CURRENT CONSERVING THEORY OF COLLISIONAL RELATIVISTIC PLASMAS

Martin Formanek PhD
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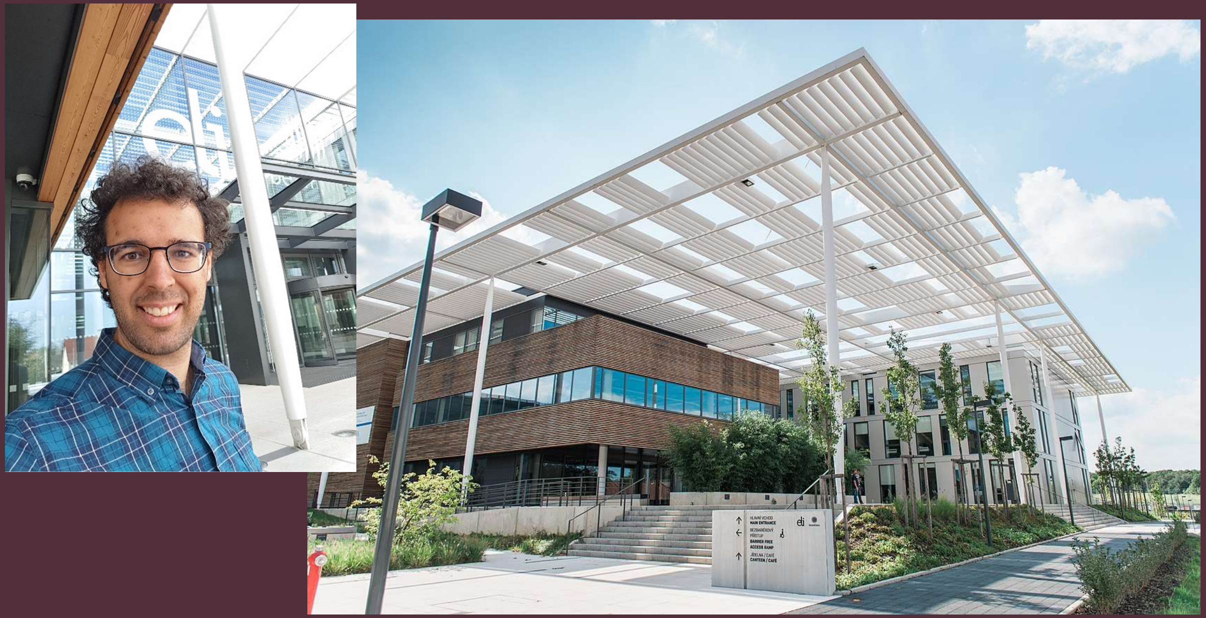

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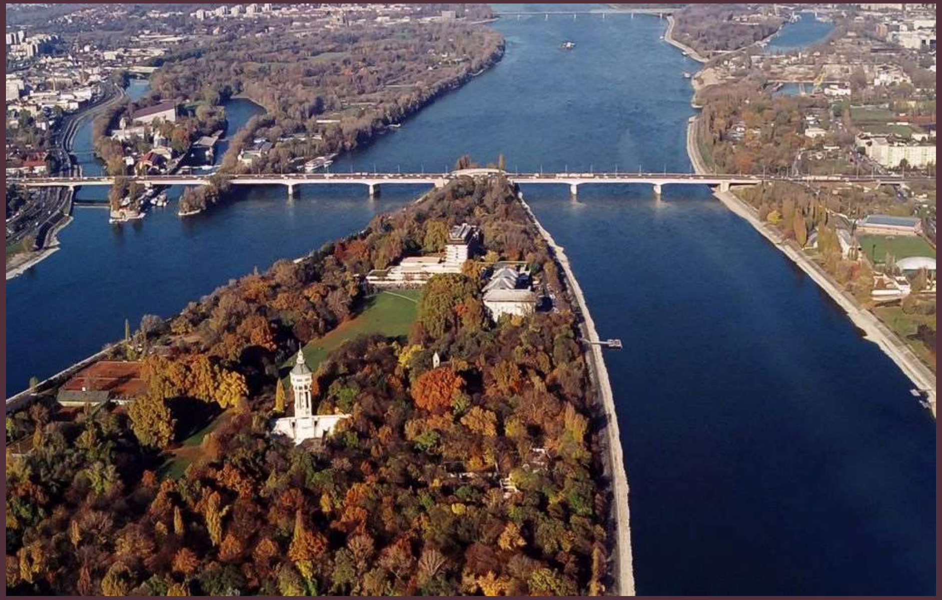
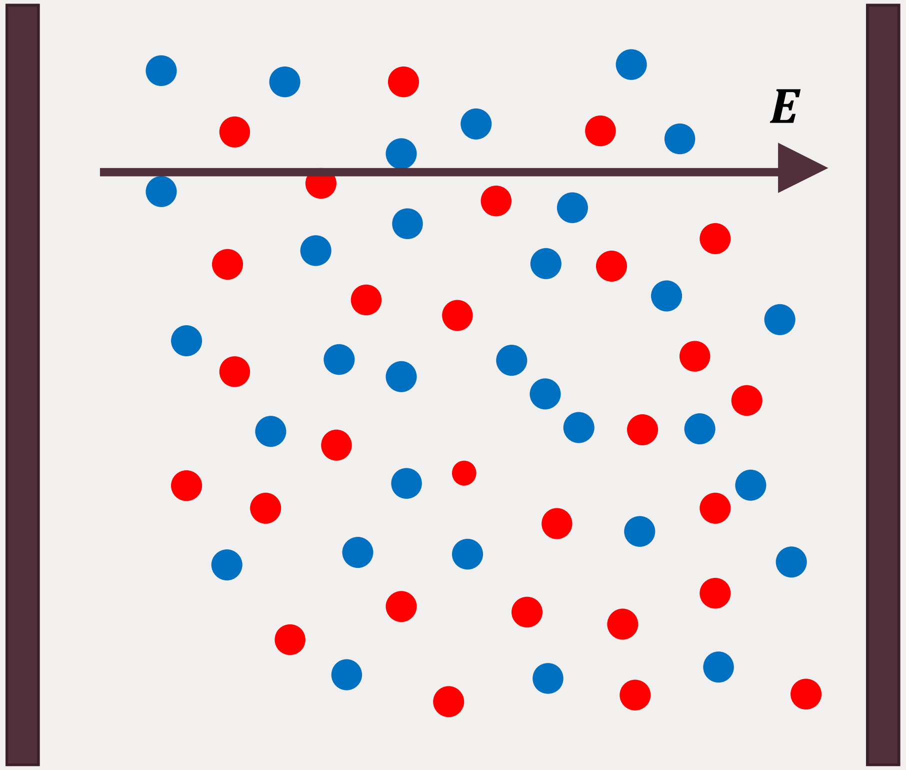


Margaret Island Symposium 2022 on
Vacuum Structure, Particles, and Plasmas

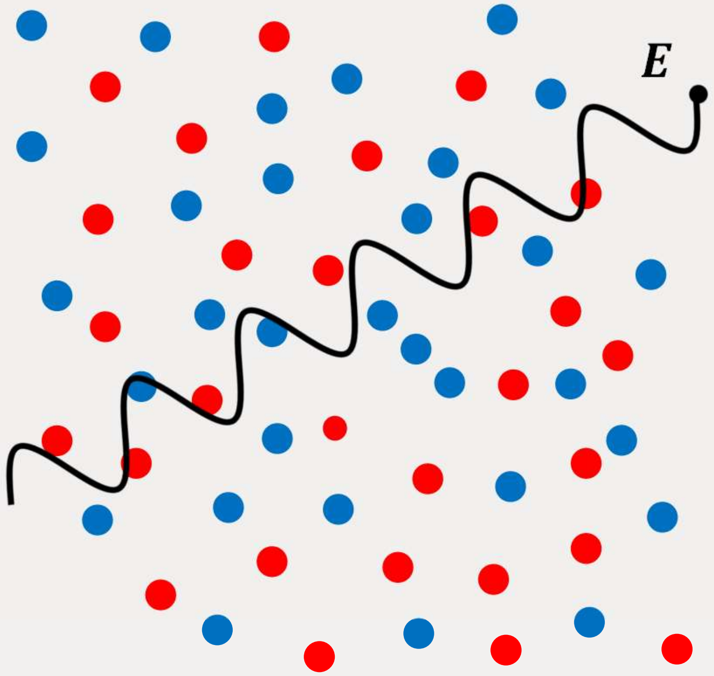
May 17th, 2022

Linear nonequilibrium response to time dependent fields



Electron-positron plasma example

Linear nonequilibrium response to time dependent fields



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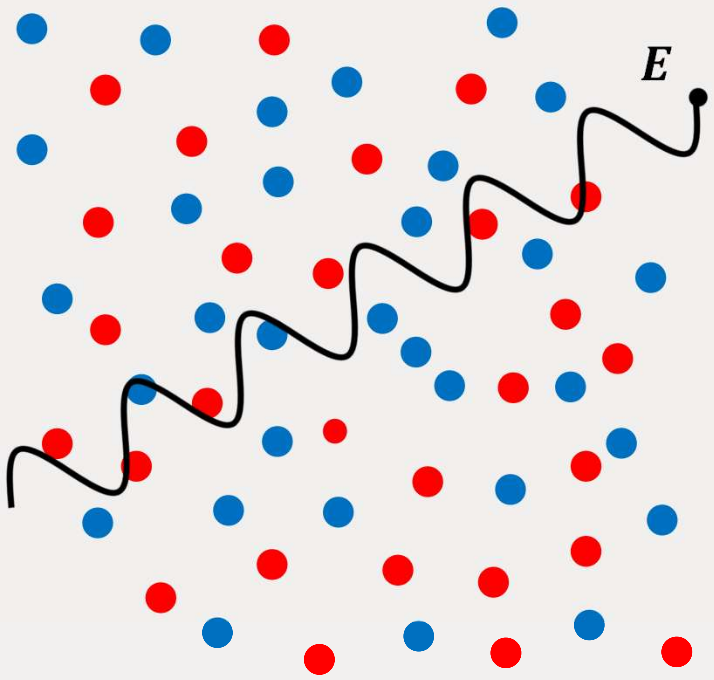

Electron-positron plasma example

Boltzmann equation for the distribution function $f(x,p)$

$$p\cdot\partial f(x,p)+\underbrace{\mathrm{qF}^{\mu\nu}\mathrm{p}_\nu\frac{\partial f(x,p)}{\partial p^\mu}}_{\text{Vlasov force term}}=\underbrace{C[f(x,p)]}_{\text{Collisions}}$$

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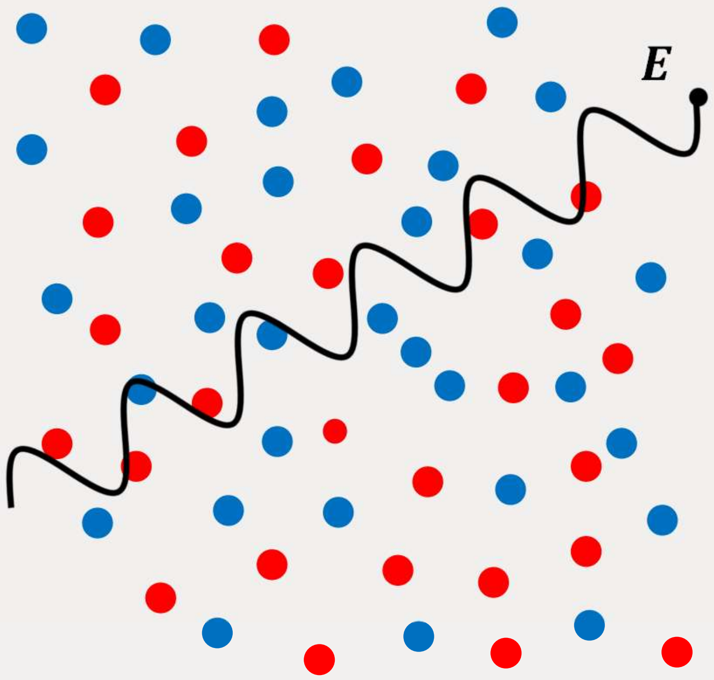

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Linear response:

$$f(x,p) = f_{eq}(p) + \boxed{\delta f(x,p)}$$

$$f_{\mathrm{eq}}(p) = \frac{1}{\exp(p \cdot u/T) + 1}$$

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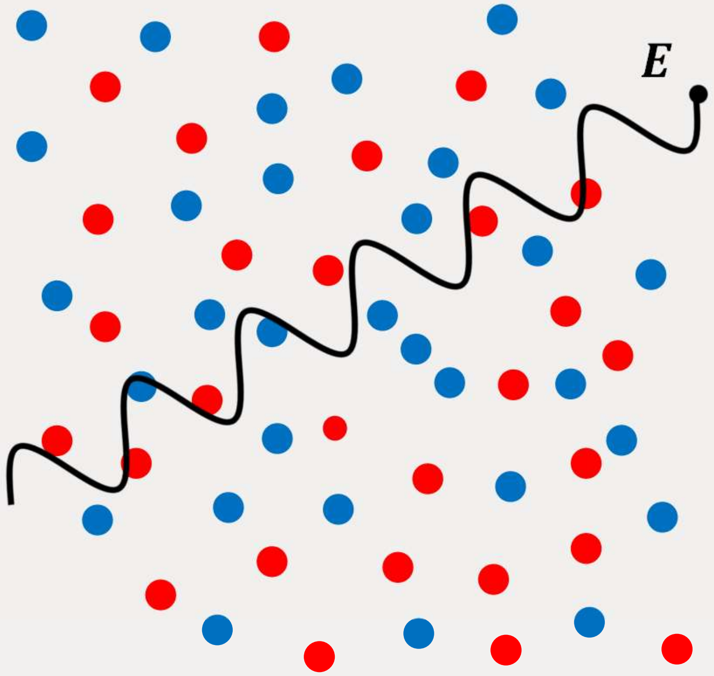

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$$j^\mu_{ind}(x) = 2q\int (dp)p^\mu f(x,p)$$

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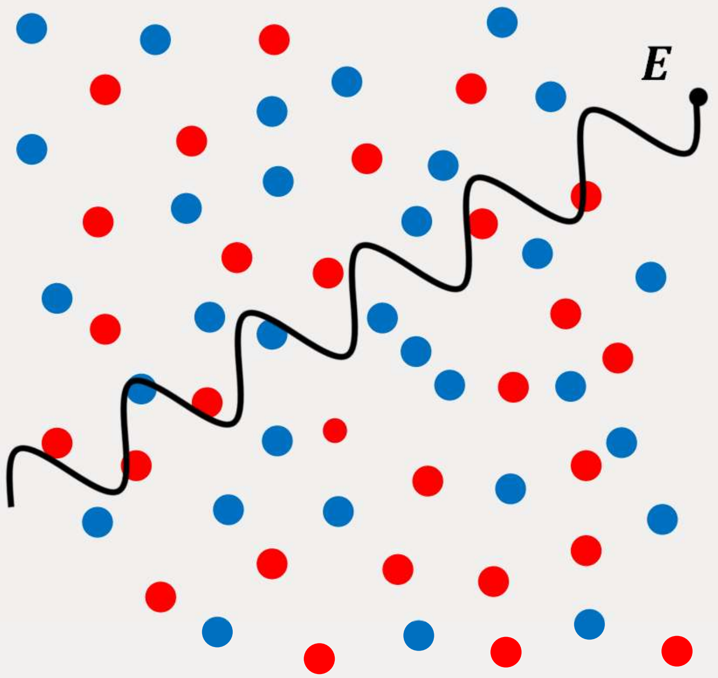
Linear nonequilibrium response to time dependent fields



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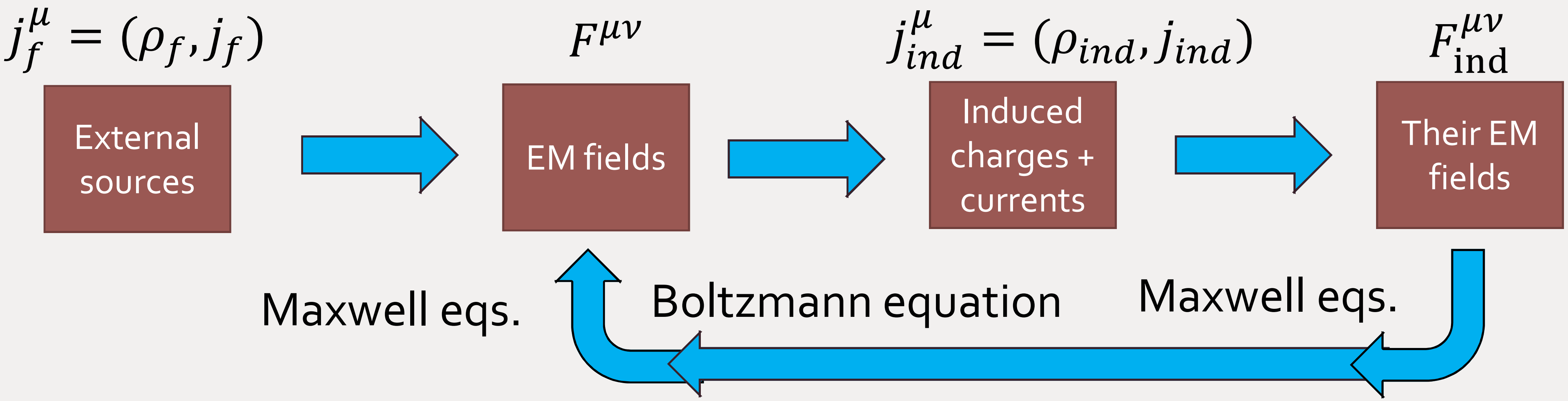
Two component plasma

$$j^\mu(x)=2q\int(dp)p^\mu[f_+(x,p)-f_-(x,p)]$$

But induced charges also generate fields!

Perturbative approach (often implicitly assumed in literature on this topic)



$j_f^\mu = (\rho_f, j_f)$

External sources

Maxwell eqs.

$F^{\mu\nu}$

EM fields

Boltzmann equation

$j_{ind}^\mu = (\rho_{ind}, j_{ind})$

Induced charges + currents

Maxwell eqs.

$F_{\text{ind}}^{\mu\nu}$

Their EM fields

Works only if the EM fields of induced charges and currents are much smaller than the fields producing the induced charges and currents in the medium!

Can we do better?

Self consistent solution in Fourier space

$$x^\mu = (t, \boldsymbol{x}) \Rightarrow k^\mu = (\omega, \boldsymbol{k})$$

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$\tilde{A}^\nu(k) = (\tilde{\phi}, \widetilde{\boldsymbol{A}})$ - all the fields including the induced ones

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Maxwell equations in medium: $\boxed{k_\mu \tilde{F}^{\mu\nu} = \mu_0(\tilde{j}^\nu_f + \tilde{j}^\nu_{\text{ind}})}$

$\tilde{F}^{\mu\nu}(k) = k^\mu \tilde{A}^\nu(k) - k^\nu \tilde{A}^\mu(k)$

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Solution for components $\tilde{\phi}, \widetilde{\boldsymbol{A}}$ where $\widetilde{\boldsymbol{A}} = \tilde{A}_\parallel \boldsymbol{k} + \widetilde{\boldsymbol{A}}_\perp$:

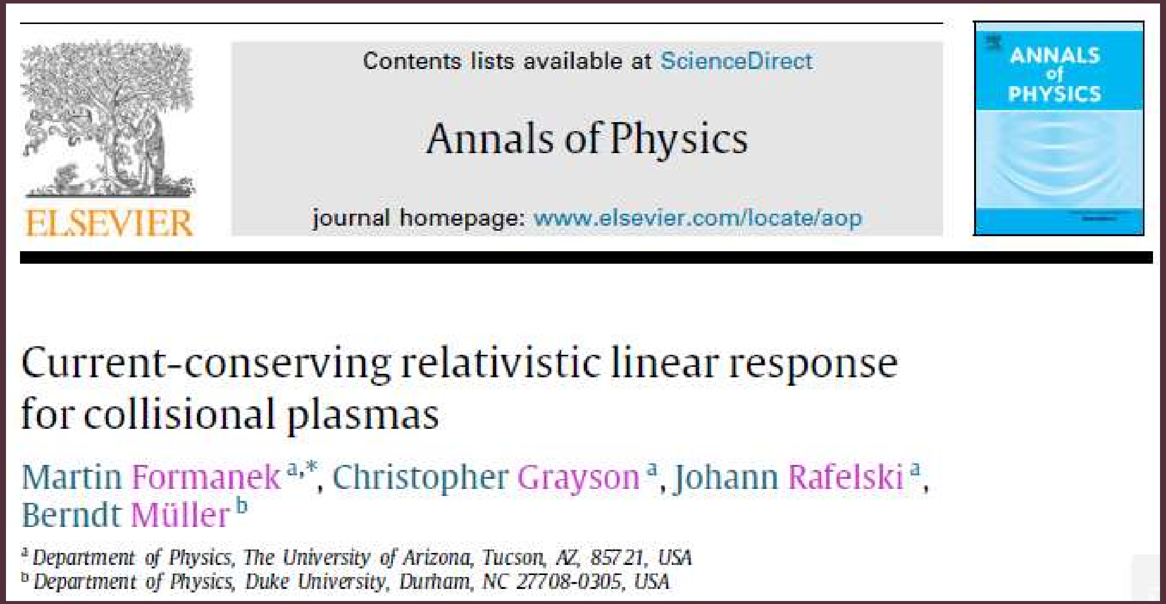
$$\tilde{\phi}(k) = \frac{\tilde{\rho}_f(k)}{(k^2 - \omega^2)(\Pi_L(k)/\omega^2 - 1)} \qquad \widetilde{\boldsymbol{A}}_\perp(k) = \frac{\tilde{\boldsymbol{J}}_{\perp,f}\,(k)}{k^2 - \omega^2 - \Pi_T(k)} \qquad \tilde{A}_\parallel(k) = \frac{\omega}{|\boldsymbol{k}|}\tilde{\phi}(k)$$

Depends only on external charges / currents and polarization tensor properties

Π_T and Π_L - transverse and longitudinal projections of Π^μ_ν (Weldon, PRD 26 (1982) 1394)

Product of remote collaboration

Annals of Physics 434, 168605 (2021):



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ANNALS of PHYSICS

Current-conserving relativistic linear response for collisional plasmas

Martin Formanek[a,*], Christopher Grayson[a], Johann Rafelski[a], Berndt Müller[b]

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Important “sponsor” I should mention:

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Collision term

In general complicated (Groot, Leeuwen, Weert, *Relativistic kinetic theory*, 1980)

$$C[f,f] = \frac{1}{2}\int \frac{\mathrm{d}^3 p_1}{p_1^0}\frac{\mathrm{d}^3 p'}{p'^0}\frac{\mathrm{d}^3 p'_1}{p'^0_1}(f'f'_1 - ff_1)\,W(p', p'_1|p, p_1)\,.$$

Transition rate

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Transition rate

Simplest model-relaxation time approximation (RTA) (Anderson, Witting, Physica 74 (1974) 466)

$$C[f(x,p)] = (p^\mu u_\mu)\kappa[f_{\text{eq}}(p) - f(x,p)] \qquad \kappa = 1/\text{relaxation rate}$$

Does not conserve 4-current $\partial_\mu j^\mu_{ind} \neq 0$! Better (BGK) (Bhatnagar, Gross, Krook, Phys. Rev. 94 (1954) 511)

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$$C[f(x,p)]=(p^u u_\mu)\kappa\left[f_{\text{eq}}(p)\frac{n(x)}{n_{\text{eq}}}-f(x,p)\right]$$

Designed explicitly to conserve current:

$$\left.\begin{aligned} n(x)&=2\int(dp)(p\cdot u)f(x,p)\\ n_{eq}&=2\int(dp)(p\cdot u)f_{\text{eq}}(p)\end{aligned}\right\} 2\int(dp)(p\cdot u)C[f(x,p)]=2\int(dp)(p\cdot u)\left[f_{\text{eq}}(p)\frac{n(x)}{n_{\text{eq}}}-f(x,p)\right]=0$$

Doesn't conserve energy and momentum!

$$\partial_\mu T^{\mu\nu} = \partial_\mu 2\int (dp)p^\mu p^\nu f(x,p) \neq 0$$

But can be fixed too by adding more terms as shown very recently (Rocha, Denicol, Noronha, PRL 127 (2021) 042301) For particle-antiparticle plasma $T^{\mu\nu}$ conserved (Grayson, Formanek, Rafelski, Muller (2022) arXiv: 2204.14186)

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(2\int (dp)p^\mu p^\nu (f_-(x,p) + f_+(x,p)) \right) = 0$$

Because the equilibrium distribution doesn't depend on position and the position dependent perturbation changes sign with charge.

$$\delta f_\pm(x,p) = \pm q\delta f(x,p)$$

Assuring cancelation of this contribution.

Altogether we are solving Boltzmann equation:

$$p\cdot\partial f(x,p)+\mathrm{qF}^{\mu\nu}\mathrm{p}_\nu\frac{\partial f(x,p)}{\partial p^\mu}=\left(p^u u_\mu\right)\kappa\left[f_{\mathrm{eq}}(p)\frac{n(x)}{n_{\mathrm{eq}}}-f(x,p)\right]$$

In the plasma rest frame: $p^\mu u_\mu=\mathrm{m}\gamma$

In the linear order in perturbations: $f(x,p)=f_{\mathrm{eq}}(p)+\delta f(x,p)$

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Without collision term solved for QGP in Blaizot, Iancu, Phys. Rep. 359 (2002) 355; Satow, PRD 90 (2014) 034018 method of characteristics PDR → ODR along trajectories $m\frac{dx^{\mu}}{d\tau} = p^{\mu}$.

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Our method – Fourier transformation $\partial_\mu \to -ik_\mu$

$$-i(p \cdot k)\widetilde{\delta f}(k,p) + q\tilde{F}^{\mu\nu}p_\nu \frac{\partial f_{\mathrm{eq}}(p)}{\partial p^\mu} = (p \cdot u)\kappa \left[\frac{f_{\mathrm{eq}}(p)}{n_{\mathrm{eq}}}\widetilde{\delta n}(k) - \widetilde{\delta f}(k,p) \right]$$

Our solution (main result)

We want Fourier transformed current! In the linear order algebraic equation for $\widetilde{\delta f}(k,p)$ which can be solved.

4-current: $$\tilde{j}^{\mu}_{\mathrm{ind}}(k) = 2q\int (dp)p^{\mu}\left[\widetilde{f_+}(k,p) - \widetilde{f_-}(k,p)\right] = 4q\int (dp)p^{\mu}\widetilde{\delta f}(k,p)$$

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Where:

$$Q^{\mu}(k) = -\frac{4qi\kappa}{n_{\text{eq}}}\int (dp)\frac{(p\cdot u)f_{eq}(p)}{p\cdot k + i(p\cdot u)\kappa}p^{\mu} \qquad R^{\mu}_{\nu}(k) = -4q^2\int (dp)f'_{eq}(p)\frac{(u\cdot k)p^{\mu}p_{\nu} - (k\cdot p)p^{\mu}u_{\nu}}{p\cdot k + i(p\cdot u)\kappa}$$

$$Q(k) = -\frac{2i\kappa}{n_{\text{eq}}}\int (dp)\frac{(p\cdot u)^2 f_{\text{eq}}(p)}{p\cdot k + i(p\cdot u)\kappa} \qquad H_{\nu}(k) = -2q\int (dp)(p\cdot u)f'_{eq}(p)\frac{(u\cdot k)p_{\nu} - (k\cdot p)u_{\nu}}{p\cdot k + i(p\cdot u)\kappa}$$

Properties:

Gauge invariance:

$$\left.\begin{array}{r}\tilde{j}'^{\mu}_{\mathrm{ind}} = \Pi^{\mu}_{\nu}\widetilde{A}'^{\nu} = \Pi^{\mu}_{\nu}(\widetilde{A}^{\nu} - ik^{\nu}\widetilde{\chi}),\\ \tilde{j}^{\mu}_{\mathrm{ind}} = \Pi^{\mu}_{\nu}\widetilde{A}^{\nu}\end{array}\right\} \Rightarrow \Pi^{\mu}_{\nu}k^{\nu} = 0$$

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✓

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Current conservation:

$$k_{\mu}\widetilde{j}^{\mu} = 0 = k_{\mu}\Pi^{\mu}_{\nu}\widetilde{A}^{\nu}\,, \quad \Rightarrow \quad k_{\mu}\Pi^{\mu}_{\nu} = 0\,,$$

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Electron positron plasma application

All the integrals can be calculated in ultra-relativistic limit – massless particles. Only two Independent components:

$$\Pi_T(k) = \frac{m_D^2\omega}{4|\boldsymbol{k}|}\left(\frac{\omega'^2}{|\boldsymbol{k}|^2}\Lambda - \Lambda - \frac{2\omega'}{|\boldsymbol{k}|}\right)$$

$$\Pi_L(k) = \frac{m_D^2\omega^2}{|\boldsymbol{k}|^2}\left(1 - \frac{\omega'}{2|\boldsymbol{k}|}\Lambda\right)\frac{2|\boldsymbol{k}|}{2|\boldsymbol{k}| - i\kappa\Lambda}$$

Where:

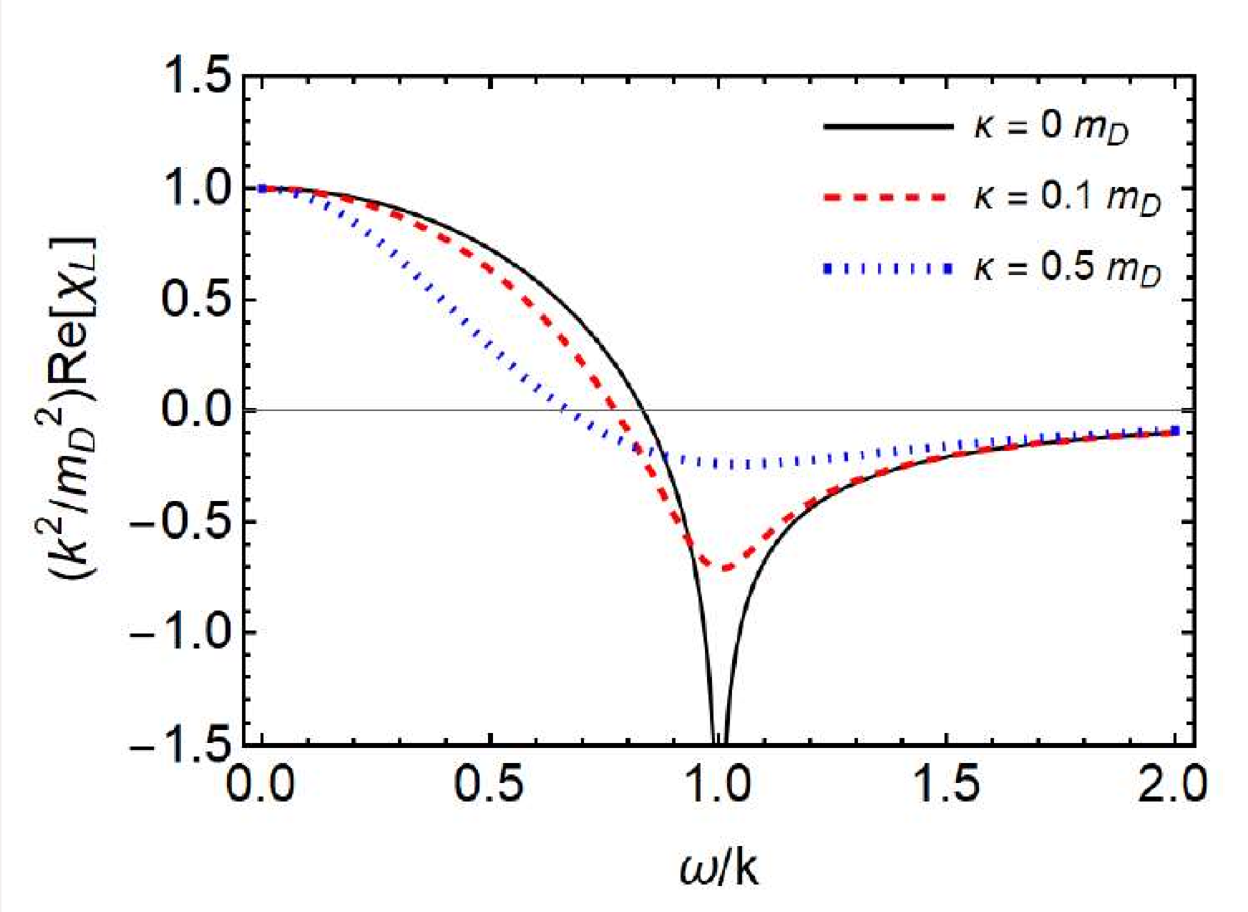
$$\omega' = \omega + i\kappa \qquad \Lambda = \ln\frac{\omega' + |\boldsymbol{k}|}{\omega' - |\boldsymbol{k}|}$$

Debay mass:

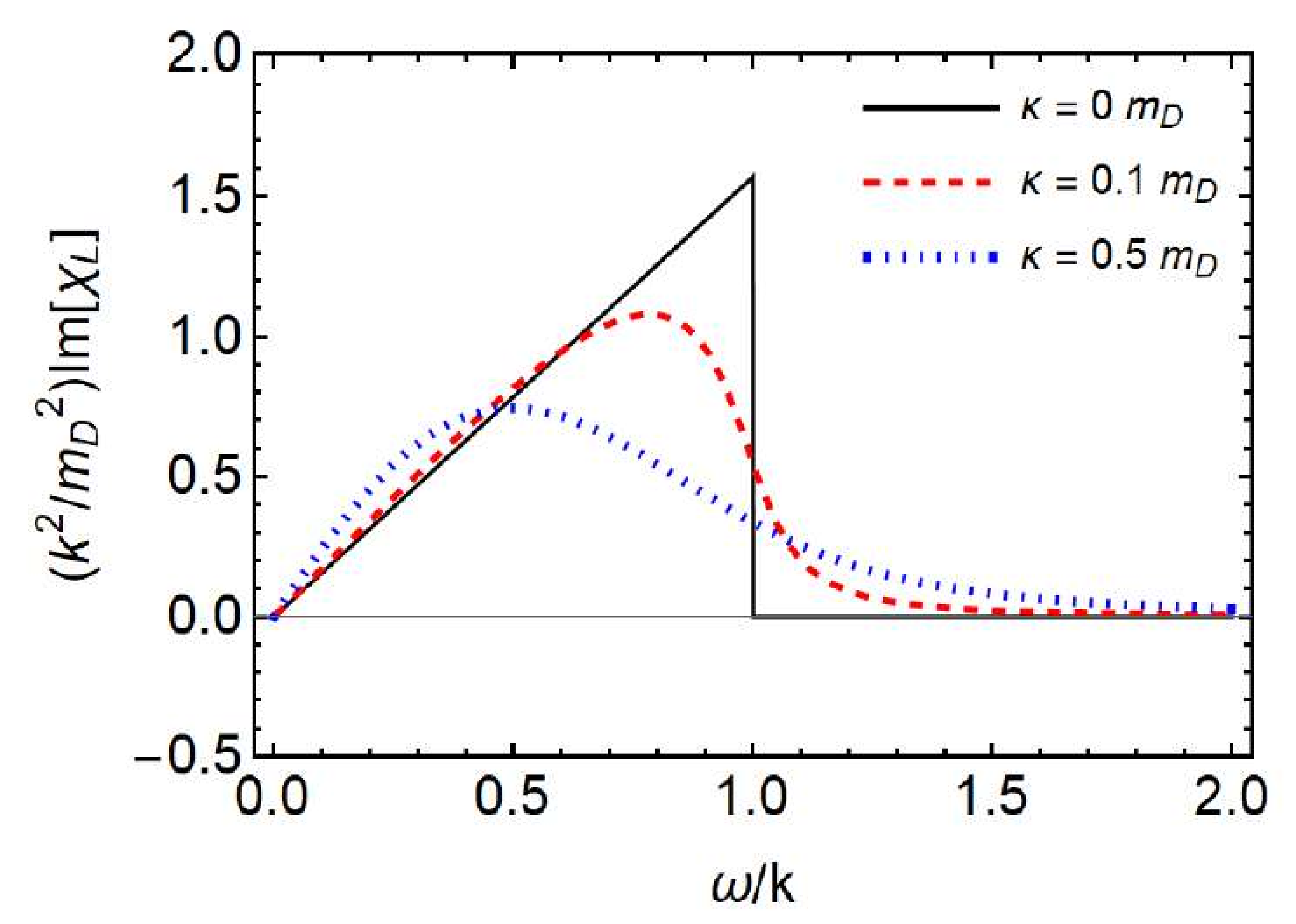
$$m_D^2 = -\frac{2q^2}{\pi^2}\int_0^\infty |\boldsymbol{p}|^2 d|\boldsymbol{p}| f'_{eq}(|\boldsymbol{p}|) = \frac{q^2T^2}{3}$$

Reduces to standard result Weldon, PRD 26 (1982) 1394 in the limit $\kappa \to 0$

Susceptibility $\chi_L = \Pi_L/\omega^2$ $\boldsymbol{D}_L = \varepsilon_0(1+\chi_L)\boldsymbol{E}_\mathrm{L}$



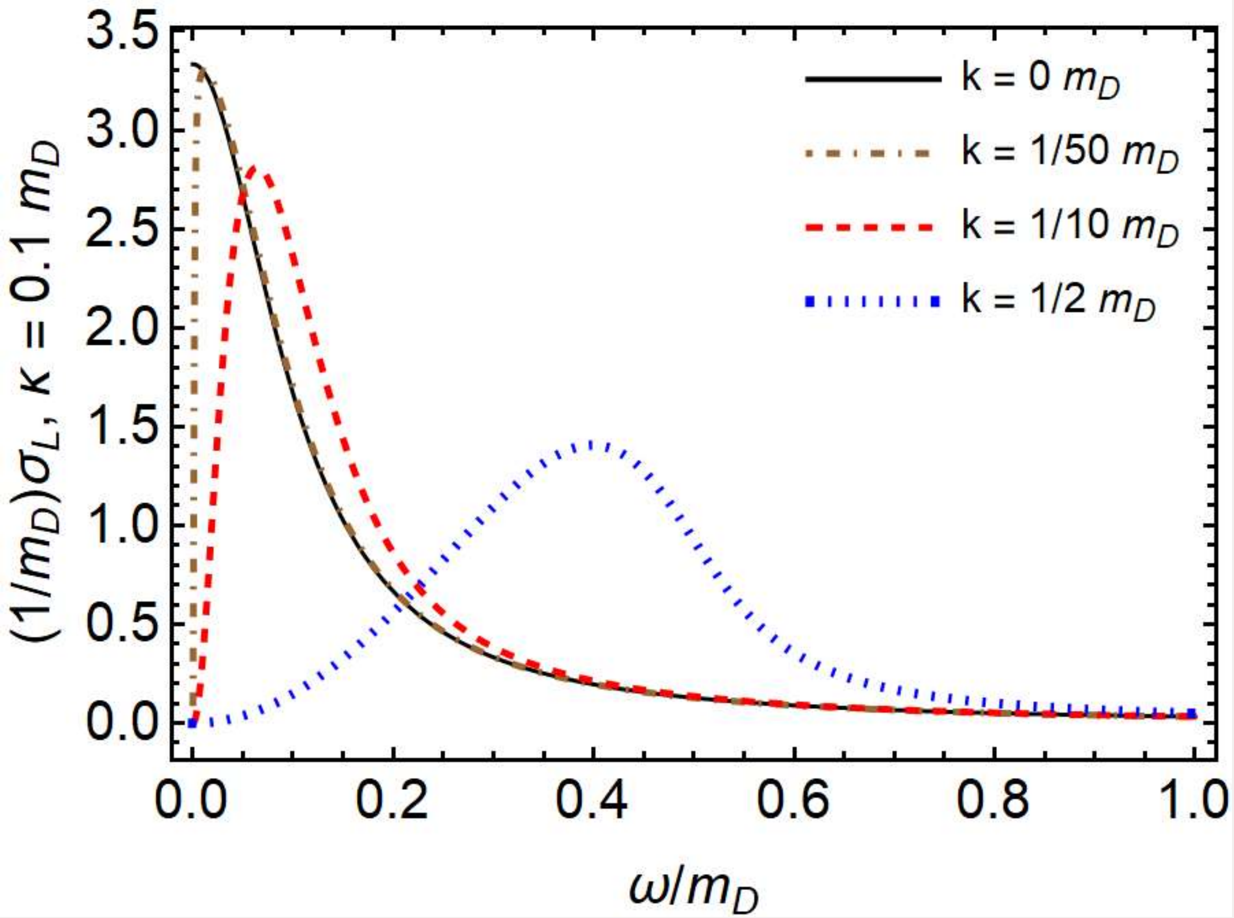
Real part



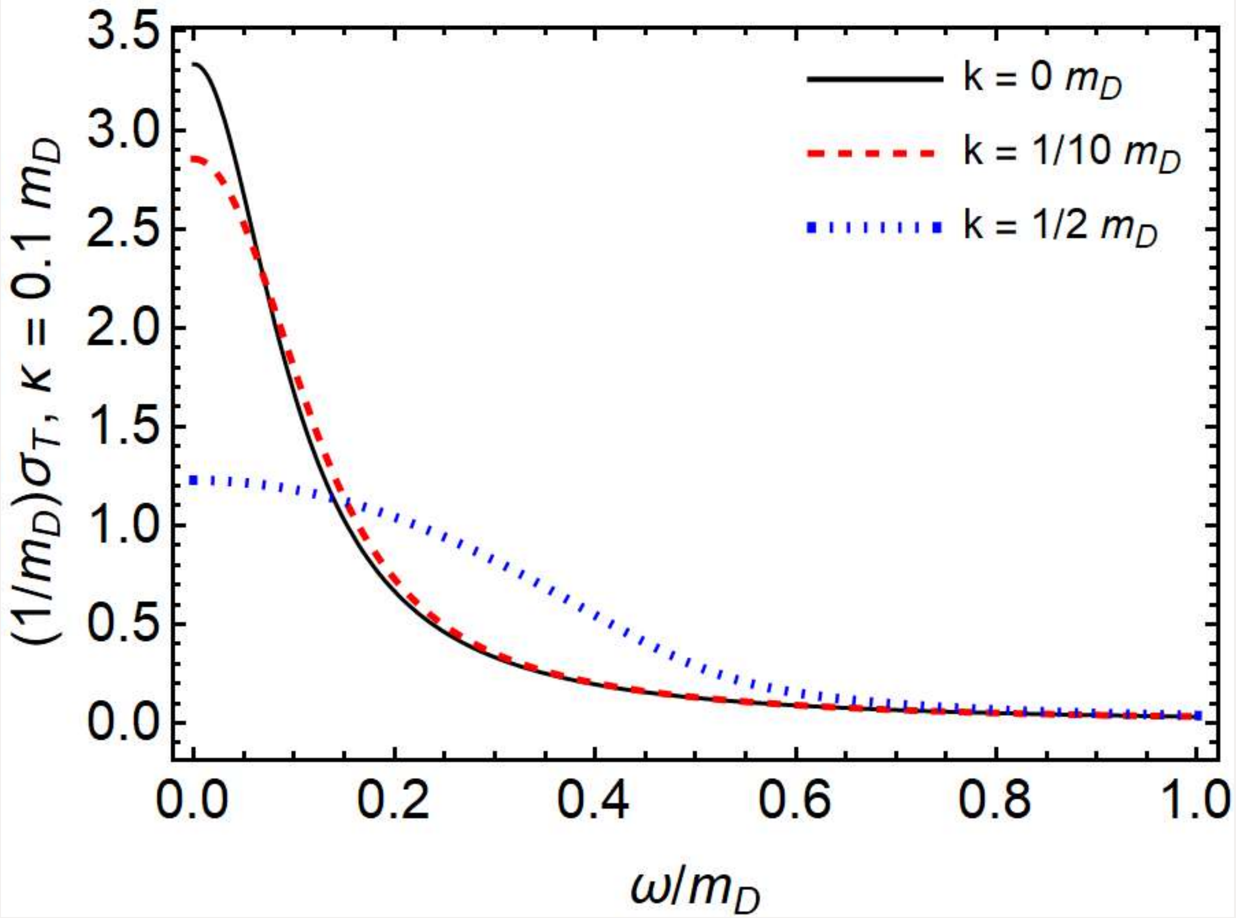
Imaginary part

For $\kappa = 0$ we reproduce Weldon, PRD 26 (1982) 1394

Conductivity $\sigma_{T/L} = -i\omega\Pi_{T/L}$



Real part of longitudinal conductivity



Real part of transversal conductivity

Discontinuity at $k = 0$ comes from infinite extent of plasma (Baranger, 1989)

Dispersion relations - $\boldsymbol{\omega(k)}$

$$\Big[\underbrace{(k\cdot u)^2+\mu_0\Pi_L(k)}_{\text{Longitudinal modes}}\Big)\Big[\underbrace{k^2+\mu_0\Pi_T(k)}_{\text{Transverse modes}}\Big]=0$$

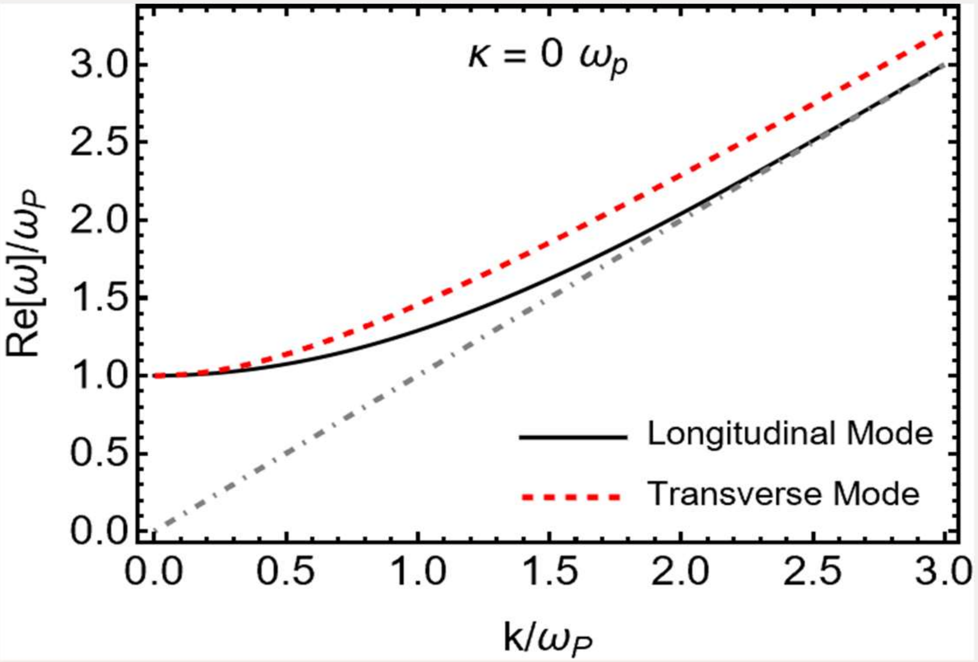
For $|\boldsymbol{k}| \to 0$ we have for both L,T:

$$\omega_{\pm}=-\frac{i\kappa}{2}\pm\sqrt{\omega_p^2-\frac{\kappa^2}{4}}$$

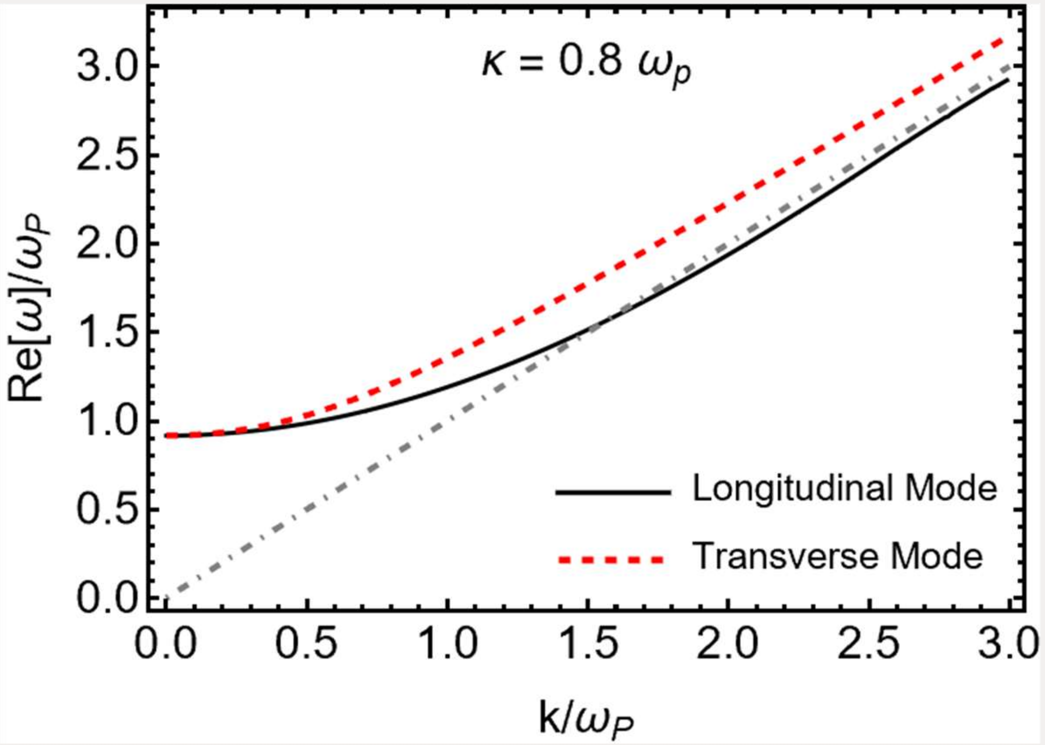
Modified plasma frequency $\quad \omega_p=\frac{1}{3}m_D^2$

- $\kappa \ll \omega_p$ weekly damped case
- $\kappa > 2\omega_p$ overdamped case

$\kappa = 0, |\boldsymbol{k}| \neq 0$



$|\boldsymbol{k}| \neq 0, \kappa \neq 0$



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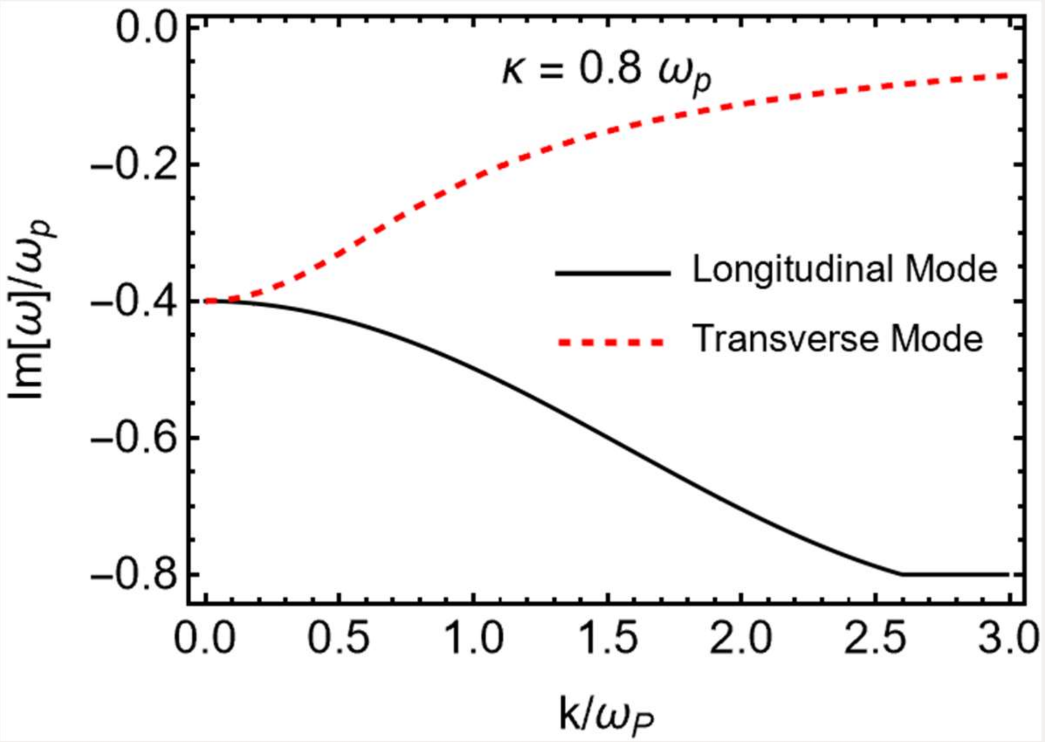

Imaginary part

See also results of Carrington, Fugleberg, Pickering, Thoma, Can. J. Phys. 82 (2004) 671; B. Schenke, M. Strickland, C. Greiner, M.H. Thoma, Phys. Rev. D 73 (2006) 125004

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- Manifestly Lorentz covariant formulation of the plasma perturbation
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Outlook:

- Generalization to laser light interaction with matter
- application to QGP in heavy ion collisions (following talk by Chris M. Grayson)
- Early universe $e^{-}e^{+}$ plasma (talk tomorrow by Cheng-Tao Yang)
- Finite physical systems
- Generalization for multicomponent plasma

Thank you for your attention!

Happy birthday Prof. Rafelski!

2014 - today