma\frac{\mathrm{d}}{\mathrm{d}t}\Delta z^{(\mathrm{P})} \qquad \frac{1}{2}m\bar{v}^2 - \frac{\mathrm{d}}{\mathrm{d}t}s^{(\mathrm{P})} - \frac{\hbar^2}{2m}\frac{1}{\left(\Delta z^{(\mathrm{P})}\right)^3} = \gamma s^{(\mathrm{P})}$$



(a)
(b)
(c)
$\gamma v(0)$
$\gamma\bar{v}(t)$
0
0
$-\frac{m}{2}\bar{v}^2(0)$
$-\frac{m}{2}\bar{v}^2(t)$
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}
$\omega(t)/\gamma$
10^{0} 10^{-1} 10^{-2}
$\Delta z(t)/\Delta z(0)$
10^{1} 10^{0}
(i) (ii) (iii) (iv)
γt



$|\psi(z,t)|^2\,\Delta z(0)$
0.5
0
(i)
$\gamma t = 0$
0
$-\frac{p^2(0)}{2m}$
$V(z,t)$
(ii)
$\gamma t = 0.4$
(iii)
$\gamma t = 1.1$
(iv)
$\gamma t = 4.0$
0
z_∞
z

On the Quantum Correction For Thermodynamic Equilibrium

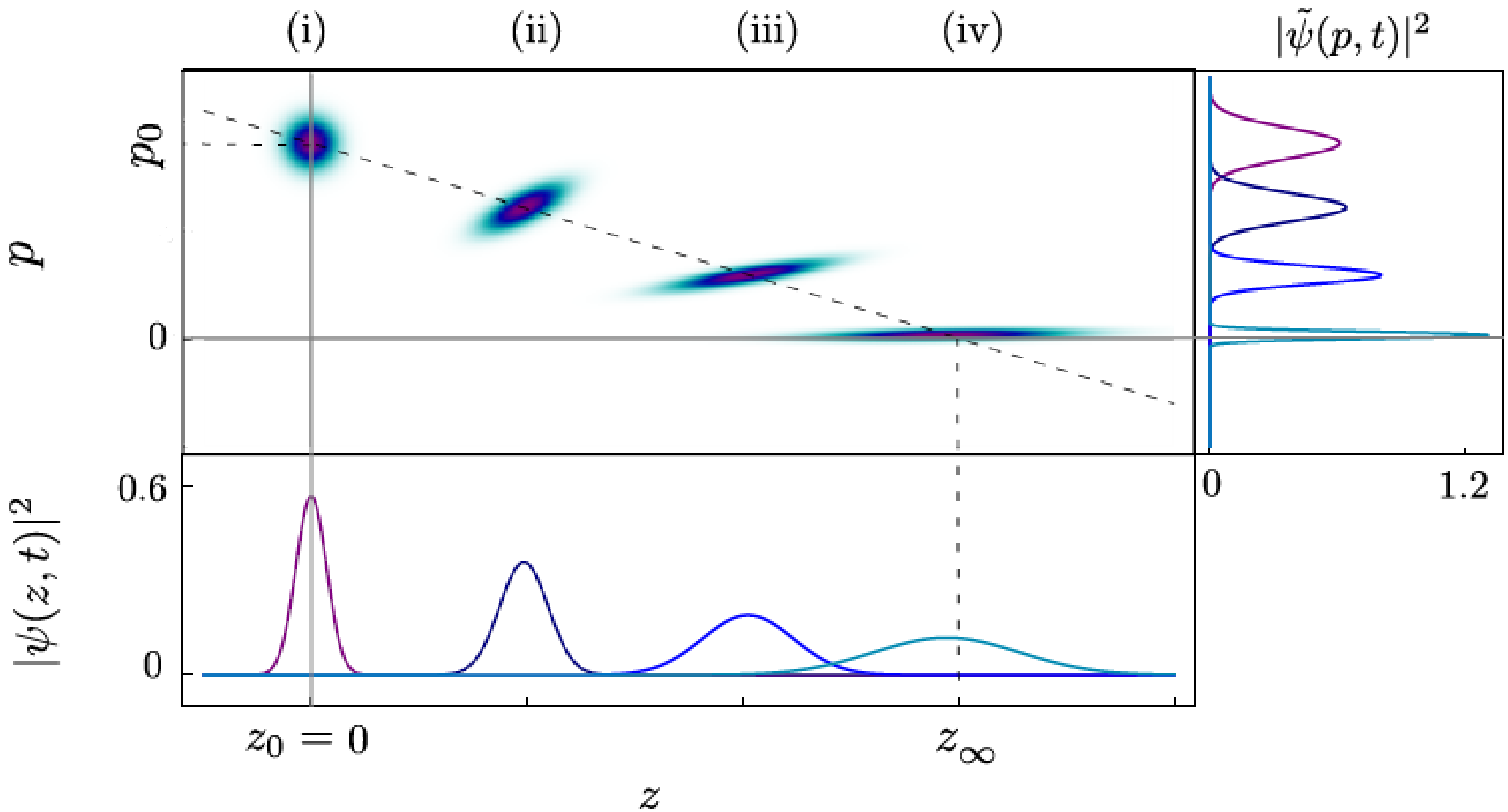
By E. WIGNER
Department of Physics, Princeton University
(Received March 14, 1932)

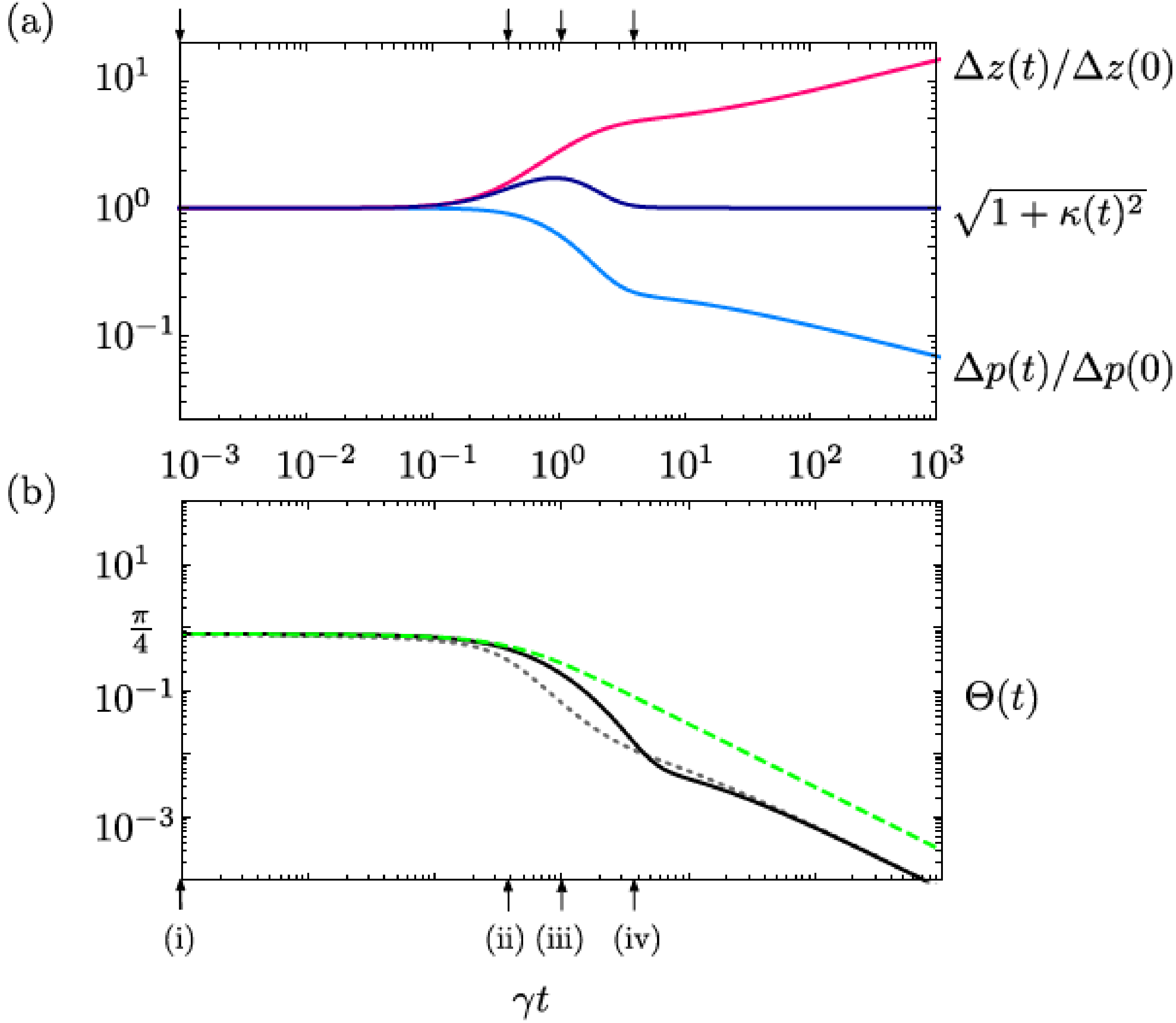
If a wave function $\psi(x_1 \cdots x_n)$ is given one may build the following expression[2]

$$P(x_1, \cdots, x_n; p_1, \cdots, p_n) = \left(\frac{1}{h\pi}\right)^n \int_{-\infty}^{\infty} \cdots \int dy_1 \cdots dy_n \psi(x_1 + y_1 \cdots x_n + y_n)^* \psi(x_1 - y_1 \cdots x_n - y_n) e^{2i(p_1 y_1 + \cdots + p_n y_n)/h} \quad (5)$$

and call it the probability-function of the simultaneous values of $x_1 \cdots x_n$ for the coordinates and $p_1 \cdots p_n$ for the momenta.

[2] This expression was found by L. Szilard and the present author some years ago for another purpose.


(i)
(ii)
(iii)
(iv)
$|\tilde{\psi}(p,t)|^2$
p_0
p
0
0
1.2
0.6
$|\psi(z,t)|^2$
0
$z_0 = 0$
z_∞
z



(a)
10^1
10^0
10^{-1}
$\Delta z(t)/\Delta z(0)$
$\sqrt{1+\kappa(t)^2}$
$\Delta p(t)/\Delta p(0)$
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
(b)
10^1
$\frac{\pi}{4}$
10^{-1}
10^{-3}
$\Theta(t)$
(i)
(ii)
(iii)
(iv)
γt

Summary

- Deceleration of a classical particle
- Deceleration of a quantum particle
- Nonlinear Schrödinger equation
- Deceleration à la Kostin