The QCD phase diagram from the lattice

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- Considerable progress over last few years
- Results begin to be phenomenologically relevant

CRC-TR 211 Strong-interaction matter under extreme conditions

ELEMENTS

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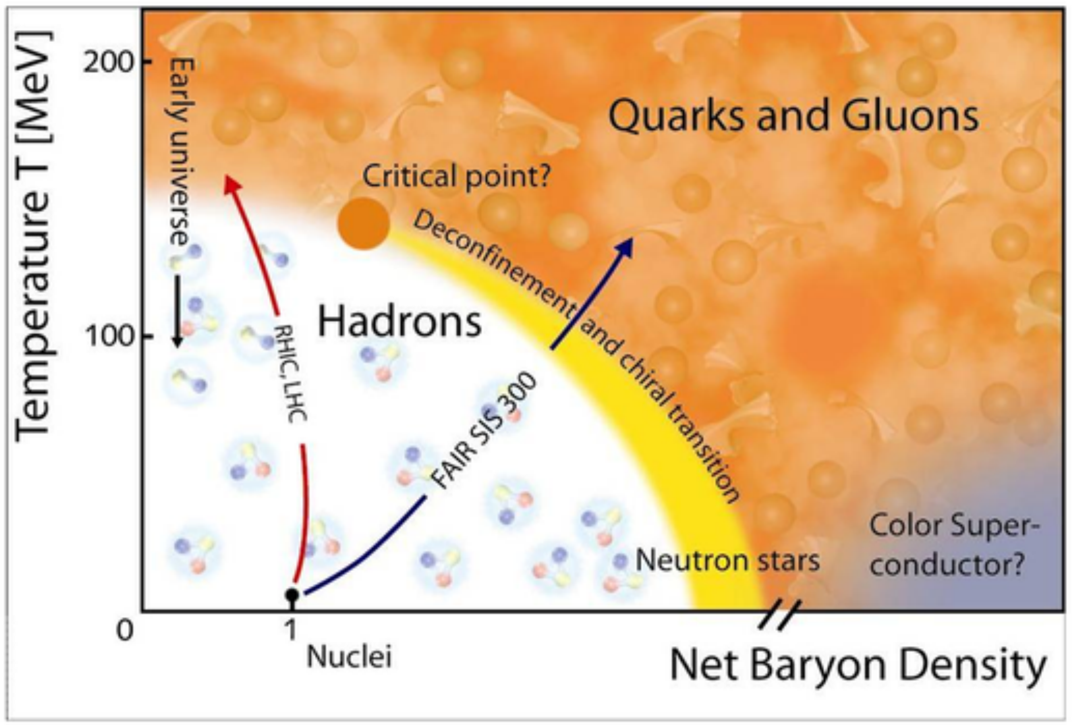
The QCD phase diagram

- Fundamental for particle-, nuclear-, astro- physics
- Future textbook knowledge
- Non-perturbative problem
- “Sign problem” prevents Monte Carlo simulation (NP-hard problem?)



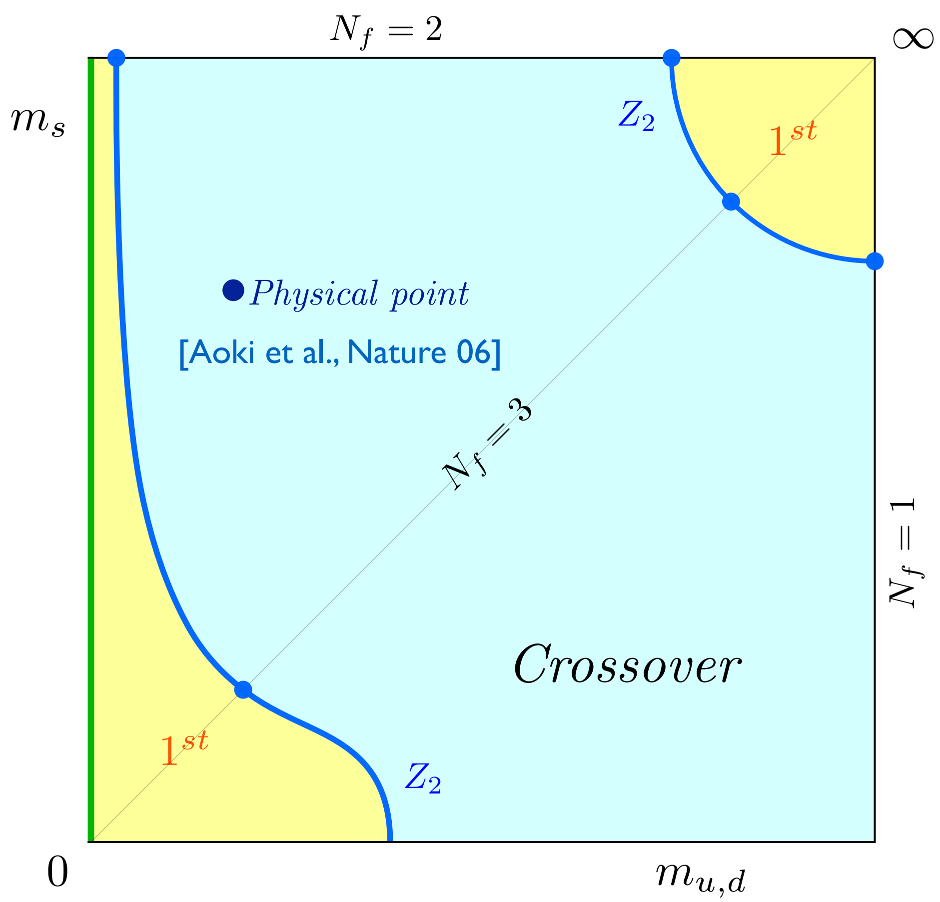
from GSI

→ Collect evidence of QCD parameter regions away from physical point

Vary $T, \mu_B, m_q, N_f, N_c, g^2, a$ → constraints, coherent picture starts to emerge

The nature of the QCD thermal transition at zero density

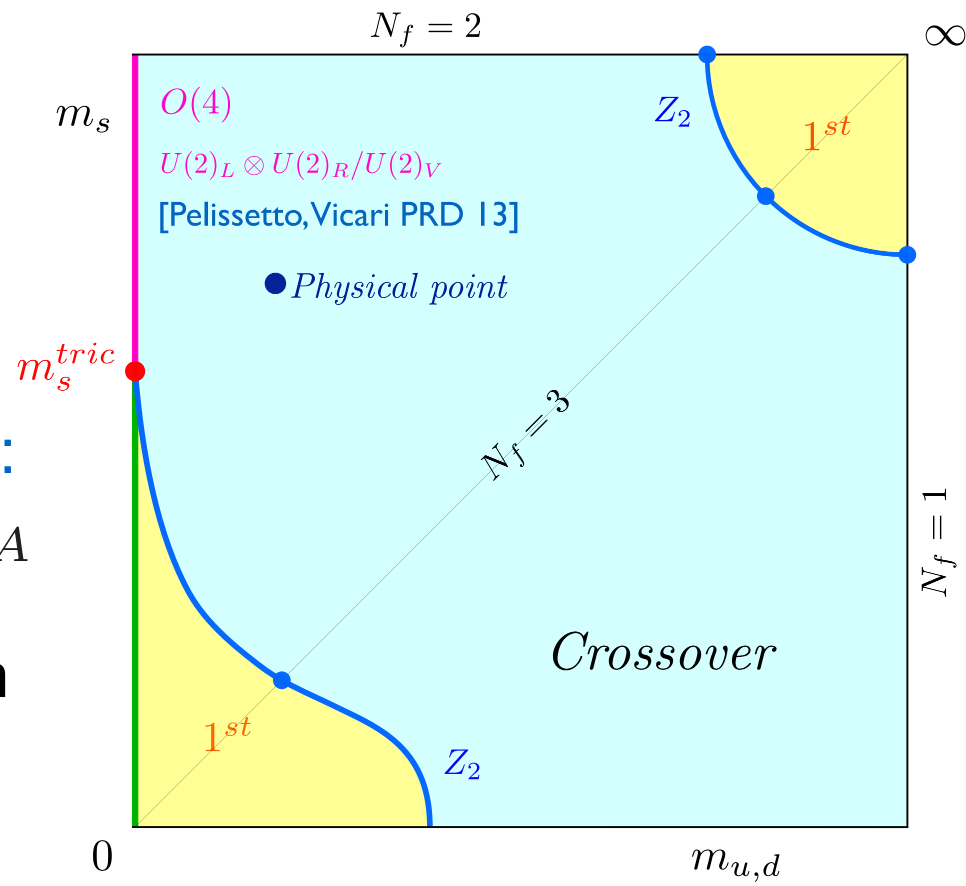
deconfinement p.t.:
breaking of global $Z(3)$ symmetry



[Pisarski, Wilczek, PRD 84]:
$N_f = 2$ depends on $U(1)_A$

restored broken

$N_f \geq 3$ 1st order



chiral p.t.
restoration of global symmetry in flavour space

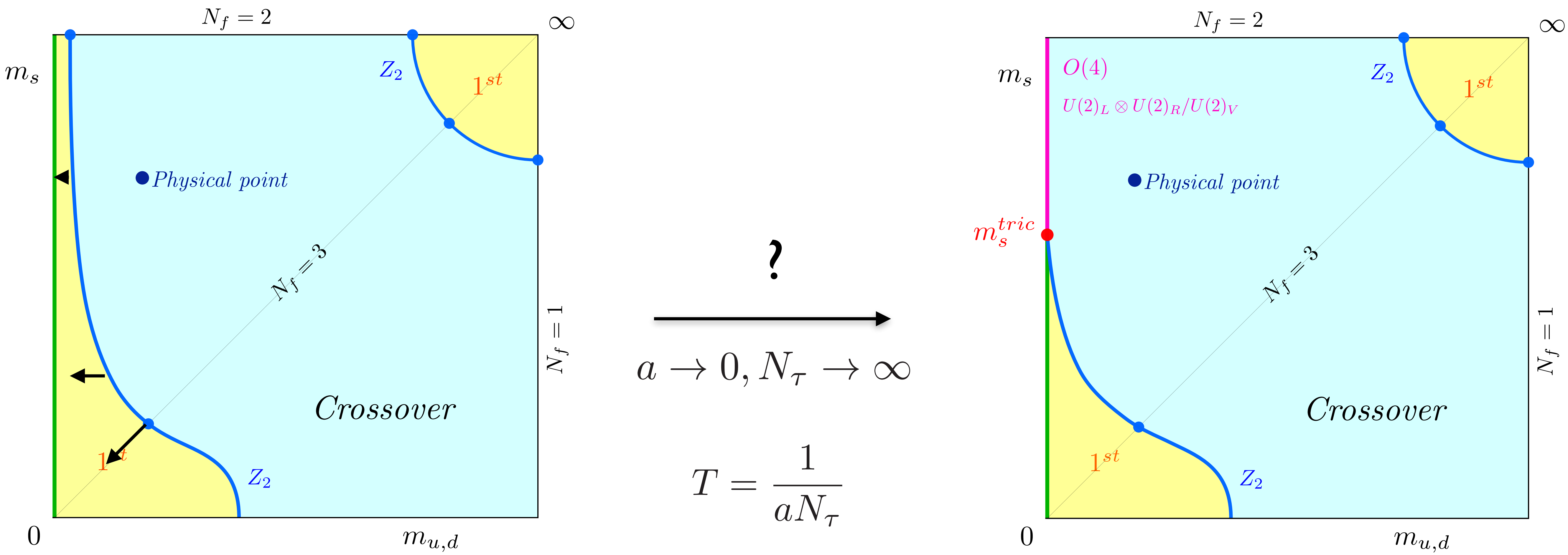
$SU(2)_L \times SU(2)_R \times U(1)_A$

anomalous

Can a trace of the chiral transition (scaling) be detected experimentally?

The nature of the QCD chiral transition

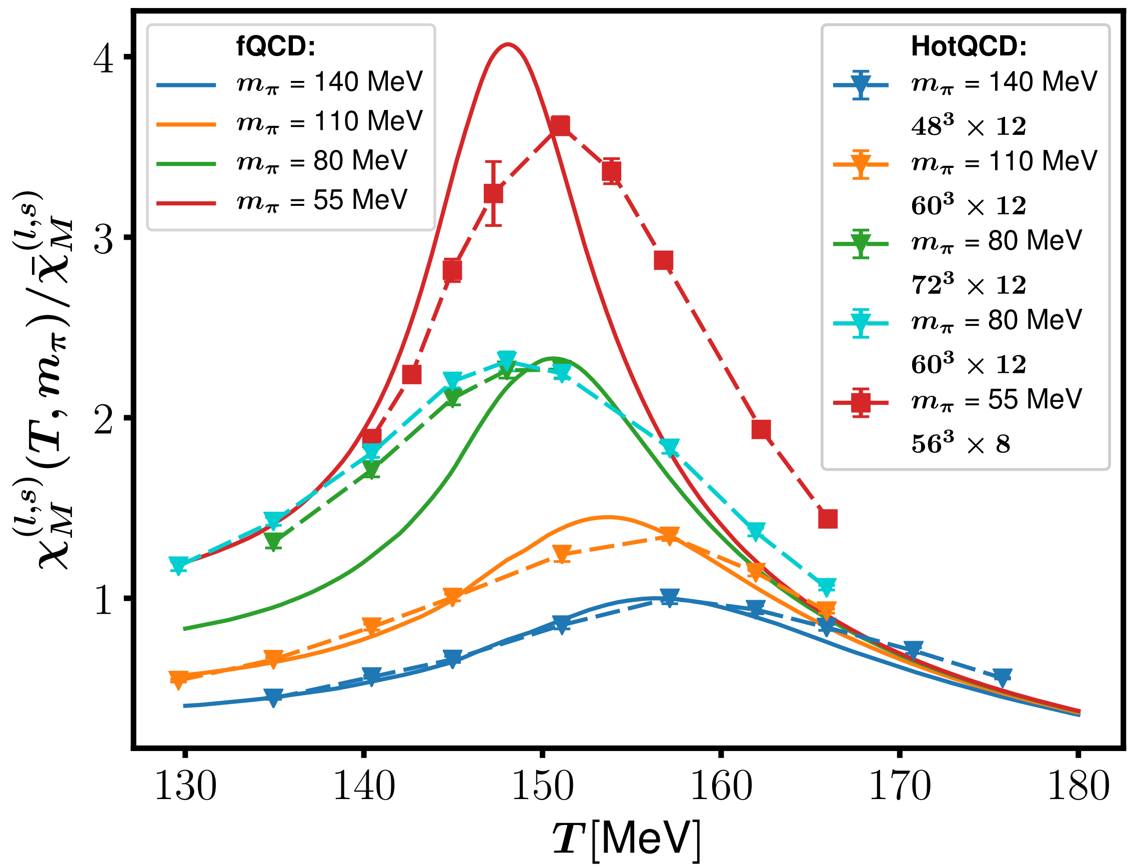
…is elusive, massless limit not simulable!

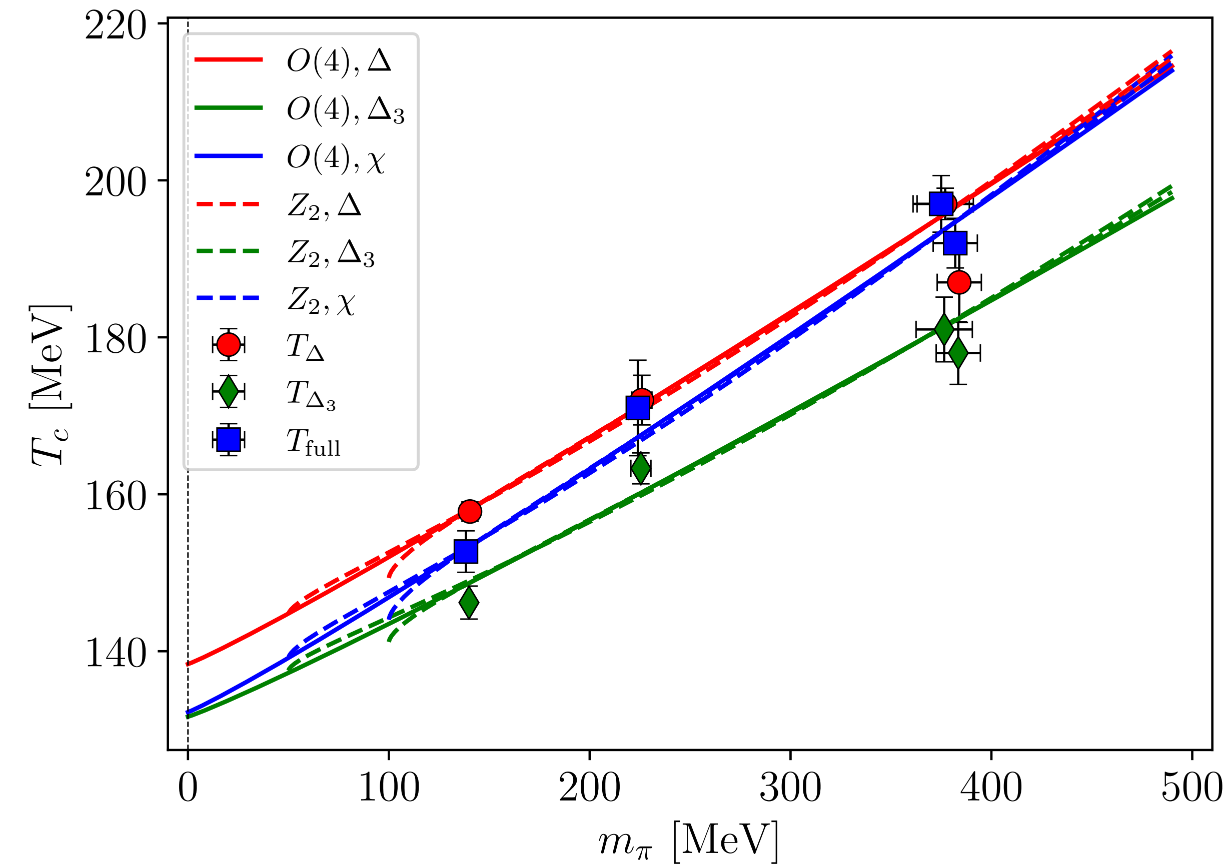


- Coarse lattices or unimproved actions: 1st order for $N_f = 2, 3$
- 1st order region shrinks rapidly as $a \to 0$
- Improved staggered actions: no 1st order region so far, even for $N_f = 3$ $m_{PS} > 45\text{MeV}$

Details and reference list: [O.P., Symmetry 13, 2021]

From the physical point to the chiral limit





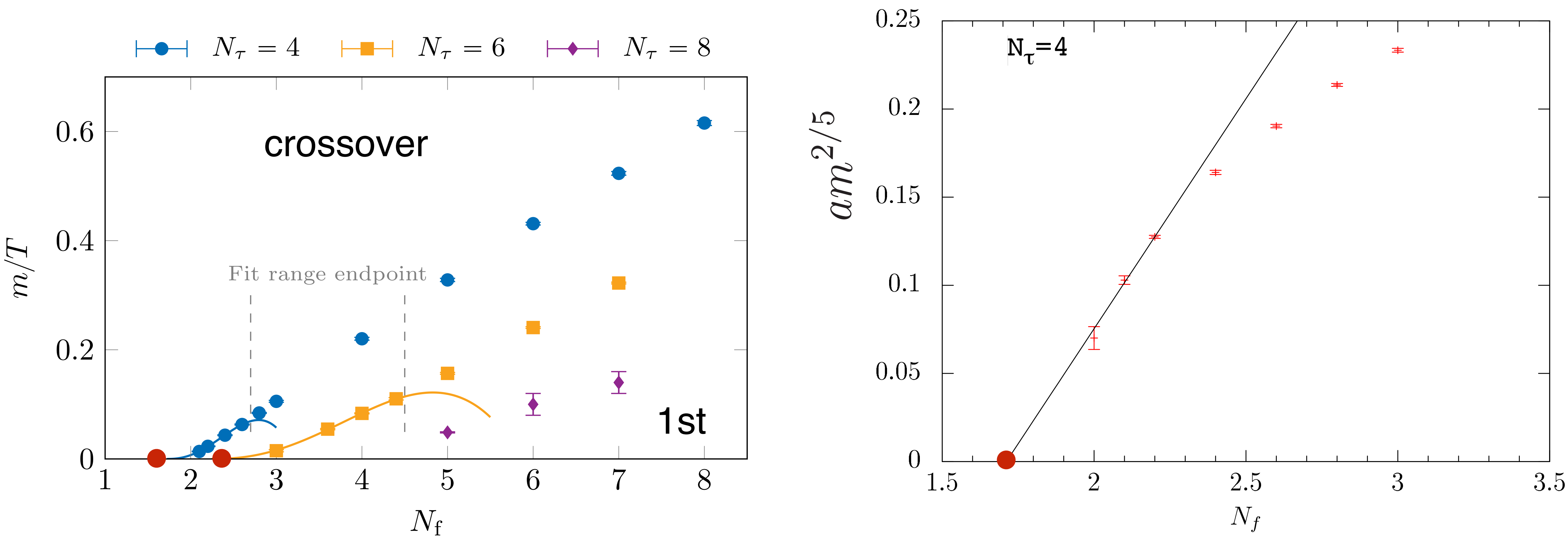
[HotQCD, PRL 19] HISQ (staggered) [Kotov, Lombardo, Trunin, PLB 21] Wilson twisted mass

$$T_c^0 = 132^{+3}_{-6} \text{ MeV} \qquad T_{pc}(m_l) = T_c^0 + K\, m_l^{1/\beta\delta} \qquad T_c^0 = 134^{+6}_{-4} \text{ MeV}$$

- Keep strange quark mass fixed, crossover gets stronger as chiral limit approached
- Cannot distinguish between Z(2) vs. O(4) exponents, need exponential accuracy!
- Determination of chiral critical temperature possible, but not the order of the transition
- Comparison with fRG: $T_c^0 \approx 142\text{MeV}$, "most likely O(4)" [Braun et al., PRD 20,21]

Bare parameter space of unimproved staggered LQCD

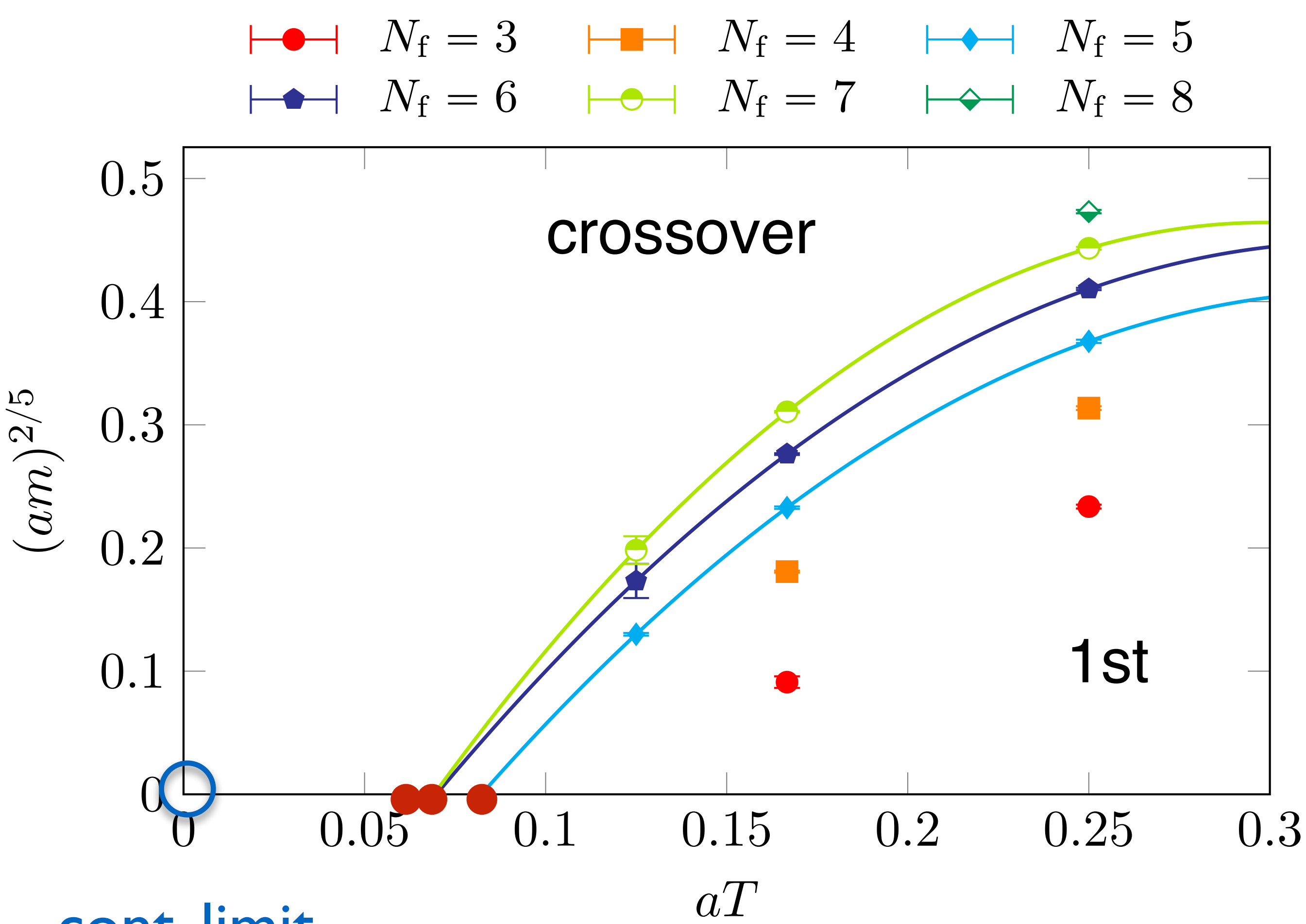
[Cuteri, O.P., Sciarra, JHEP 21]



- Employ non-integer N_f to resolve tricritical point, ($(\det(D(m))^{N_f}$ in partition fcn.)
- Observe tric. scaling in N_f (also in imaginary μ [Bonati et al. PRD 14])
- Old question: $m_c/T = 0 \text{ or } \neq 0$? Answered for $N_f = 2$
- New question: will $N_f^{\rm tric}(N_\tau)$ slide beyond $N_f = 3$?

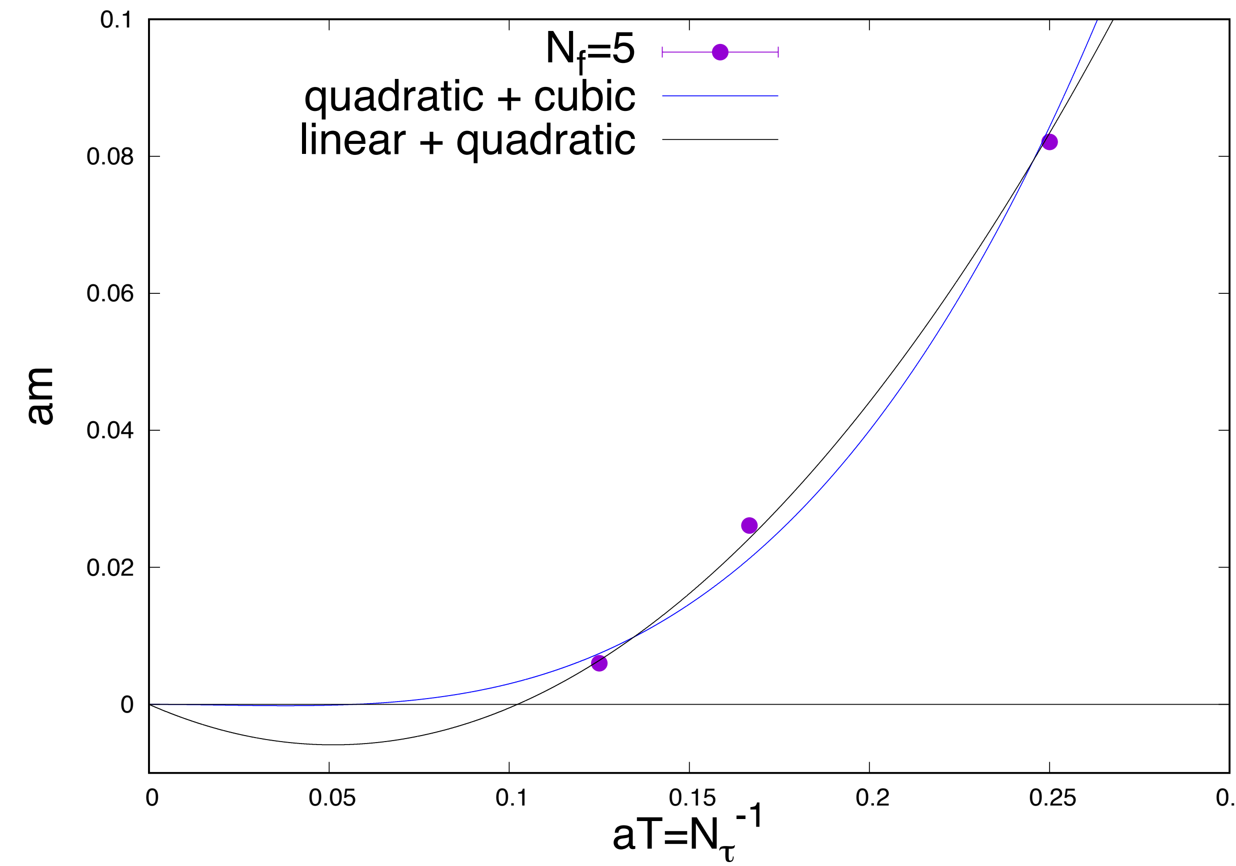
Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra, JHEP 21]



cont. limit

1st order scenario does not fit!

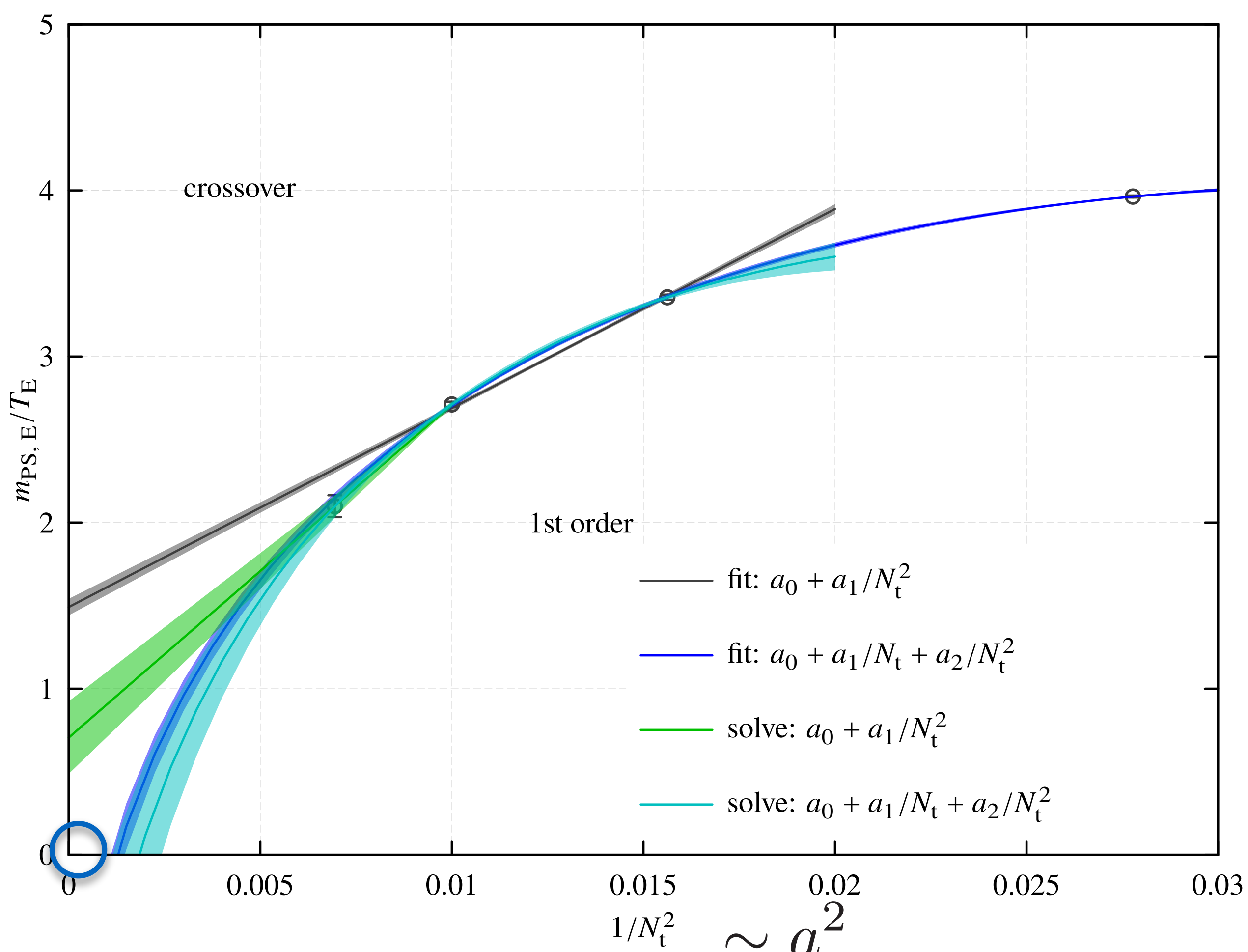


- Tricritical scaling, $N_f^{\rm tric}(N_\tau)$ implies: 1st order region does not extend to continuum
- First-order scenario Incompatible with data! $\chi^2_{\rm dof} > 10$
- $N_f = 2 - 7$ all have 2nd order chiral phase transitions in the continuum!

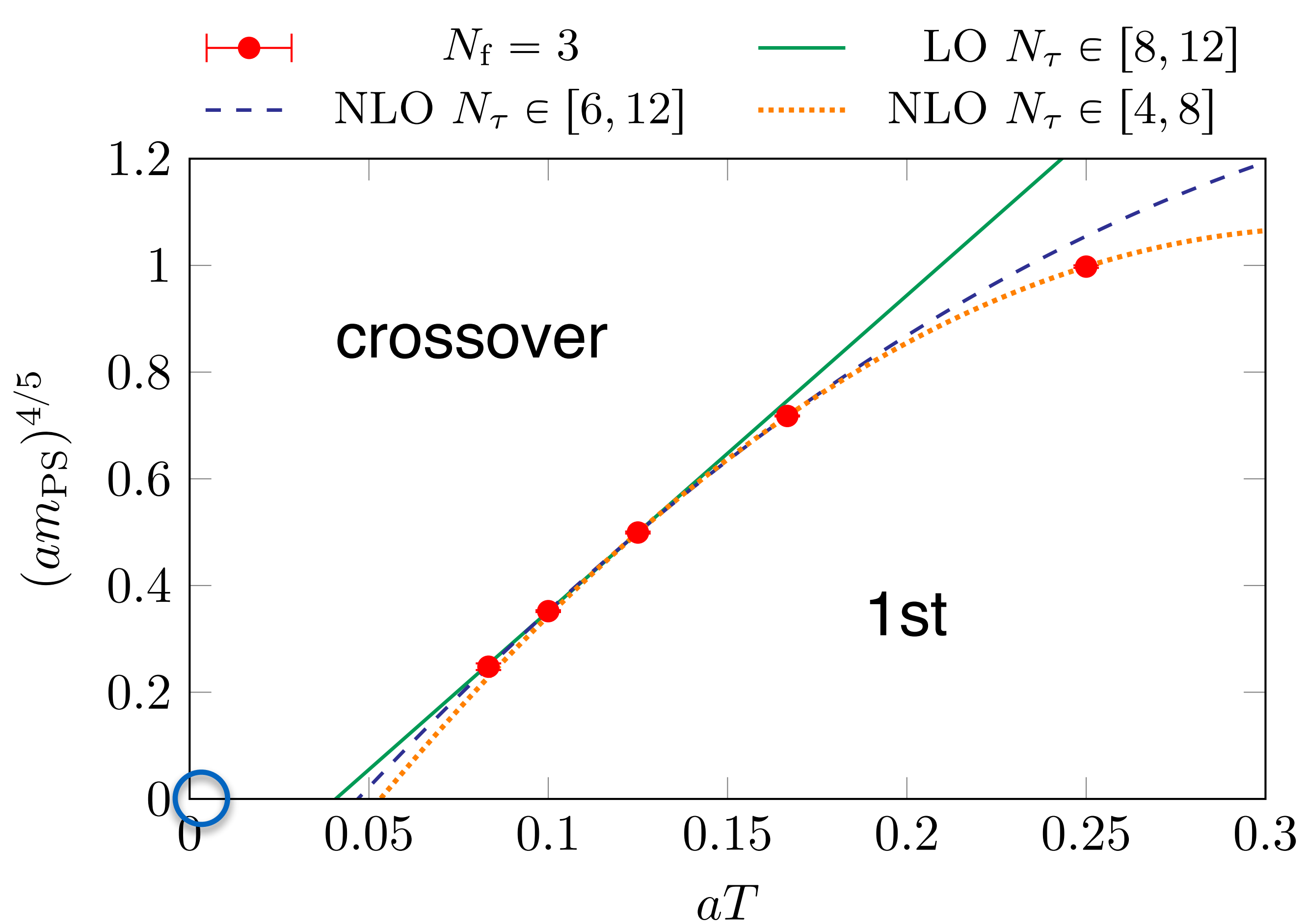
Nf=3 O(a)-improved Wilson fermions

[Kuramashi et al. PRD 20] $m_\pi^c \leq 110\ \mathrm{MeV} \quad N_\tau = 4, 6, 8, 10, 12$

Re-analysis using: $am_{PS}^2 \propto am_q$



cont. limit



Tricritical scaling! [Cuteri, O.P., Sciarra, JHEP 21]

Nf=3 consistent with staggered, 2nd order in chiral continuum limit!

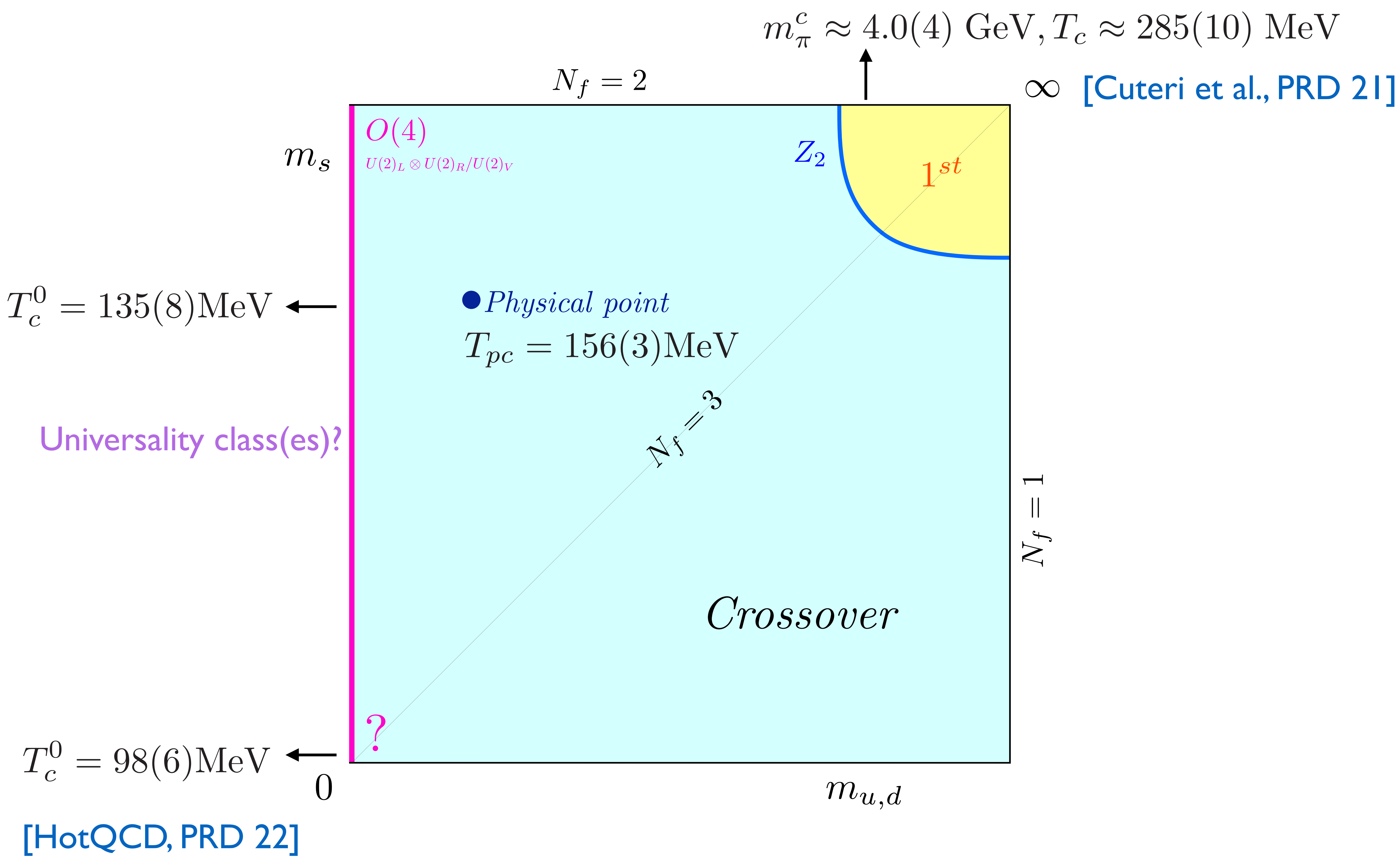
What about Pisarski, Wilczek?

- Investigated 3d ϕ^4 sigma model,
  i.e. Ginzburg-Landau-Wilson theory for chiral condensate

  Results based on epsilon expansion about $\epsilon = 1$
- Conclusions confirmed by [Butti, Pelisetto, Vicari, JHEP 03]
  (High order perturbative expansion in fixed d=3)
- Support also from simulation of 3d sigma model [Gausterer, Sanielovici, PLB 88]

- fRG: 3d ϕ^6 has infrared fixed points and 2nd order transitions [Litim, Tetradis, NPB 96]
- Conformal bootstrap methods: fixed point also with O(4)xO(2)
  [Nakayama, Ohtsuki PRD 14]
- 3d ϕ^6 with t'Hooft term: 2nd order transition for restored anomaly! [Fejos, PRD 22]

The emerging Columbia plot in the continuum



The Columbia plot with real and imaginary μ

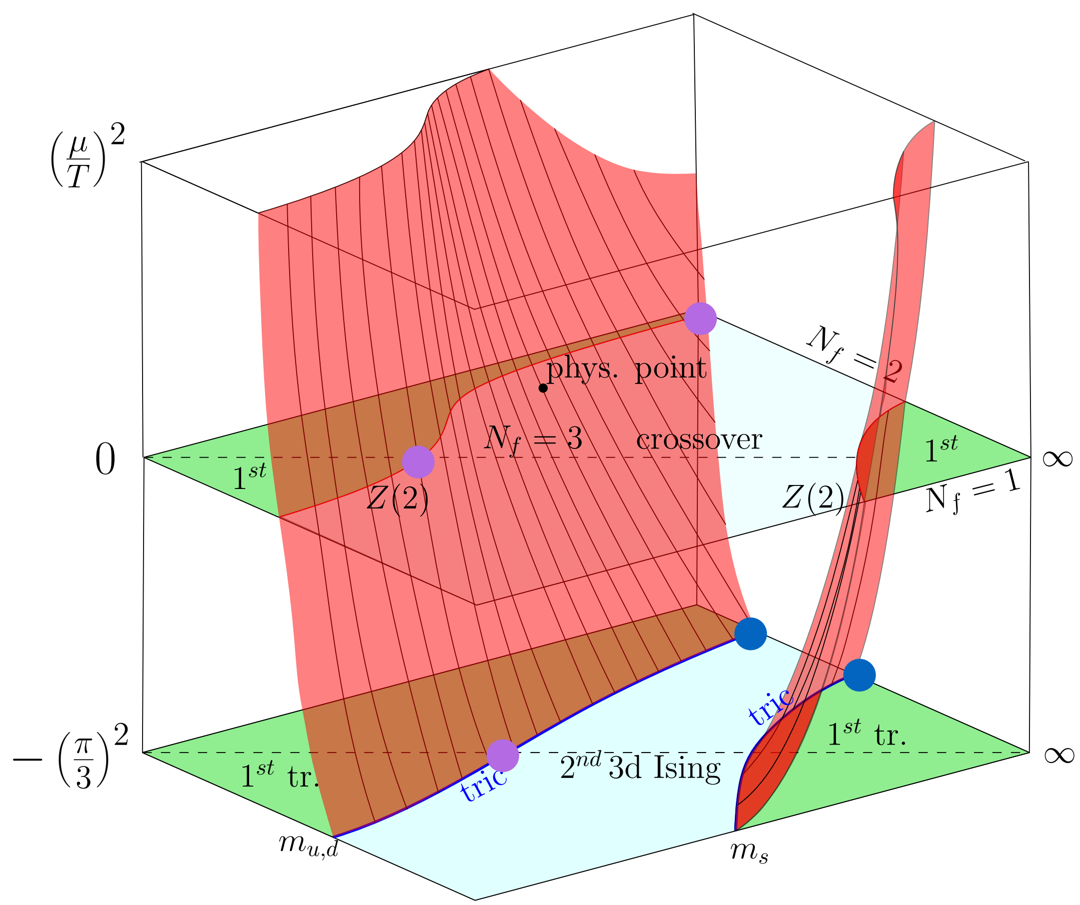
Exact symmetries:

$$Z(\mu) = Z(-\mu)$$

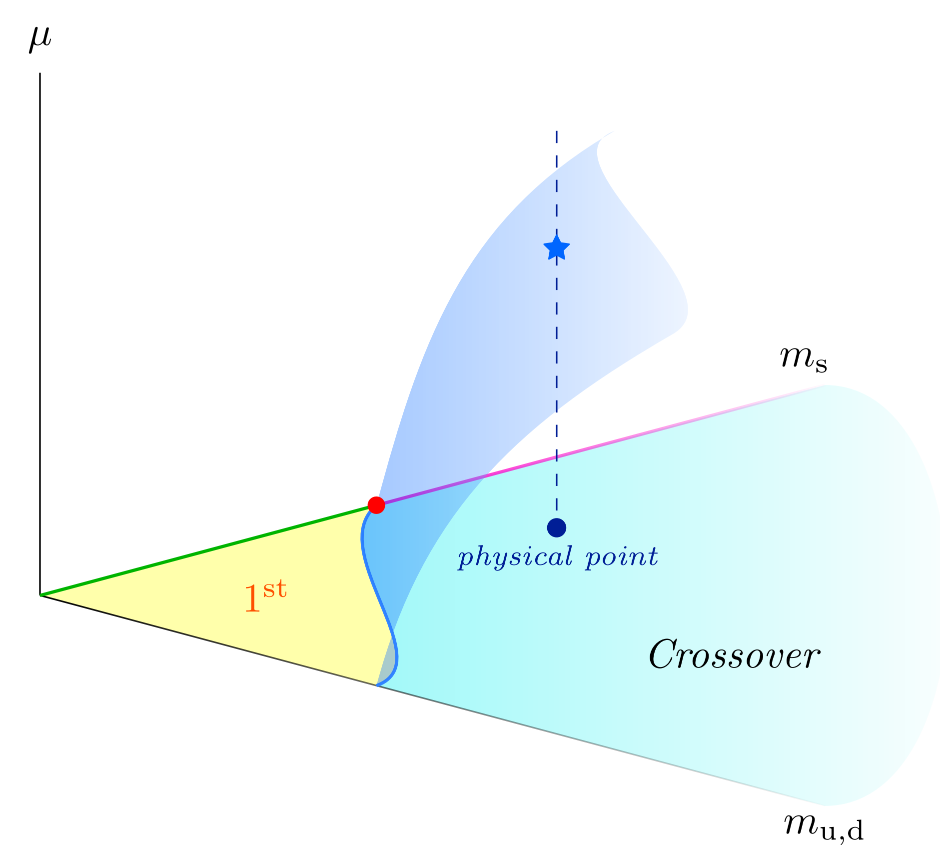
[Roberge, Weiss NPB 86]

$$Z\left(T, i\frac{\mu_i}{T}\right) = Z\left(T, i\frac{\mu_i}{T} + i\frac{2n\pi}{N_c}\right)$$

Finding on coarse lattices:



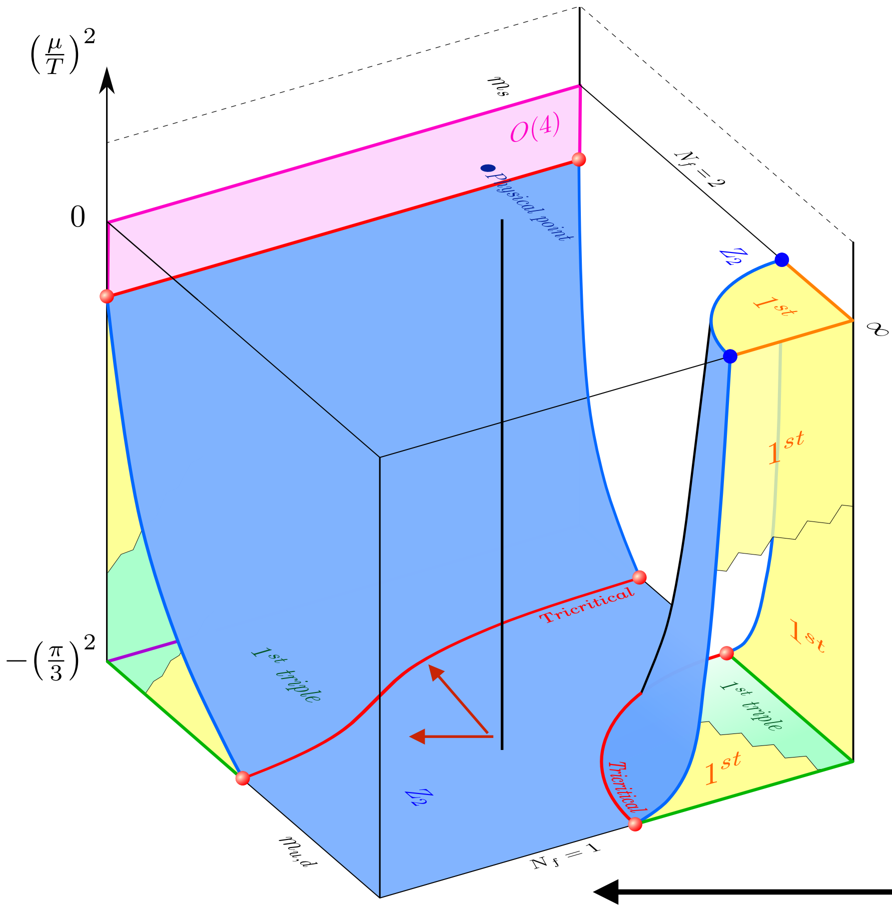
Required for phase diagram with critical endpoint:

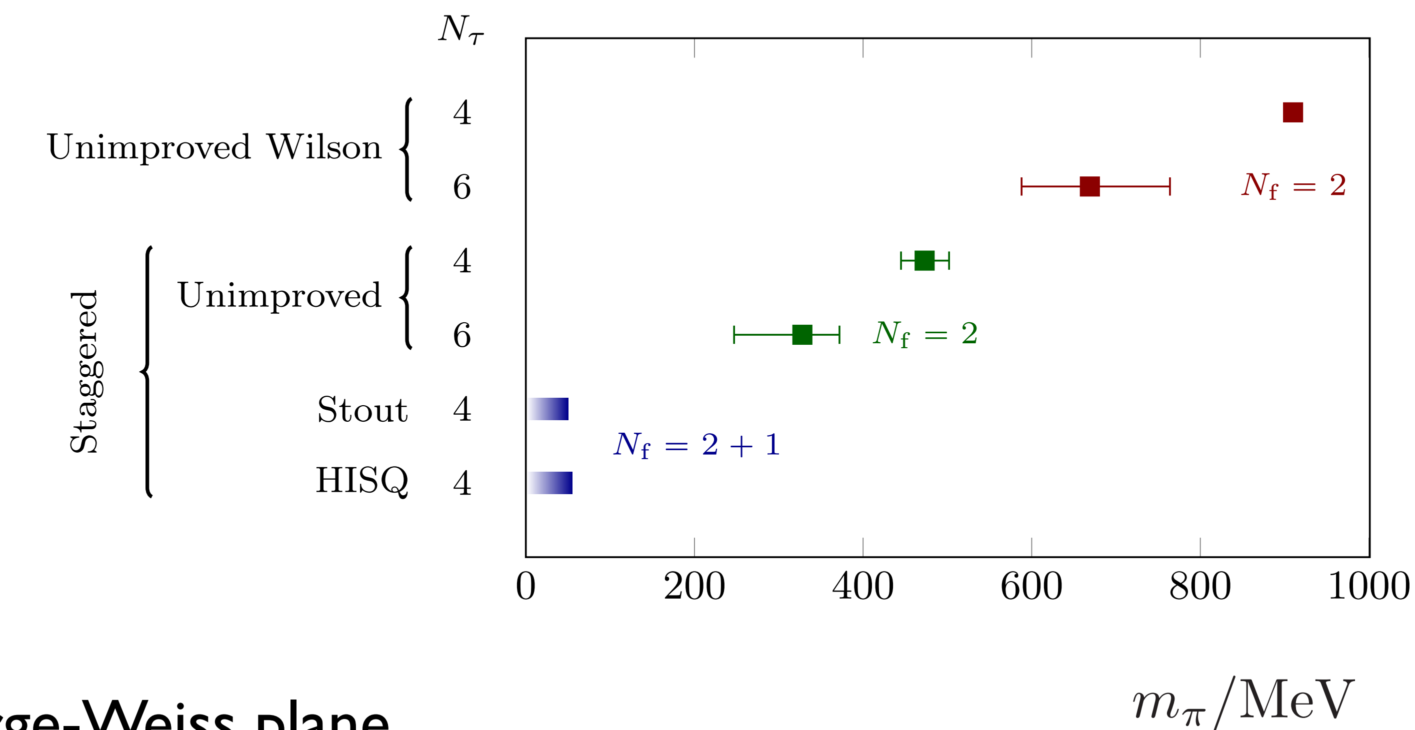


unimproved staggered [de Forcrand, O.P., PRL 10, Bonati et al., PRD 11]

unimproved Wilson [O.P., Pinke, PRD 14]

The Columbia plot with imaginary μ





Roberge-Weiss plane

- Unimproved actions: first-order region shrinks on finer lattices
  [Pinke, O.P. PRD 14, O.P. Sciarra PRD 20]

- Improved staggered actions: no first-order region seen, upper bounds:

  [Bonati et al., PRD 19]: stout smearing, light quark mass down to $m_\pi \approx 50$ MeV
  [HotQCD, PoS LAT 19]: HISQ, light quark mass down to $m_\pi \approx 55$ MeV

➡ Entire chiral critical surface shifts towards chiral limit! Any continuum dependence on μ_B ?

The physical point at small baryon density

Taylor expansion of the pressure:

Baryon number fluctuations, known up to 2n=8 on $N_\tau = 16$

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{2n!} \chi_{2n}^B(T) \left(\frac{\mu_B}{T}\right)^{2n}, \quad \chi_{2n}^B(T) = \left.\frac{\partial^{2n}(\frac{p}{T^4})}{\partial(\frac{\mu_B}{T})^{2n}}\right|_{\mu_B=0}$$

- Calculate derivatives [Allton et al., PRD 2002;…]
- Calculate full function at imaginary μ_B, fit Taylor coefficients [de Forcrand, O.P., NPB 2002, D'Elia, Lombardo, PRD 2003;…]

Strong check of systematics, fully consistent!

[Bonati et al., PRD 18, NPA 19]

Pseudo-critical temperature:

$$\frac{T_{pc}(\mu_B)}{T_{pc}(0)} = 1 + \kappa_2 \left(\frac{\mu_B}{T}\right)^2 + \kappa_4 \left(\frac{\mu_B}{T}\right)^4 + \ldots$$

κ_2	Action	
0.0158(13)	imag. μ, stout-smeared staggered	[Bellwied et al, PLB 15]
0.0135(20)	imag. μ, stout-smeared staggered	[Bonati et al, NPA 19]
0.0145(25)	Taylor, stout-smeared staggered	[Bonati et al, PRD 18]
0.016(5)	Taylor, HISQ	[HotQCD, PLB 19
0.0153(18)	imag. μ, stout-smeared staggered	[Borsanyi et al, PRL 20]

consistent with 0

The search for a critical endpoint

(T_c, μ_B^c) : upper bound on radius of convergence of Taylor expansion in chem.pot. or fugacity

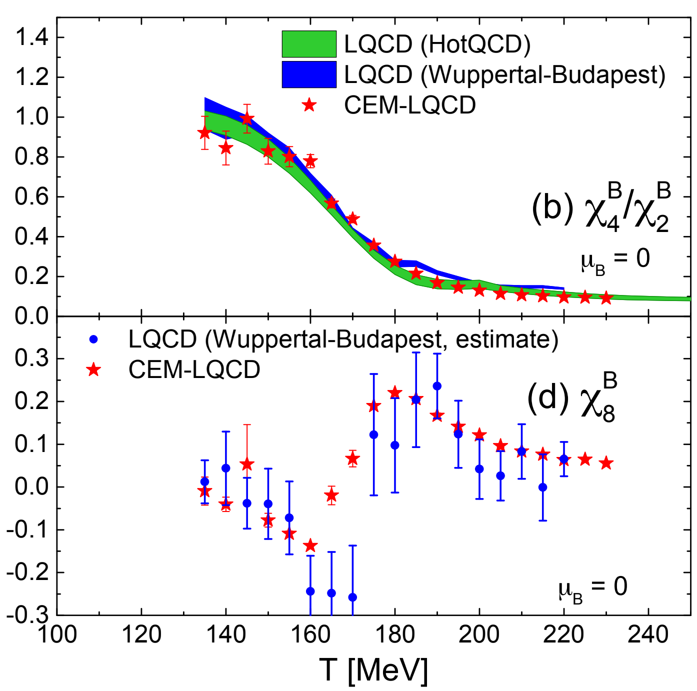
Radius of convergence $r = \lim_{n\to\infty} r_{2n}\,, \quad r_{2n} = \left|\frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B}\right|$ $\qquad r_n = \left|\frac{c_{n+1}\,c_{n-1} - c_n^2}{c_{n+2}\,c_n - c_{n+1}^2}\right|^{1/4}$

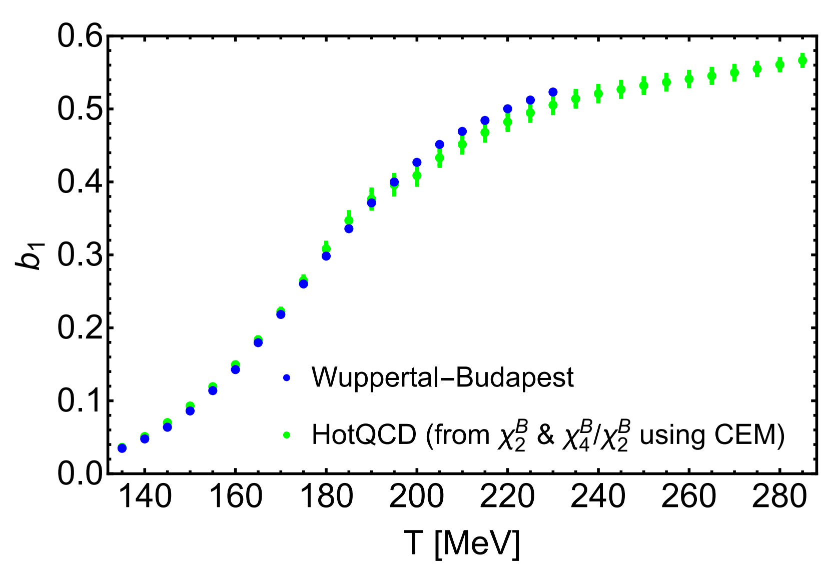
ratio estimator $\qquad$ Mercer-Roberts estimator

Cluster Expansion Model [Vovchenko et al., PRD, NPA 2018]

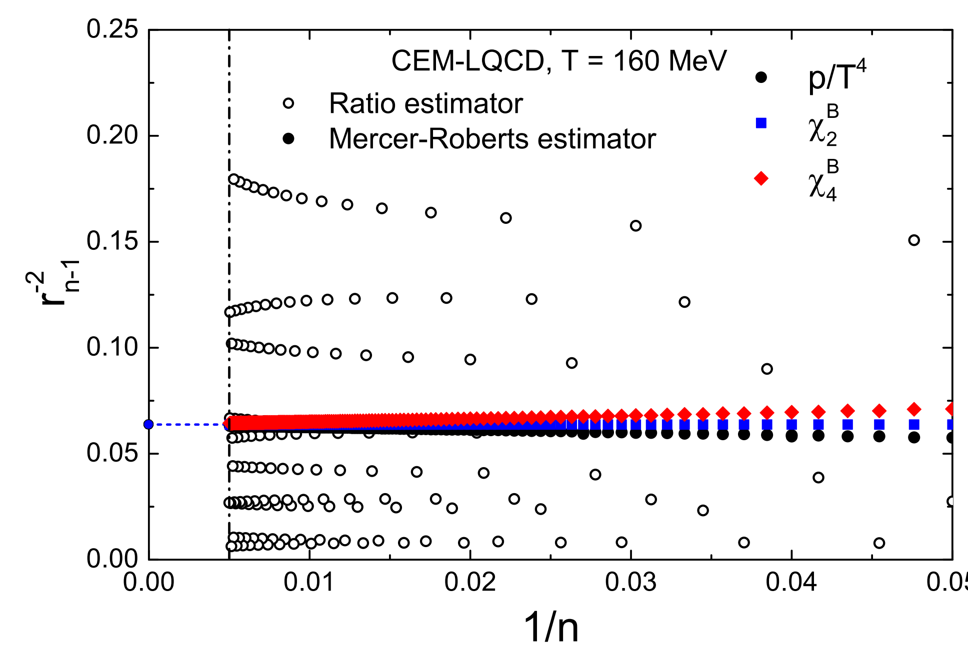
Recursive relation between fugacity coeffs, matched to LQCD

$$\frac{n_B(T,\mu_B)}{T^3} = -\frac{2}{27\pi^2}\frac{\hat{b}_1^2}{\hat{b}_2}\left\{4\pi^2\left[\mathrm{Li}_1(x_+) - \mathrm{Li}_1(x_-)\right] + 3\left[\mathrm{Li}_3(x_+) - \mathrm{Li}_3(x_-)\right]\right\}$$





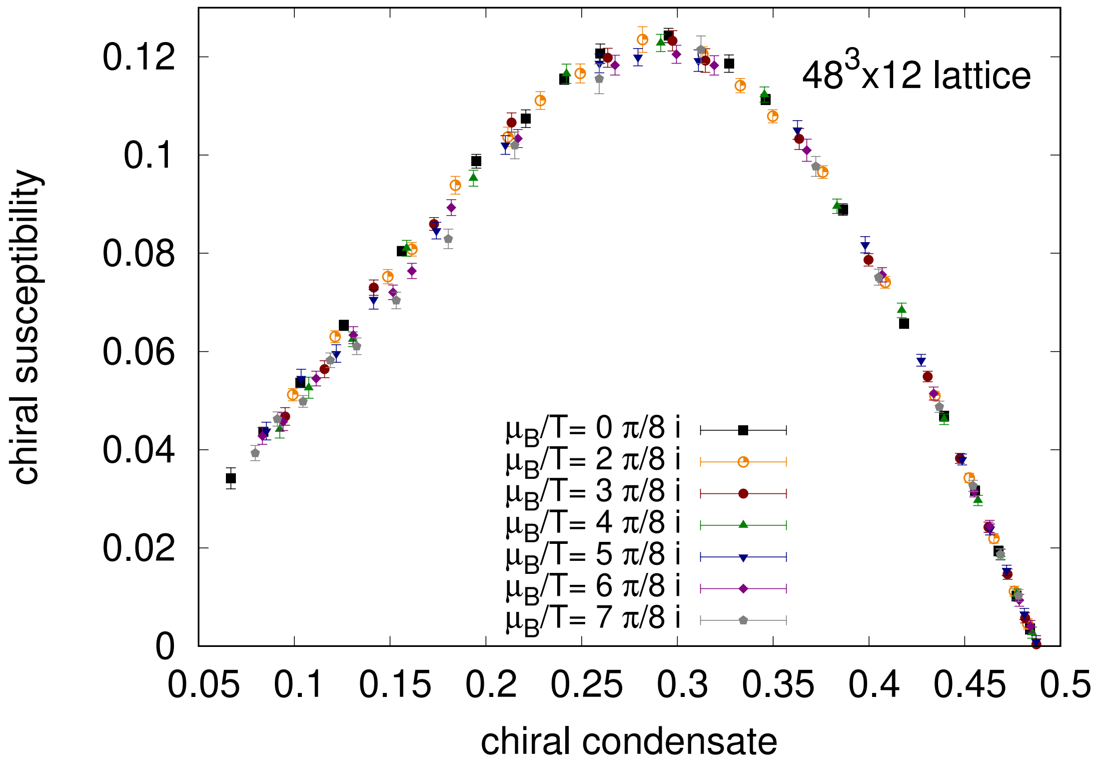
Ratio test fails! M-R works



[Giordano, Pasztor, PRD 2019]: ratio fails, improved estimators based on M-R, Cauchy-Hadamard

CEM prediction: closest singularity in complex plane is Roberge-Weiss transition at imag. μ_B

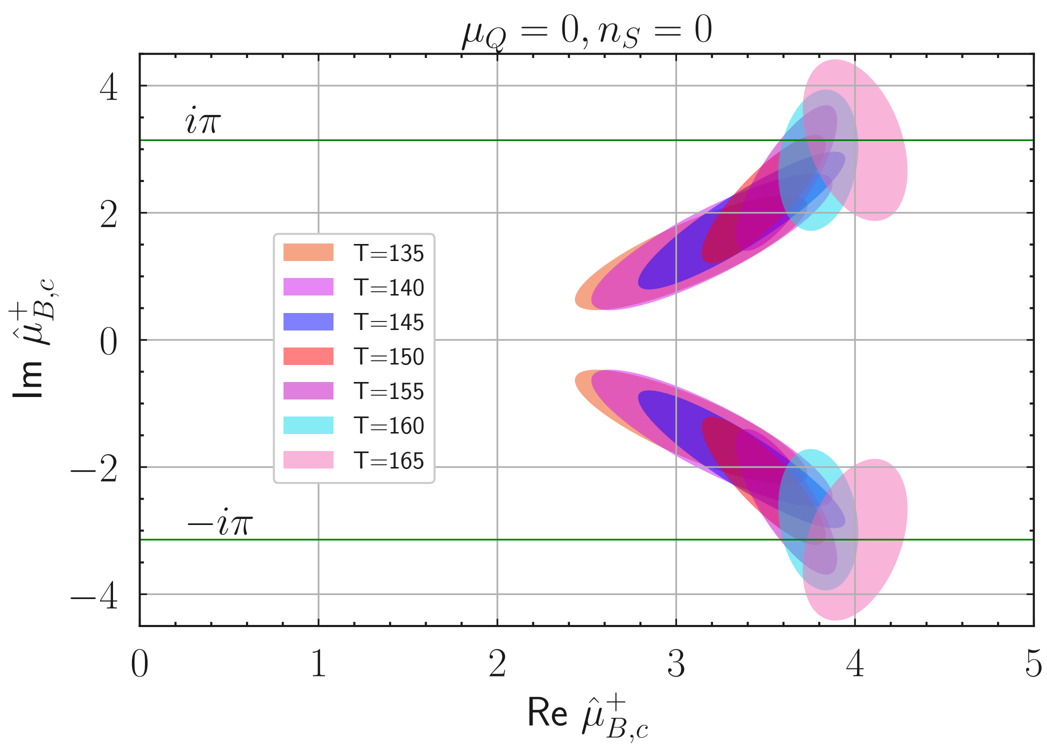
→ No critical point for real $\mu_B/T \lesssim \pi$



[Borsanyi et al., PRL 2020]

No sign of strengthening transition with imag. μ_B

Described by simple polynomial model in μ_B/T



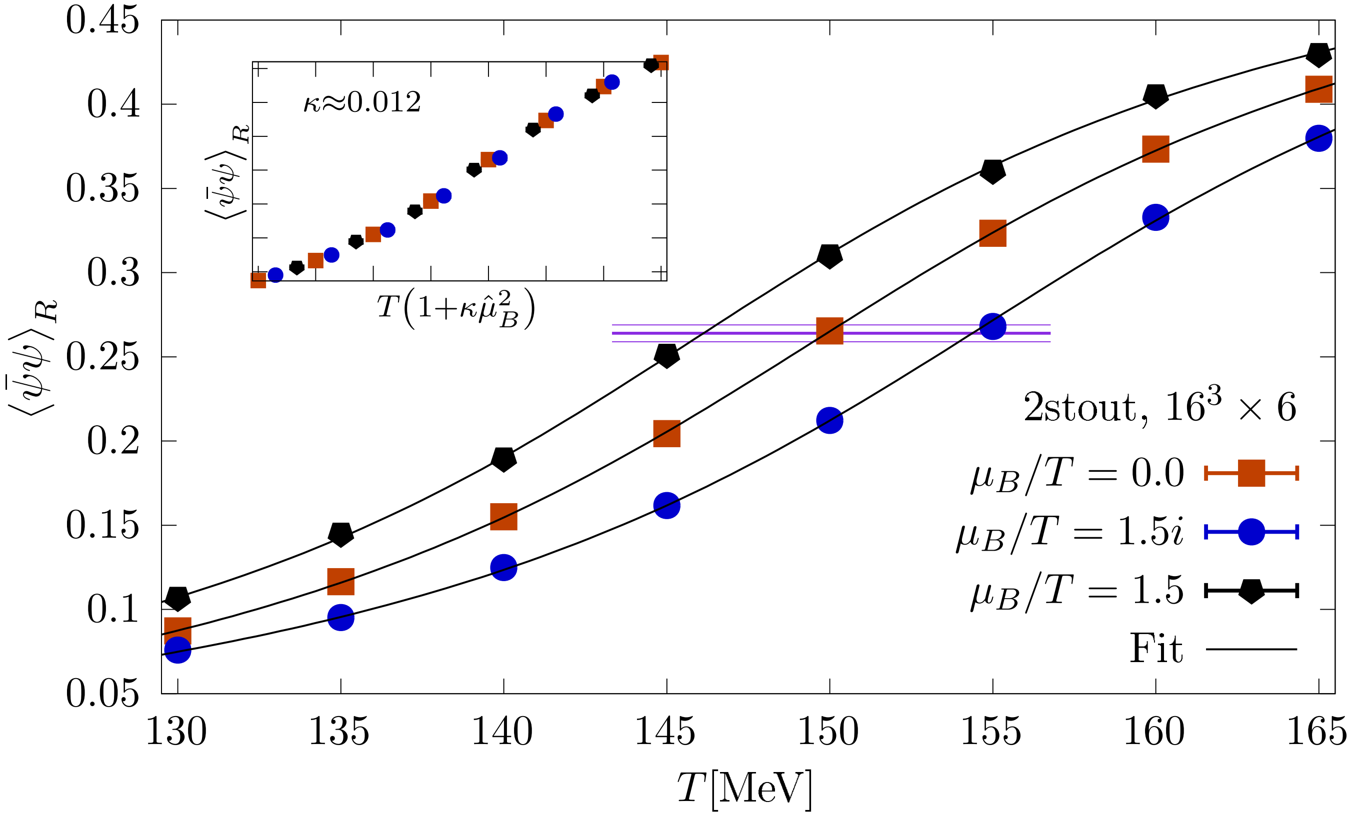
[HotQCD, PRD 2022]

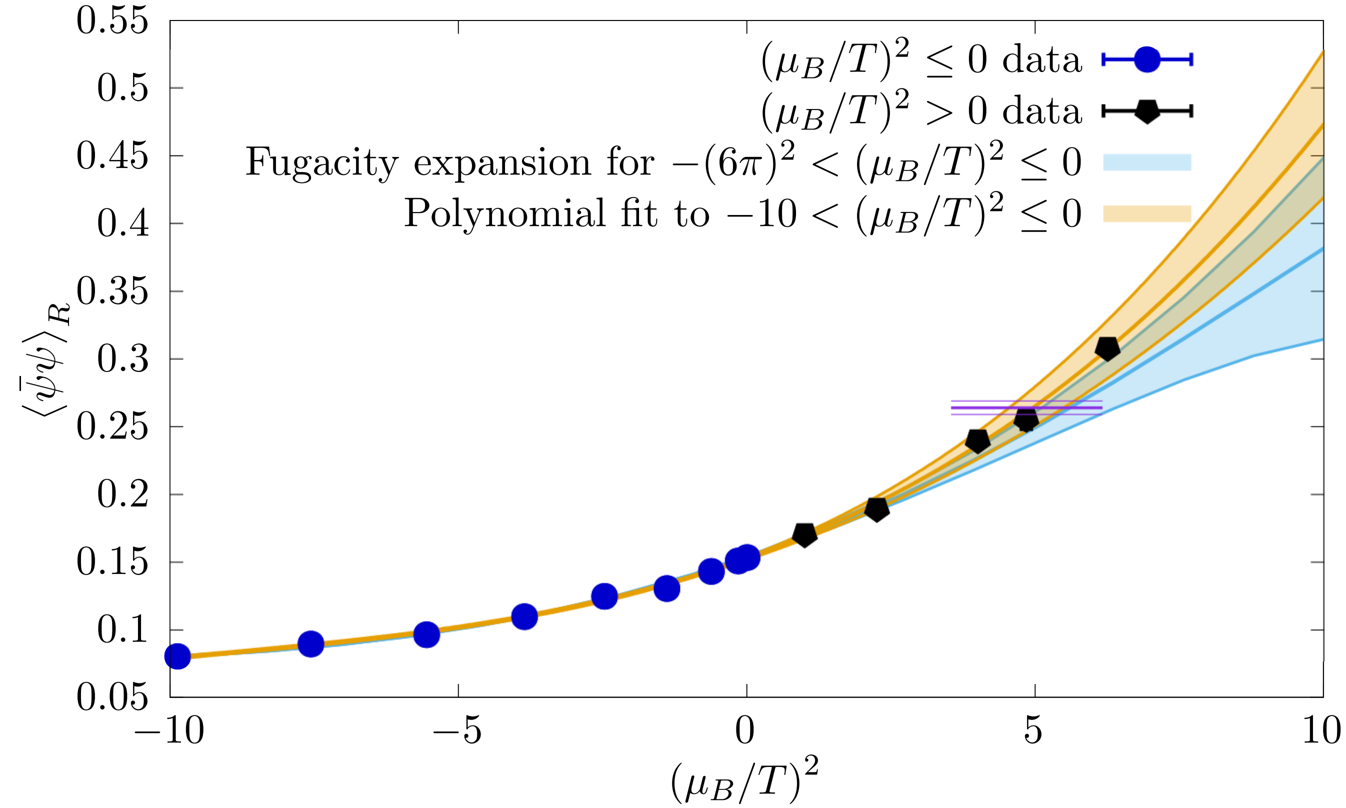
Radius of convergence by Mercer-Roberts

High order resummation in μ_B/T
by (multi-point) Pade approximants

$T_c < 125 MeV, \mu_B^c/T > 2.5T$

Critical endpoint: reweighting LQCD revisited





[Borsanyi et al., PRD 22]

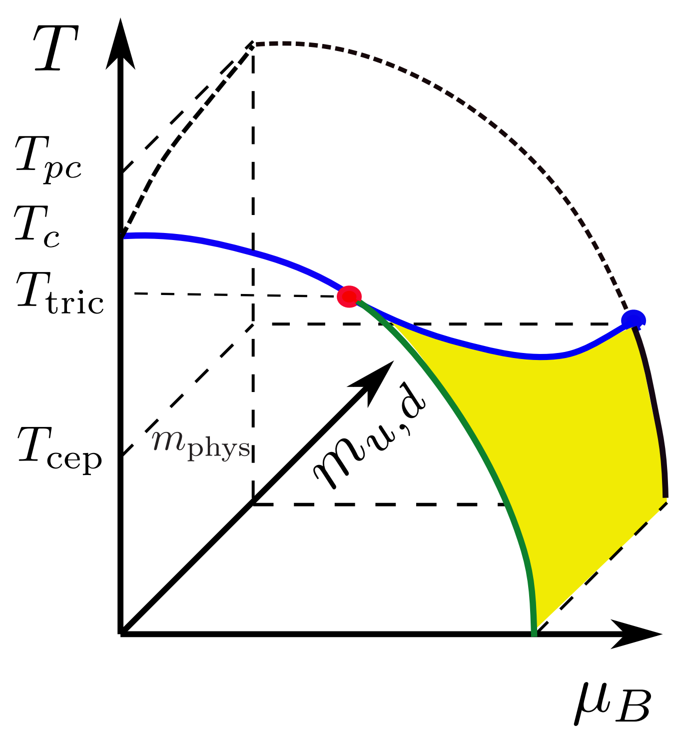
$$\langle\bar\psi\psi\rangle_R(T,\mu) = -\frac{m_{ud}}{f_\pi^4}\left[\langle\bar\psi\psi\rangle_{T,\mu} - \langle\bar\psi\psi\rangle_{0,0}\right]$$

- Fodor, Katz 2001 signal: coarse lattices, entanglement with rooted staggered artefacts [Giordano et al., PRD 20]
- New treatment: determinant of averaged taste quartets + reweighting in sign only [Giordano et al. JHEP 20]
- Simulation with stout-sm. staggered action, $N_\tau = 6$: no sign of criticality for $\mu_B < 2.5T$

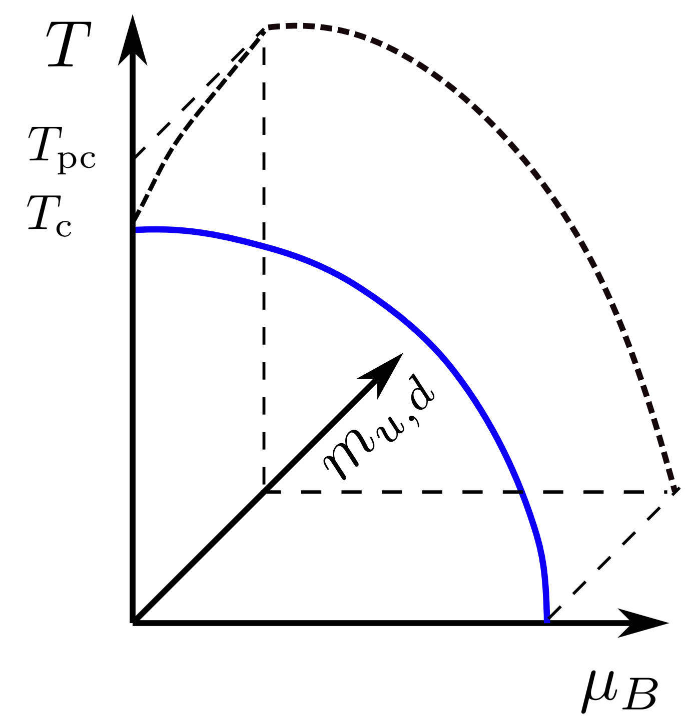
Connecting chiral limit and the physical point

The "standard scenario":

[Halasz et al., PRD 98; Hatta, Ikeda, PRD 03…]



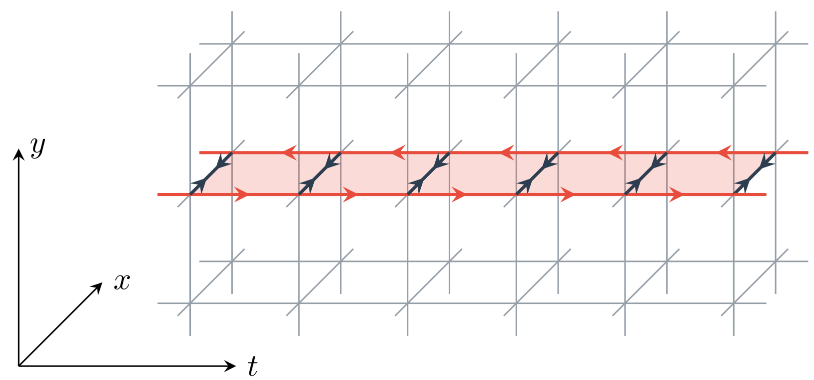
Not (yet?) ruled out by lattice data:



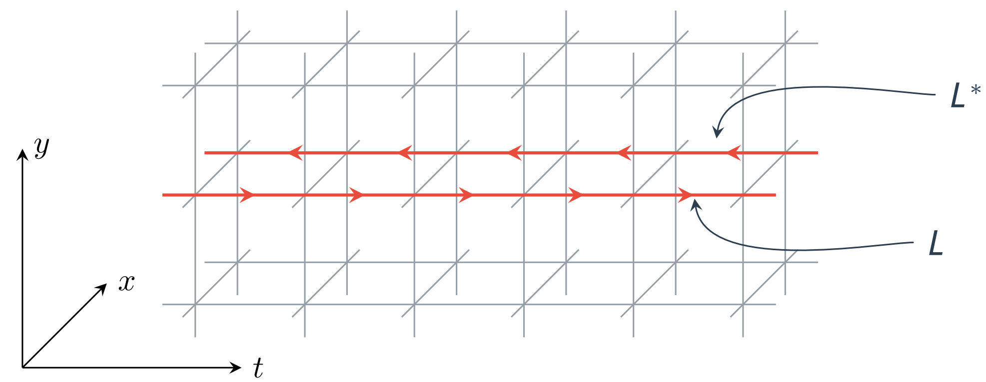
- Ordering of critical temperatures $\mu_B^{\rm cep} > 3.1\ T_{pc}(0) \approx 485\ \rm MeV$ [O.P. Symmetry 21]
- Cluster expansion model of lattice fluctuations $\mu_B^{\rm cep} > \pi T$ [Vovchenko et al. PRD 18]
- Singularities, Pade-approx. fluctuations $\mu_B^{\rm cep} > 2.5T, T < 125\ \rm MeV$ [HotQCD 21]
- Direct simulations with refined reweighting $\mu_B^{\rm cep} > 2.5T$ [Wuppertal-Budpest collaboration, 21]
- Consistent with DSE, fRG [Fischer PPNP 19; Fu, Pawlowski, Rennecke PRD 20; Gao, Pawlowski 21]

Cold and dense regime: effective lattice theory

- General idea: two-step treatment
- I. Analytic derivation of effective theory from LQCD by expansion in $\frac{1}{g^2}, \frac{1}{m_q}$
- Part of d.o.f's integrated out, sign problem becomes milder, eff. spin model
- II. Simulate effective theory (flux rep. or reweighting) or solve analytically



Integrate over all spatial gauge links



What remains is an interaction between Polyakov Loops

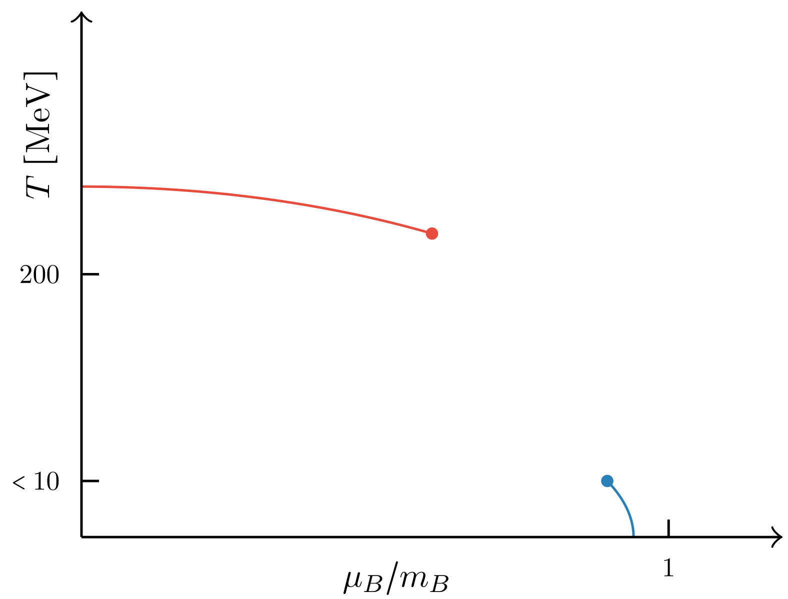
Pure gauge leading order: [Polonyi, Szachlanyi, 82; Svetitsky, Yaffe, 82]

Corrections + heavy Wilson fermions [Langelage, Lottini, O.P., JHEP 11; Fromm et al., JHEP 12]

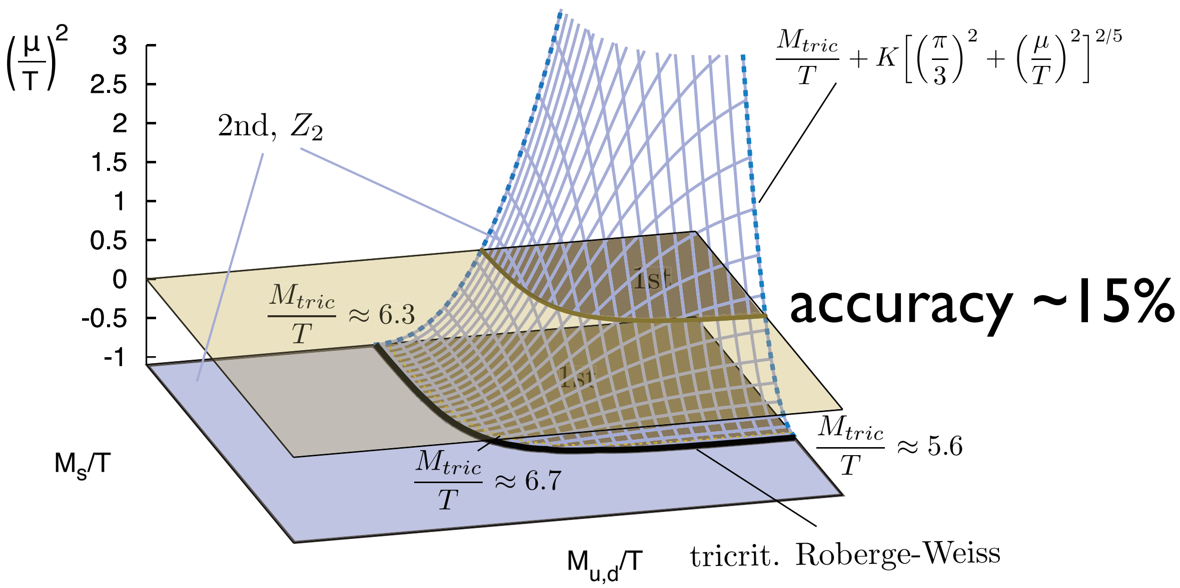
$$Z = \int DU_0 DU_i \, (\det Q)^{N_f} \; e^{S_g[U]} = \int DU_0 \; e^{S_{eff}[U_0]} = \int DL \; e^{S_{eff}[L]}$$

The phase diagram for heavy quarks, coarse lattices

Schematic phase diagram for heavy quarks

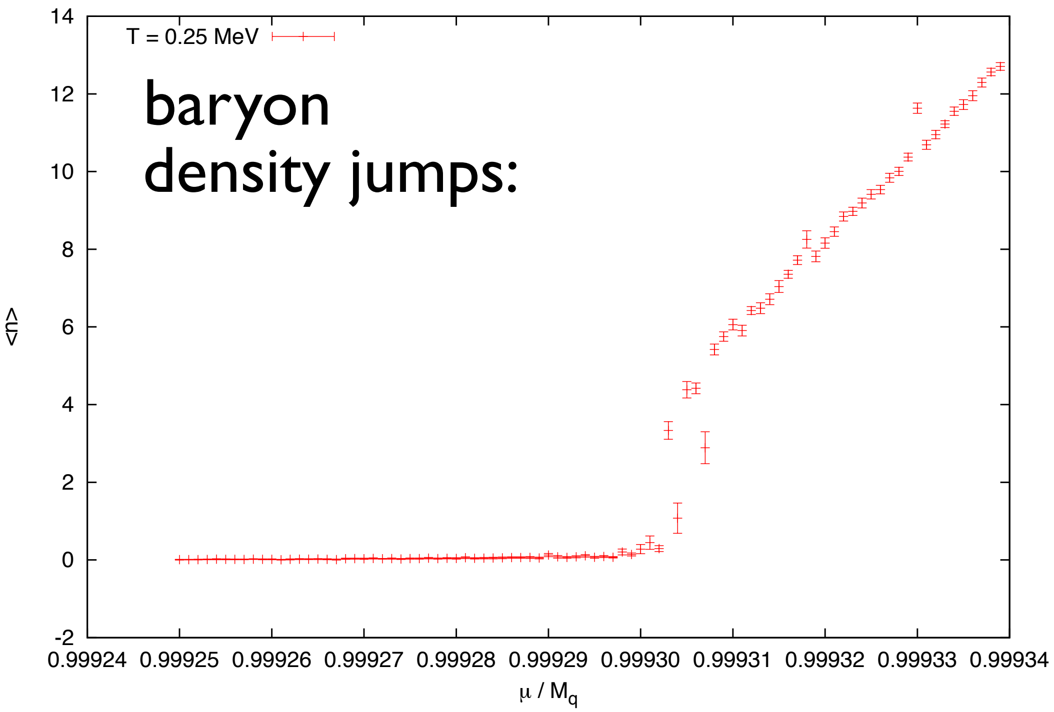


Upper right corner in Columbia plot $N_\tau = 4, 6$

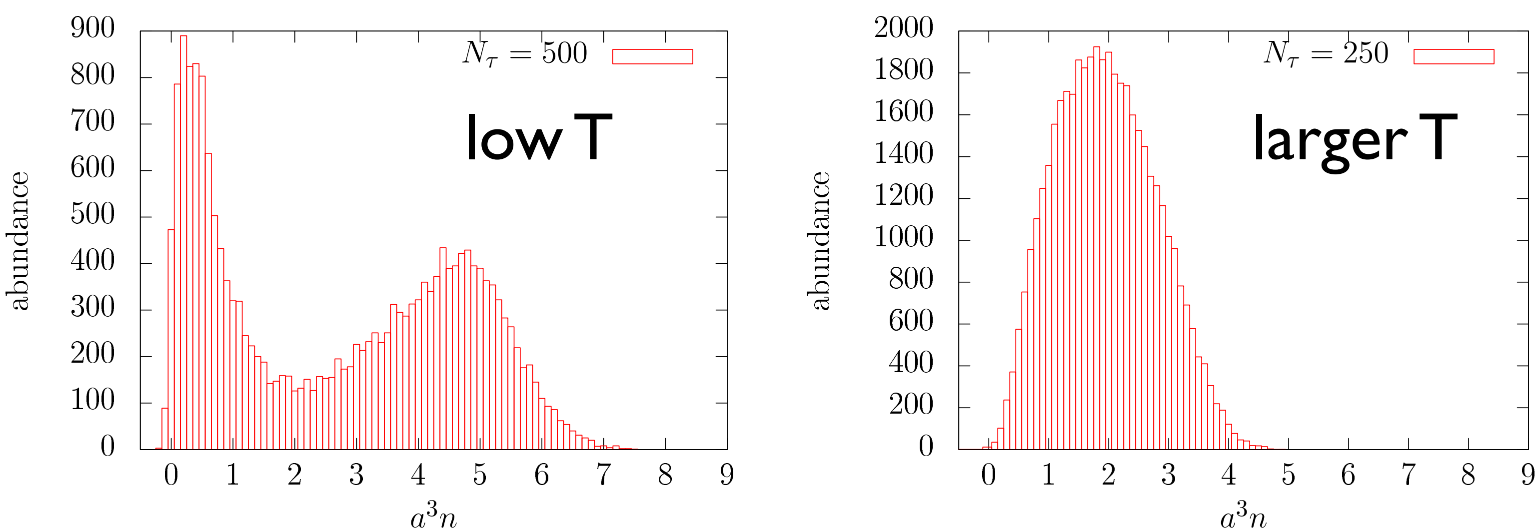


[Fromm et al., JHEP 12]

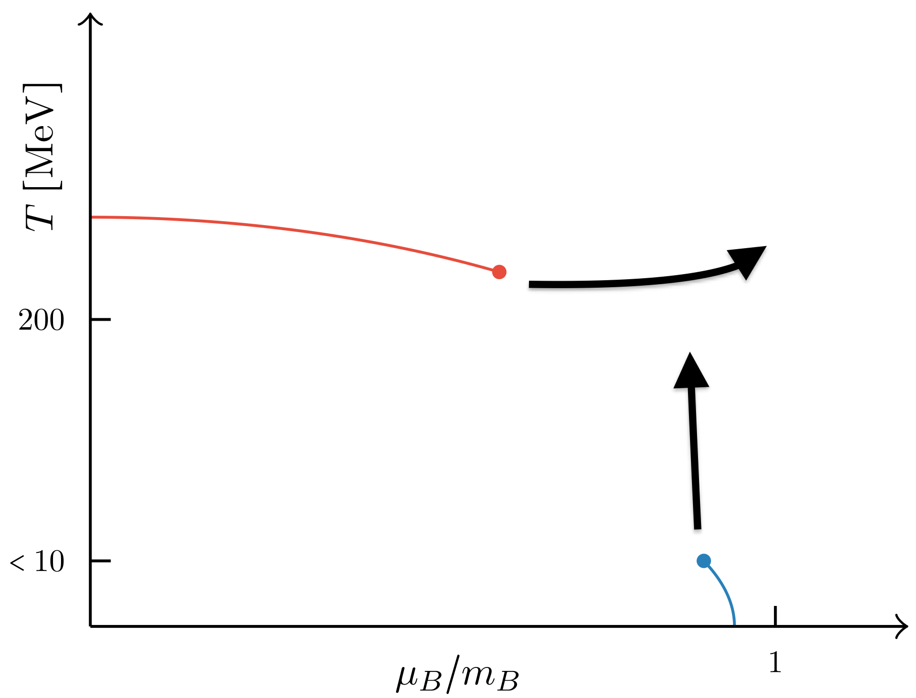
Onset transition to baryon matter (nucl. liquid gas):



[Langelage, Neuman, O.P. , JHEP 14]

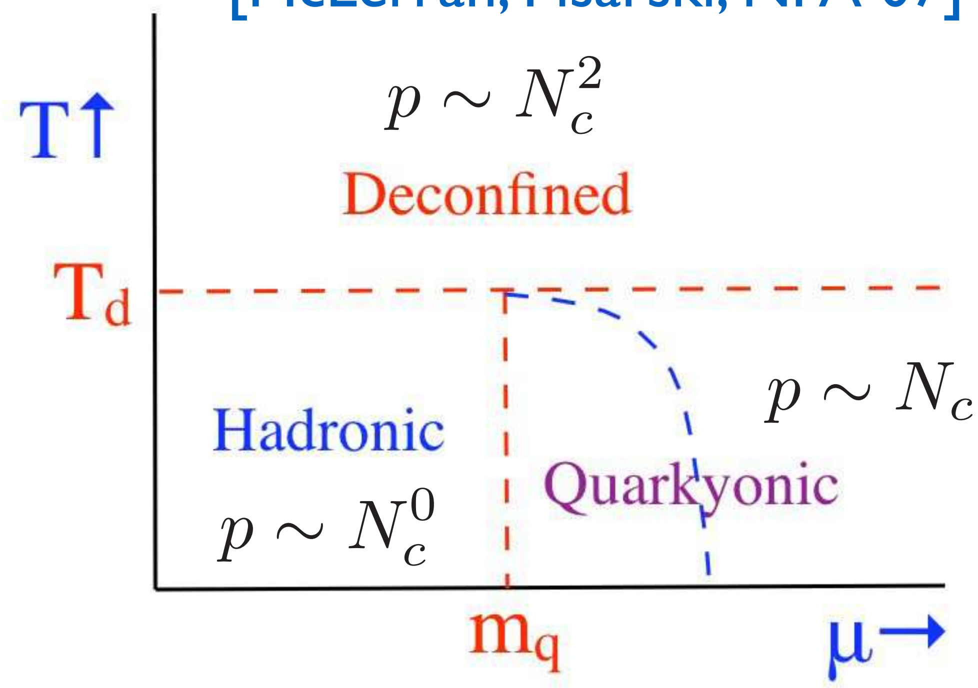


The heavy dense regime and large N_c



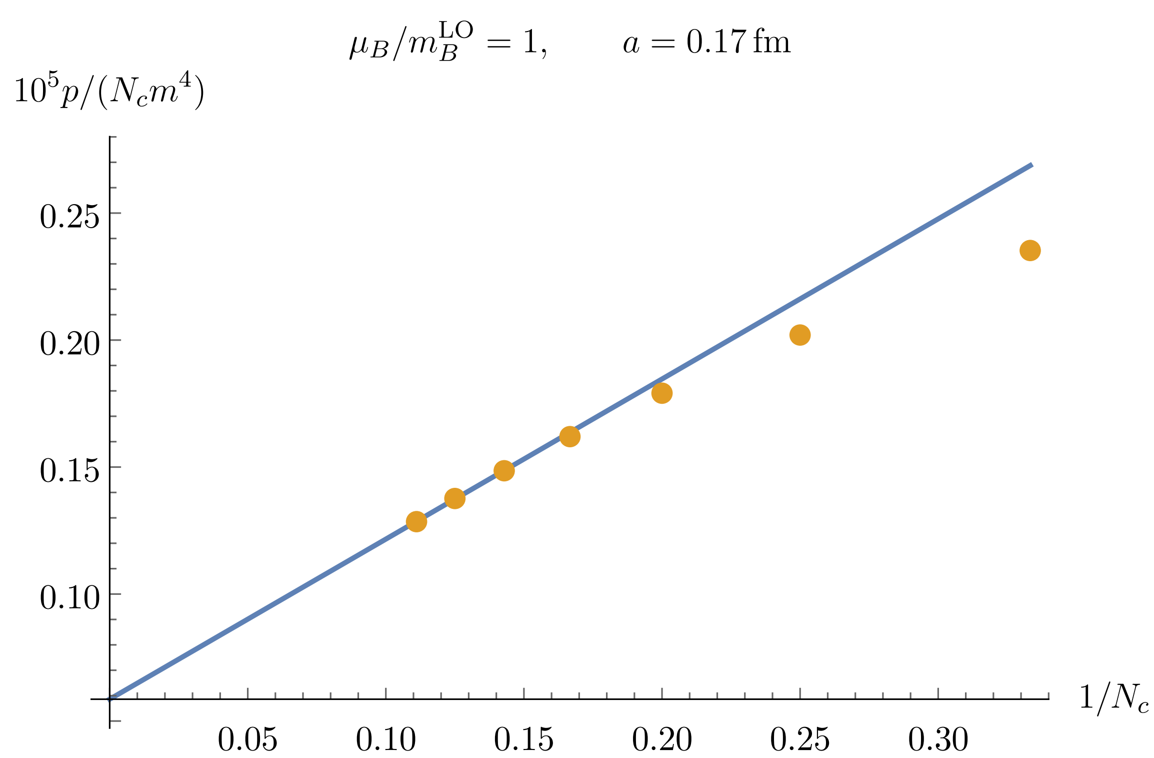
$\xrightarrow{N_c \to \infty}$

[McLerran, Pisarski, NPA 07]



[O.P. , Scheunert, JHEP 19]

- Investigate eff. th. for different N_c
- Large N_c phase diagram emerges continuously
- After baryon onset: $p \sim N_c$
  through three orders in hopping expansion $\frac{1}{m_q}$
- Consistent with quarkyonic matter!



Conclusions