FROM NONLINEAR STATISTICAL MECHANICS TO NONLINEAR QUANTUM MECHANICS: MANY-BODY SYSTEMS ET AL

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J.W. GIBBS

Elementary Principles in Statistical Mechanics - Developed with Especial Reference to the Rational Foundation of Thermodynamics

C. Scribner's Sons, New York, 1902; Yale University Press, New Haven, (1981), page 35

In treating of the canonical distribution, we shall always suppose the multiple integral in equation (92) [the partition function, as we call it nowadays] *to have a finite valued, as otherwise the coefficient of probability vanishes, and the law of distribution becomes illusory. This will exclude certain cases, but not such apparently, as will affect the value of our results with respect to their bearing on thermodynamics. It will exclude, for instance, cases in which the system or parts of it can be distributed in unlimited space […]. It also excludes many cases in which the energy can decrease without limit, as when the system contains material points which attract one another inversely as the squares of their distances. […]. For the purposes of a general discussion, it is sufficient to call attention to the assumption implicitly involved in the formula (92).*

Enrico FERMI *Thermodynamics* (Dover, 1936)

The entropy of a system composed of several parts is very often equal to the sum of the entropies of all the parts. This is true if the energy of the system is the sum of the energies of all the parts and if the work performed by the system during a transformation is equal to the sum of the amounts of work performed by all the parts. Notice that these conditions are not quite obvious and that in some cases they may not be fulfilled. Thus, for example, in the case of a system composed of two homogeneous substances, it will be possible to express the energy as the sum of the energies of the two substances only if we can neglect the surface energy of the two substances where they are in contact. The surface energy can generally be neglected only if the two substances are not very finely subdivided; otherwise, it can play a considerable role.

POSTULATED ENTROPIC FUNCTIONALS

	$p_i = \frac{1}{W} \quad (\forall i)$ **equiprobability**	$\forall p_i \; (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG entropy (*q =1*)	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* (*q real*)	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production)

Composable

Topsoe-factorizable

Amari-Ohara-Matsuzoe conformally invariant geometry

Biro-Barnafoldi-Van thermostat universal independence

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

[C.T., J. Stat. Phys. **52**, 479 (1988)]

DEFINITIONS: *q-logarithm*: $\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0;\ \ln_1 x = \ln x)$

q-exponential: $e_q^x \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \quad (e_1^x = e^x)$

Hence, the entropies can be rewritten:

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k\ \ln W$	$k \sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k\ \ln_q W$	$k \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$

TYPICAL SIMPLE SYSTEMS:

e.g., $W(N) \propto \mu^N \;\; (\mu > 1)$

Short-range space-time correlations

Markovian processes (short memory), Additive noise

Strong chaos (positive maximal Lyapunov exponent), **Ergodic**, Riemannian geometry

Short-range many-body interactions, weakly quantum-entangled subsystems

Linear/homogeneous Fokker-Planck equations, Gausssians

→ Boltzmann-Gibbs entropy (additive)

→ Exponential dependences (Boltzmann-Gibbs weight, ...)

TYPICAL COMPLEX SYSTEMS:

e.g., $W(N) \propto N^\rho \;\; (\rho > 0)$

Long-range space-time correlations

Non-Markovian processes (long memory), Additive and multiplicative noises

Weak chaos (zero maximal Lyapunov exponent), **Nonergodic**, Multifractal geometry

Long-range many-body interactions, strongly quantum-entangled sybsystems

Nonlinear/inhomogeneous Fokker-Planck equations, *q*-Gaussians

→ Entropy S*q* (nonadditive)

→ *q*-exponential dependences (asymptotic power-laws)

ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B) = S(A) + S(B)$$

Therefore, since

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)\, S_q(A)\, S_q(B)\ ,$$

S_{BG} and S_q^{Renyi} $(\forall q)$ are additive, and S_q $(\forall q \neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + \ldots + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N.

An entropy is extensive if

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty\ ,\ i.e.,\ S(N) \propto N \quad (N \to \infty)$$

EXTENSIVITY OF THE ENTROPY $(N \to \infty)$

If $W(N) \sim \mu^N \quad (\mu > 1)$

$$\Rightarrow \; S_{BG}(N) = k_B \ln W(N) \;\propto N$$

If $W(N) \sim N^\rho \quad (\rho > 0)$

$$\Rightarrow \; S_q(N) \; = k_B \ln_q W(N) \propto [W(N)]^{1-q} \propto N^{\rho(1-q)}$$

$$\Rightarrow \; S_{q=1-1/\rho}(N) \;\propto N$$

If $W(N) \sim \nu^{N^\gamma} \quad (\nu > 1;\; 0 < \gamma < 1)$

$$\Rightarrow \; S_\delta(N) = k_B \left[\ln W(N)\right]^\delta \propto N^{\gamma\,\delta}$$

$$\Rightarrow \; S_{\delta=1/\gamma}(N) \;\propto N$$

PHYSICAL REVIEW E **78**, 021102 (2008)

Nonadditive entropy reconciles the area law in quantum systems with classical thermodynamics

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The Boltzmann–Gibbs–von Neumann entropy of a large part (of linear size L) of some (much larger) d-dimensional quantum systems follows the so-called area law (as for black holes), i.e., it is proportional to L^{d-1}. Here we show, for $d=1,2$, that the (nonadditive) entropy S_q satisfies, for a special value of $q \neq 1$, the classical thermodynamical prescription for the entropy to be extensive, i.e., $S_q \propto L^d$. Therefore, we reconcile with classical thermodynamics the area law widespread in quantum systems. Recently, a similar behavior was exhibited in mathematical models with scale-invariant correlations [C. Tsallis, M. Gell-Mann, and Y. Sato, Proc. Natl. Acad. Sci. U.S.A. **102** 15377 (2005)]. Finally, we find that the system critical features are marked by a maximum of the special entropic index q.

SPIN ½ XY FERROMAGNET WITH TRANSVERSE MAGNETIC FIELD:

$$\hat{\mathcal{H}} = -\sum_{j=1}^{N-1} \left[(1+\gamma)\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + (1-\gamma)\hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + 2\lambda \hat{\sigma}_j^z \right]$$

$|\gamma| = 1 \quad \rightarrow \quad$ *Ising ferromagnet*

$0 < |\gamma| < 1 \quad \rightarrow \quad$ *anisotropic XY ferromagnet*

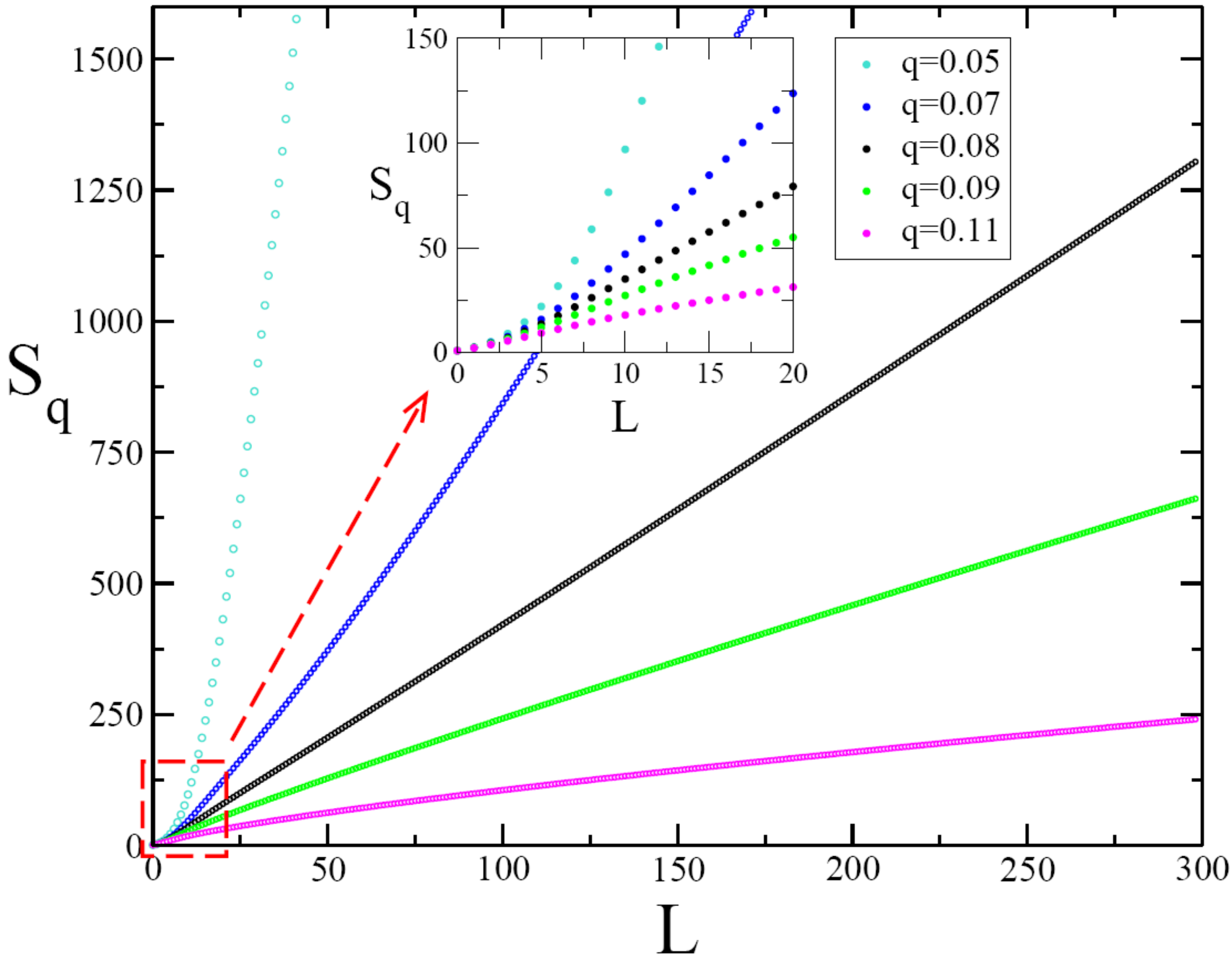
$\gamma = 0 \quad \rightarrow \quad$ *isotropic XY ferromagnet*

$\lambda \equiv$ *transverse magnetic field*

$L \equiv$ *length of a block within a* $N \rightarrow \infty$ *chain*

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

ISING MODEL



F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

Using a Quantum Field Theory result

*in P. Calabrese and J. Cardy, JSTAT P*06002 (2004)

we obtain, at the critical transverse magnetic field,

$$q_{ent} = \frac{\sqrt{9+c^2}-3}{c}$$

with $c \equiv$ central charge in conformal field theory

Hence

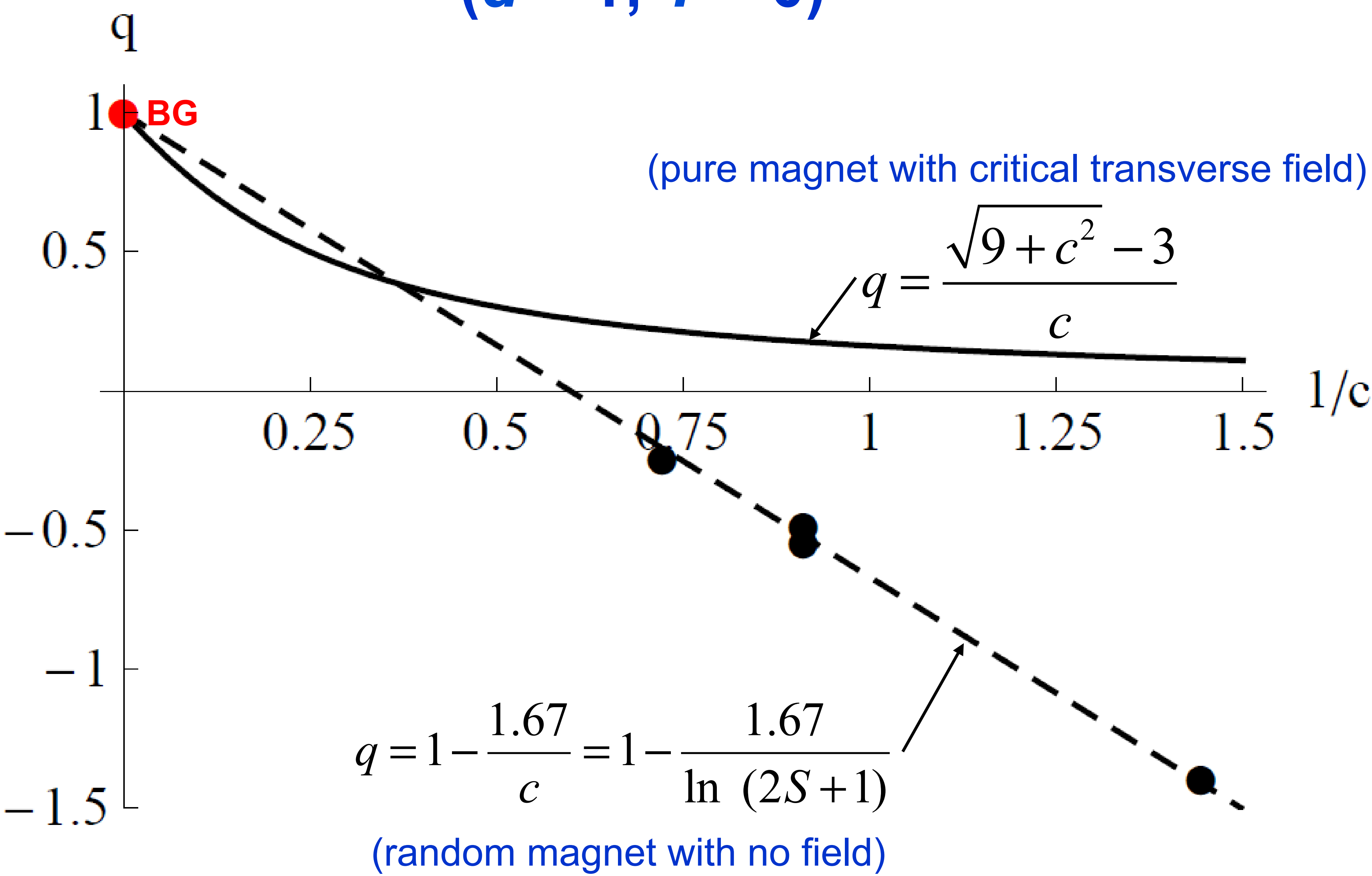
Ising and anisotropic XY ferromagnets $\Rightarrow \ c = \frac{1}{2} \ \Rightarrow \ q_{ent} = \sqrt{37}-6 \ \approx 0.0828$

and

Isotropic XY ferromagnet $\Rightarrow \ c = 1 \ \Rightarrow \ q_{ent} = \sqrt{10}-3 \ \approx 0.1623$

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

($d = 1$; $T = 0$)



A Saguia and MS Sarandy, Phys Lett A **374**, 3384 (2010)

SYSTEMS	ENTROPY S_{BG} (additive)	ENTROPY S_q (q<1) (nonadditive)
Short-range interactions, weakly entangled blocks, etc	EXTENSIVE	NONEXTENSIVE
Long-range interactions (QSS), strongly entangled blocks, etc	NONEXTENSIVE	EXTENSIVE

quarks-gluons, plasma, curved space ...?

PHYSICAL REVIEW E **84**, 021121 (2011)

Group entropies, correlation laws, and zeta functions

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(Received 15 February 2011; revised manuscript received 3 May 2011; published 10 August 2011)

The notion of group entropy is proposed. It enables the unification and generaliztion of many different definitions of entropy known in the literature, such as those of Boltzmann-Gibbs, Tsallis, Abe, and Kaniadakis. Other entropic functionals are introduced, related to nontrivial correlation laws characterizing universality classes of systems out of equilibrium when the dynamics is weakly chaotic. The associated thermostatistics are discussed. The mathematical structure underlying our construction is that of formal group theory, which provides the general structure of the correlations among particles and dictates the associated entropic functionals. As an example of application, the role of group entropies in information theory is illustrated and generalizations of the Kullback-Leibler divergence are proposed. A new connection between statistical mechanics and zeta functions is established. In particular, Tsallis entropy is related to the classical Riemann zeta function.

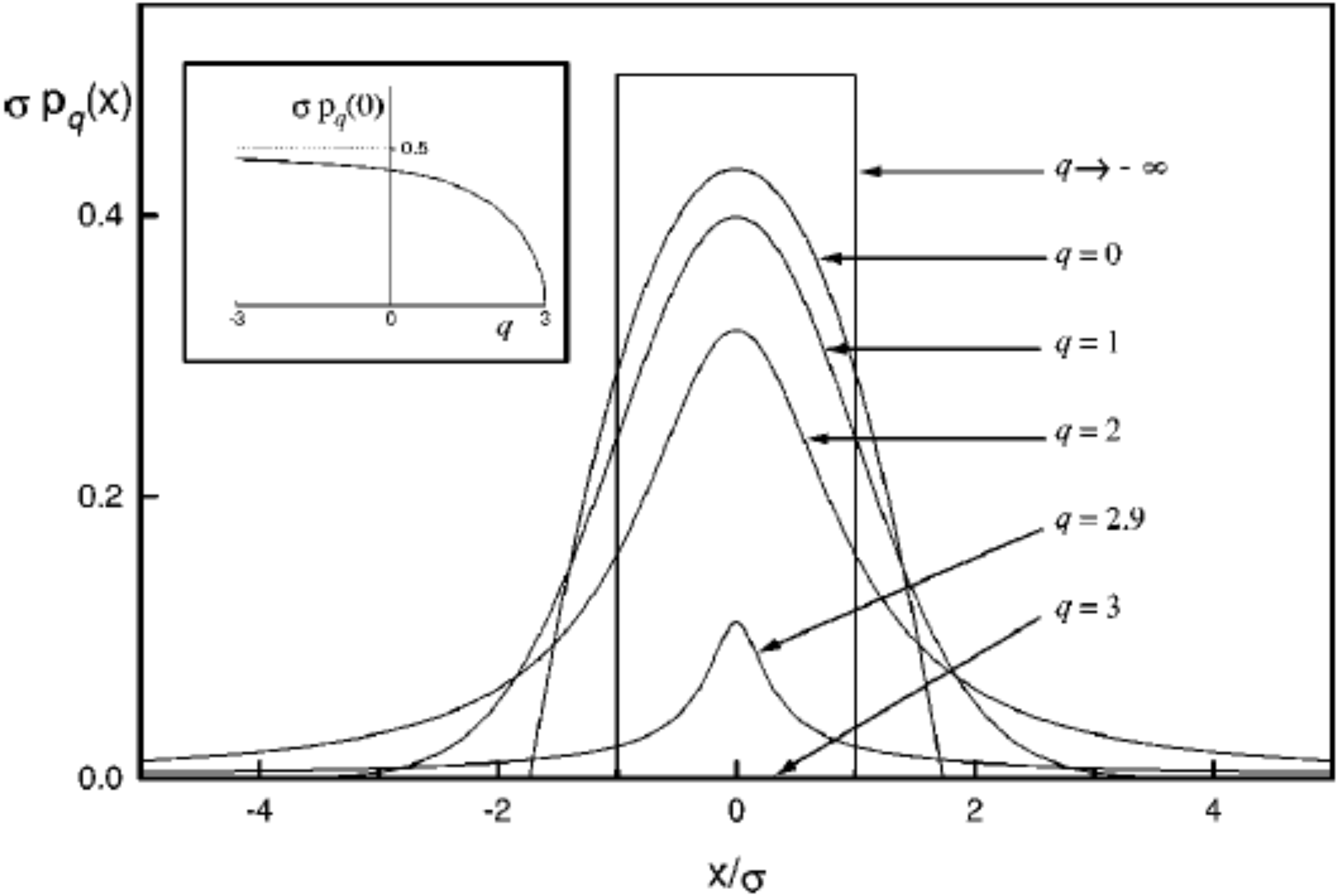
$$S_q \leftrightarrow \frac{1}{(1-q)^{s-1}}\varsigma(s) \quad (q<1)$$

$$\text{with } \varsigma(s) \equiv \sum_{n=1}^{\infty}\frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1-p^{-s}}$$

$$= \frac{1}{1-2^{-s}}\frac{1}{1-3^{-s}}\frac{1}{1-5^{-s}}\frac{1}{1-7^{-s}}\frac{1}{1-11^{-s}}\cdots$$

q-GAUSSIANS:

$$p_q(x) \propto e_q^{-(x/\sigma)^2} \equiv \frac{1}{\left[1+(q-1)\,(x/\sigma)^2\right]^{\frac{1}{q-1}}} \qquad (q<3)$$



D. Prato and C. T., Phys Rev E **60**, 2398 (1999)

Milan j. math. 76 (2008), 307–328
© 2008 Birkhäuser Verlag Basel/Switzerland
1424-9286/010307-22, *published online* 14.3.2008
DOI 10.1007/s00032-008-0087-y

Milan Journal of Mathematics

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for $q>1$

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See also:

H.J. Hilhorst, JSTAT P10023 (2010)

M. Jauregui and C. T., Phys Lett A **375**, 2085 (2011)

M. Jauregui, C. T. and E.M.F. Curado, JSTAT P10016 (2011)

A. Plastino and M.C. Rocca, Physica A and Milan J Math (2012)

CENTRAL LIMIT THEOREM

$N^{1/[\alpha(2-q)]}$ - *scaled attractor* $F(x)$ *when summing* $N \to \infty$ q-*independent identical random variables*

with symmetric distribution $f(x)$ *with* $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q$ $\left(Q \equiv 2q-1,\ q_1 = \frac{1+q}{3-q} \right)$

	$q=1$ [*independent*]	$q \neq 1$ (*i.e.*, $Q \equiv 2q-1 \neq 1$) [*globally correlated*]
$\sigma_Q < \infty$ $(\alpha = 2)$	$F(x) =$ *Gaussian* $G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$F(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \sim \begin{cases} G(x) & \text{if } \lvert x \rvert \ll x_c(q,2) \\ f(x) \sim C_q / \lvert x \rvert^{2/(q-1)} & \text{if } \lvert x \rvert \gg x_c(q,2) \end{cases}$ *with* $\lim_{q \to 1} x_c(q,2) = \infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q \to \infty$ $(0 < \alpha < 2)$	$F(x) =$ *Levy distribution* $L_\alpha(x)$, *with same* $\lvert x \rvert \to \infty$ *behavior* $L_\alpha(x) \sim \begin{cases} G(x) \\ \quad \text{if } \lvert x \rvert \ll x_c(1,\alpha) \\ f(x) \sim C_\alpha / \lvert x \rvert^{1+\alpha} \\ \quad \text{if } \lvert x \rvert \gg x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha \to 2} x_c(1,\alpha) = \infty$ Levy-Gnedenko CLT	$F(x) = L_{q,\alpha}$, *with same* $\lvert x \rvert \to \infty$ *asymptotic behavior* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\ \alpha}(x) \sim C^*_{q,\alpha} / \lvert x \rvert^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} \\ \quad (\textit{intermediate regime}) \\ G_{\frac{2\alpha q-\alpha+3}{\alpha+1},\ 2}(x) \sim C^L_{q,\alpha} / \lvert x \rvert^{(1+\alpha)/(1+\alpha q-\alpha)} \\ \quad (\textit{distant regime}) \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg J Math Phys 51, 033502 (2010)

CORRELATIONS IN COUPLED LOGISTIC MAPS AT THE EDGE OF CHAOS IN THE PRESENCE OF GLOBAL NOISE

We consider a linear chain of N coupled maps with periodic boundary conditions in a noisy environment:

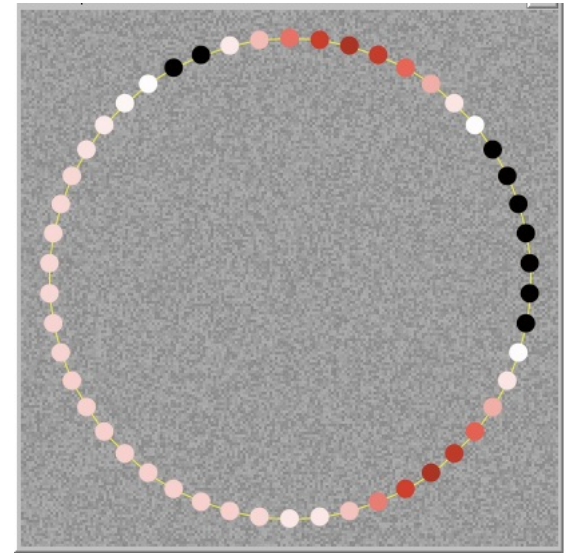
$$x_{t+1}^{i} = (1-\varepsilon) f(x_t^i) + \frac{\varepsilon}{2}[f(x_t^{i-1}) + f(x_t^{i+1})] + \sigma_t$$

$\sigma_t \in [0, \sigma_{max}]$ additive noise

with $\varepsilon \in [0,1]$ coupling strength

and $f(x_t^i) = 1 - \mu (x_t^i)^2 \quad \mu \in [0,2]$ *i*-th logistic map ($i = 1...N$)

[zero noise: N.B. Ouchi and K. Kaneko, Chaos **10**, 359 (2000)]



edge of chaos: $\mu = 1.4011551$

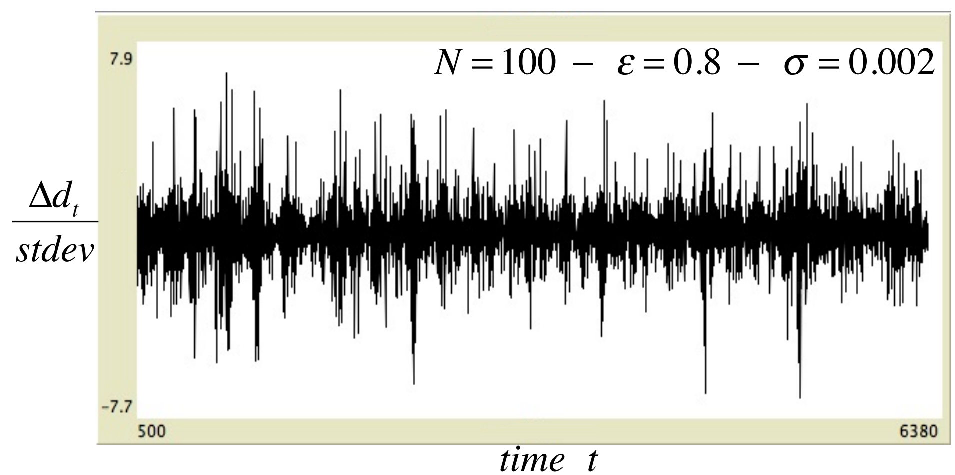
global parameter

$$d_t = \Sigma_{i=1}^{N} |x_t^i - < x_t^i > |$$

time returns

$$\Delta d_t = d_{t+\tau} - d_t$$

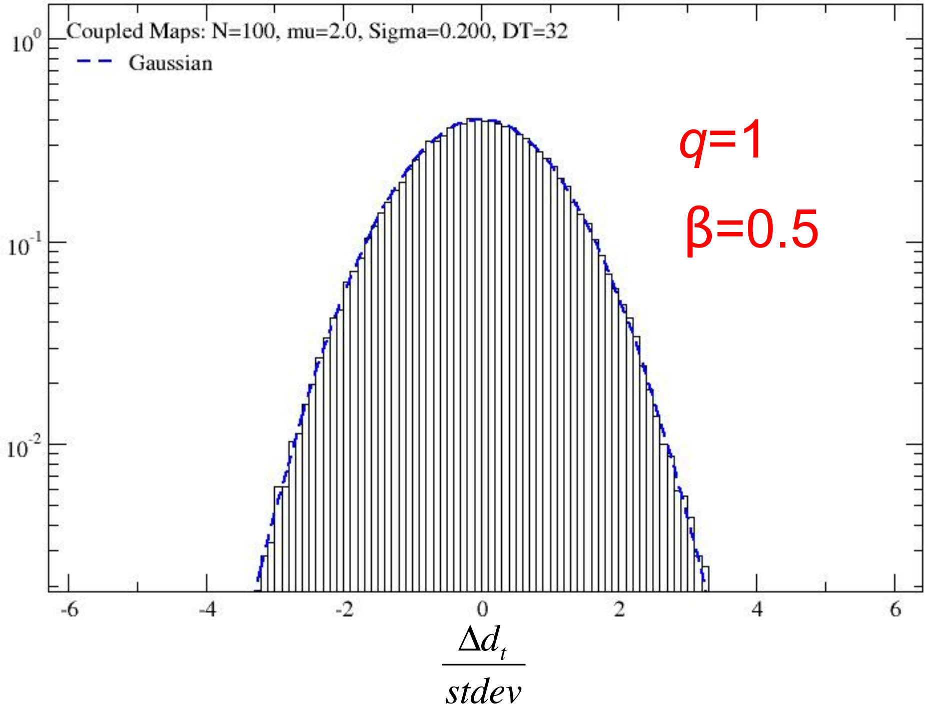
Intermittency in the normalized returns time-series



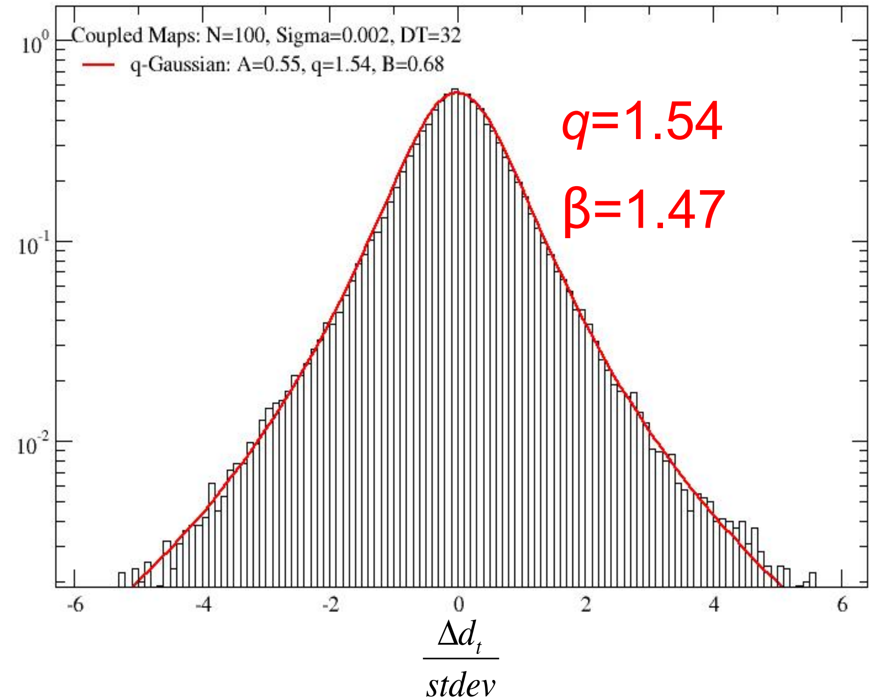
A. Pluchino, A. Rapisarda and C. T. (2012), 1206.2152 [cond-mat.stat-mech]

$$N = 100;\ \varepsilon = 0.8;\ \sigma_{\max} = 0.002;\ \tau = 32$$

Chaotic Regime: $\mu = 2.0$



Edge of chaos: $\mu = 1.4011551$



A. Pluchino, A. Rapisarda and C. T. (2012), 1206.2152 [cond-mat.stat-mech]

CONSERVATIVE MC MILLAN MAP:

$$x_{n+1} = y_n$$

$$y_{n+1} = -x_n + 2\mu \frac{y_n}{1 + y_n^2} + \varepsilon y_n$$

$\mu \neq 0 \Leftrightarrow$ nonlinear dynamics

G. Ruiz, T. Bountis and C. T.
Int J Bifurcat Chaos (2012), in press

$(\mu,\varepsilon)=(1.6,1.2)$ $(\lambda_{\max}\approx 0.05)$

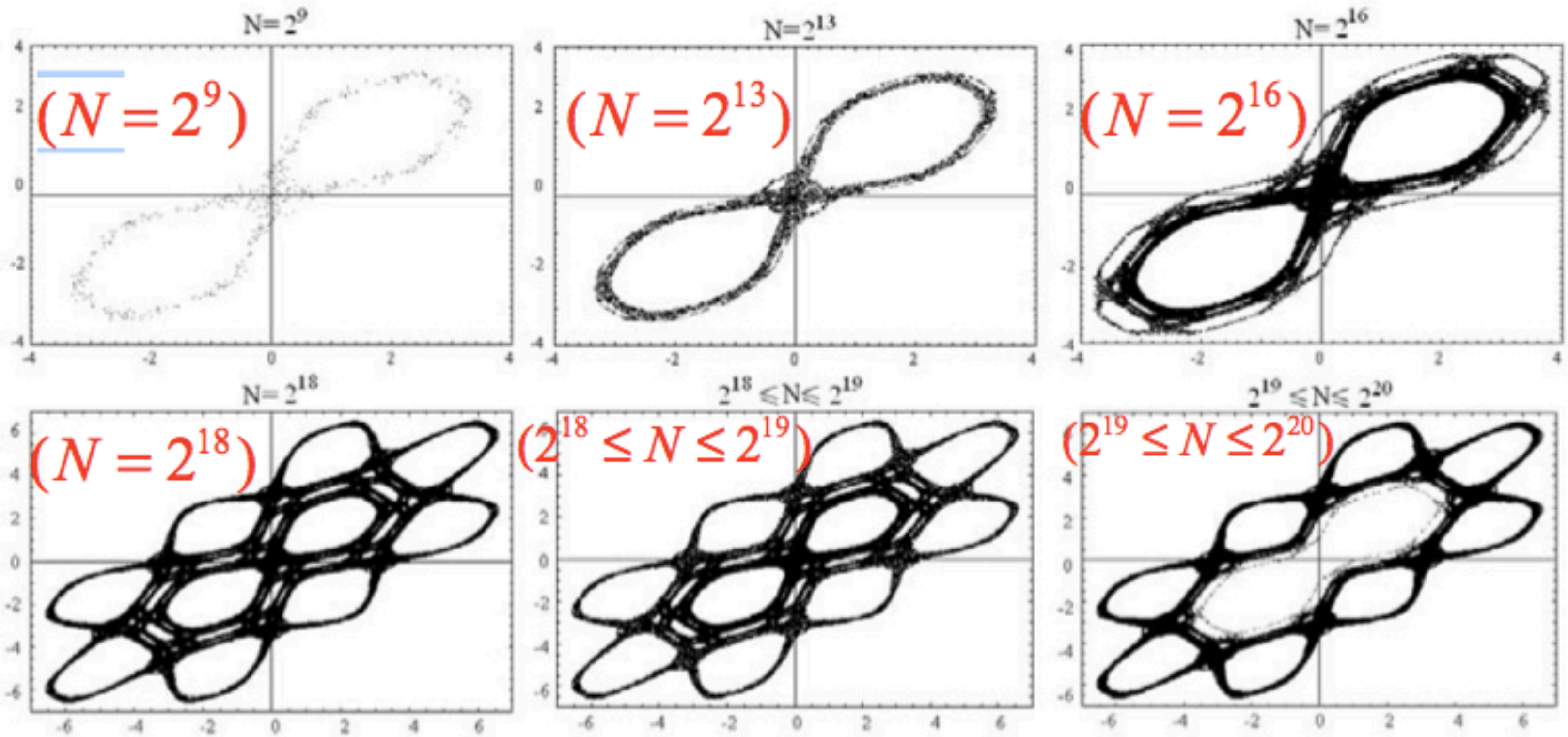
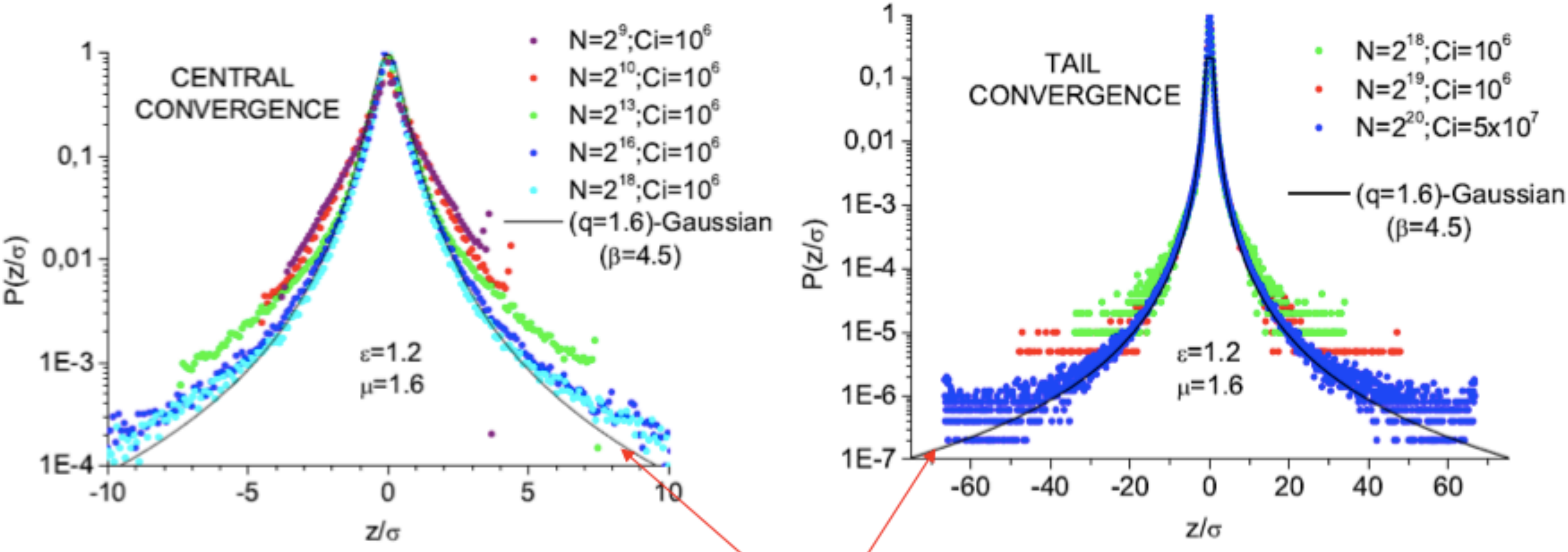


FIG. 10. Structure of phase space plot of Mc. Millan perturbed map for parameter values $\mu = 1.6$ and $\epsilon = 1.2$, starting form a randomly chosen initial condition in a square $(0, 10^{-6}) \times (0, 10^{-6})$, and for $i = 1 \ldots N$ $(N = 2^{10}, 2^{13}, N^{16}, N^{18})$ iterates.

G. Ruiz, T. Bountis and C. T.
Int J Bifurcat Chaos (2012), in press



$$p \propto e_q^{-\beta(z/\sigma)^2}$$

with $(q, \beta) = (1.6, 4.5)$

G. Ruiz, T. Bountis and C. T.
Int J Bifurcat Chaos (2012), in press

Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

Thermostatistics in the neighbourhood of the π-mode solution for the Fermi–Pasta–Ulam β system: from weak to strong chaos

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Abstract. We consider a π-mode solution of the Fermi–Pasta–Ulam β system. By perturbing it, we study the system as a function of the energy density from a regime where the solution is stable to a regime where it is unstable, first weakly and then strongly chaotic. We introduce, as an indicator of stochasticity, the ratio ρ (when it is defined) between the second and the first moment of a given probability distribution. We will show numerically that the transition between weak and strong chaos can be interpreted as the symmetry breaking of a set of suitable dynamical variables. Moreover, we show that in the region of weak chaos there is numerical evidence that the thermostatistic is governed by the Tsallis distribution.

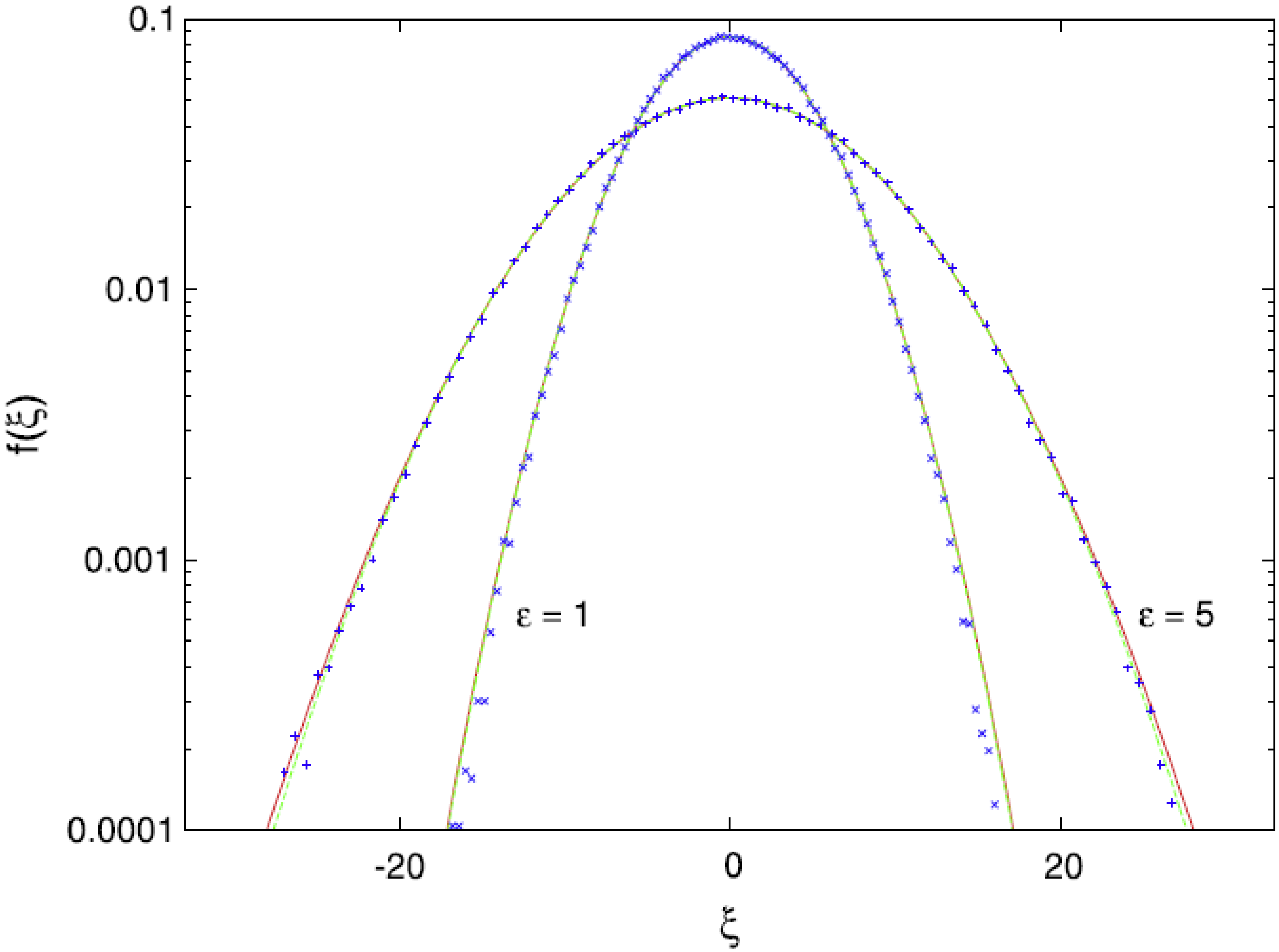

Figure 5. Plot on a linear-log scale of the numerical distribution $f(\xi)$ (blue points) fitted with a Tsallis distribution (red) and a Gauss distribution (green) for $N = 128$, $\epsilon = 1$ and 5. In both cases the Tsallis and Gaussian distributions essentially overlap.

M. Leo, R.A. Leo and P. Tempesta, J Stat Mech P04021 (2010)

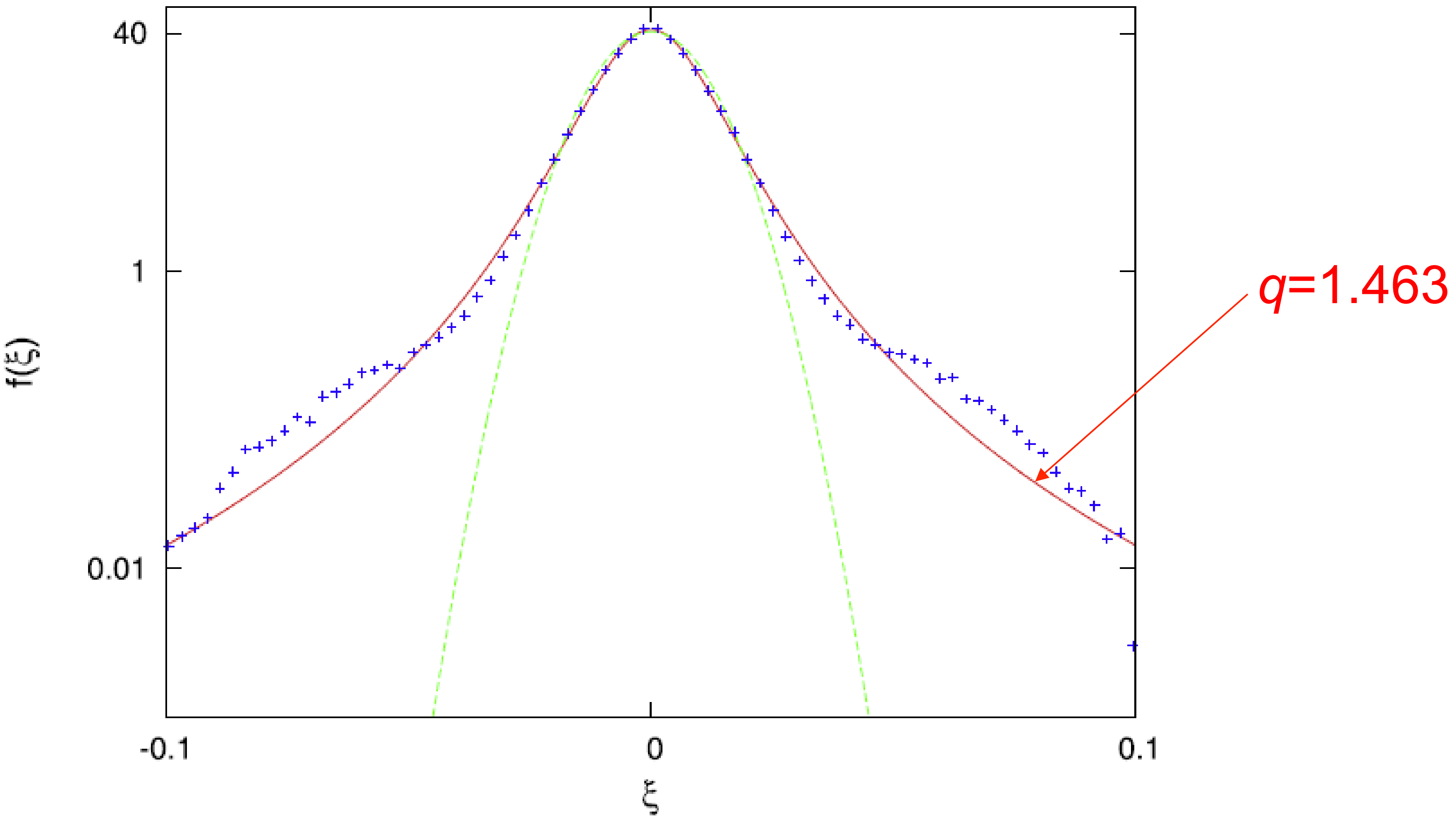

Figure 4. Plot on a linear-log scale of the numerical distribution $f(\xi)$ (blue points) fitted with a Tsallis distribution (red) and a Gauss distribution (green) for $N = 128$ and $\epsilon = 0.006$.

M. Leo, R.A. Leo and P. Tempesta, J Stat Mech P04021 (2010)

COLD ATOMS IN DISSIPATIVE OPTICAL LATTICES:

RAPID COMM

PHYSICAL REVIEW A **67**, 051402(R) (2003)

Anomalous diffusion and Tsallis statistics in an optical lattice

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(Received 26 February 2003; published 27 May 2003)

We point out a connection between anomalous transport in an optical lattice and Tsallis' generalized statistics. Specifically, we show that the momentum equation for the semiclassical Wigner function which describes atomic motion in the optical potential, belongs to a class of transport equations recently studied by Borland [Phys. Lett. A **245**, 67 (1998)]. The important property of these ordinary linear Fokker-Planck equations is that their stationary solutions are exactly given by Tsallis distributions. An analytical expression of the Tsallis index q in terms of the microscopic parameters of the quantum-optical problem is given and the spatial coherence of the atomic wave packets is discussed.

(i) The distribution of atomic velocities is a q-Gaussian;

(ii) $q = 1 + \frac{44 E_R}{U_0}$ where $E_R \equiv$ recoil energy

$U_0 \equiv$ potential depth

PRL **96,** 110601 (2006) PHYSICAL REVIEW LETTERS week ending 24 MARCH 2006

Tunable Tsallis Distributions in Dissipative Optical Lattices

P. Douglas, S. Bergamini, and F. Renzoni
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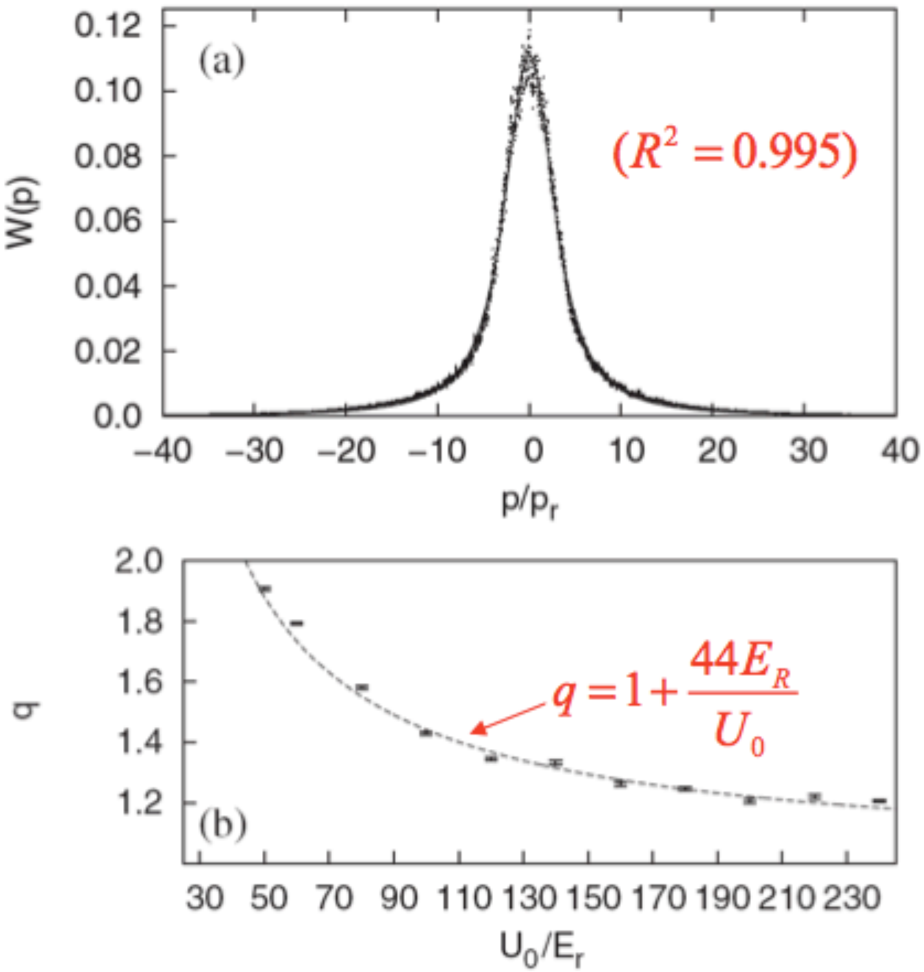
(Received 10 January 2006; published 24 March 2006)

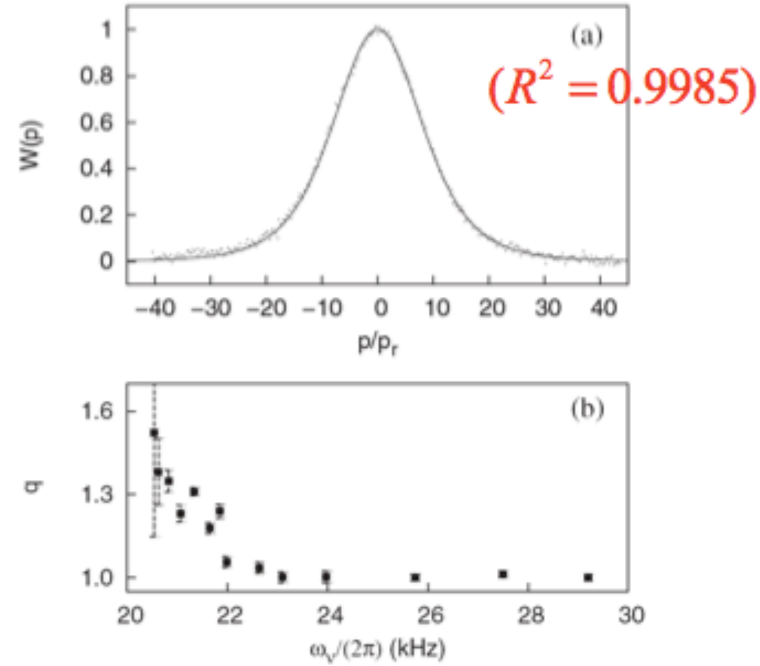
We demonstrated experimentally that the momentum distribution of cold atoms in dissipative optical lattices is a Tsallis distribution. The parameters of the distribution can be continuously varied by changing the parameters of the optical potential. In particular, by changing the depth of the optical lattice, it is possible to change the momentum distribution from Gaussian, at deep potentials, to a power-law tail distribution at shallow optical potentials.

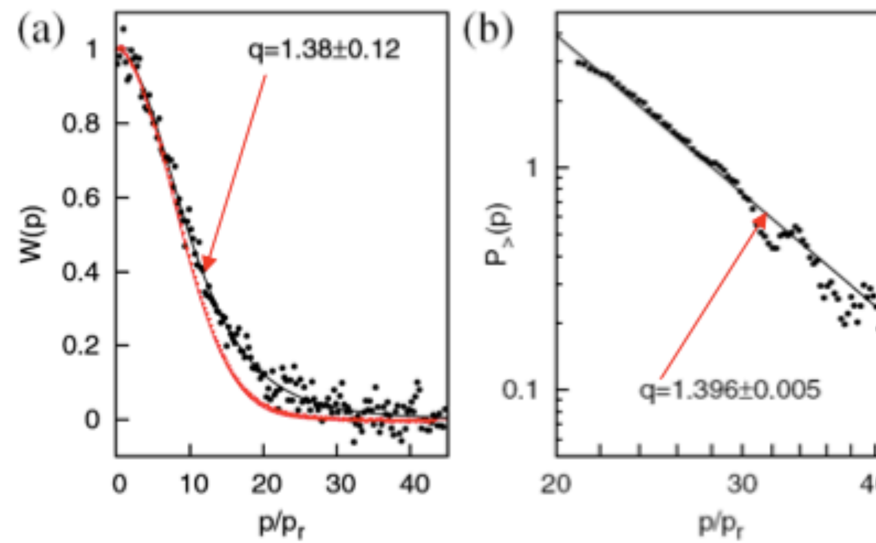
Experimental and computational verifications

by P. Douglas, S. Bergamini and F. Renzoni, Phys Rev Lett **96**, 110601 (2006)



(Computational verification: quantum Monte Carlo simulations)





(Experimental verification: Cs atoms)

PRL **105,** 260601 (2010) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2010

Thermostatistics of Overdamped Motion of Interacting Particles

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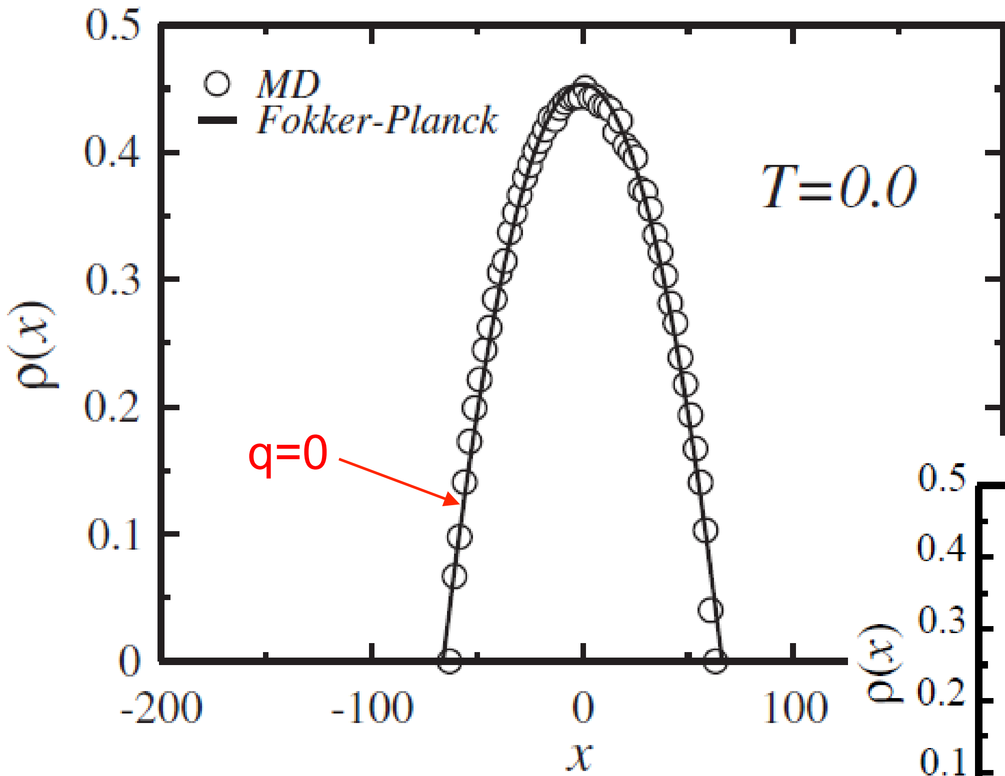
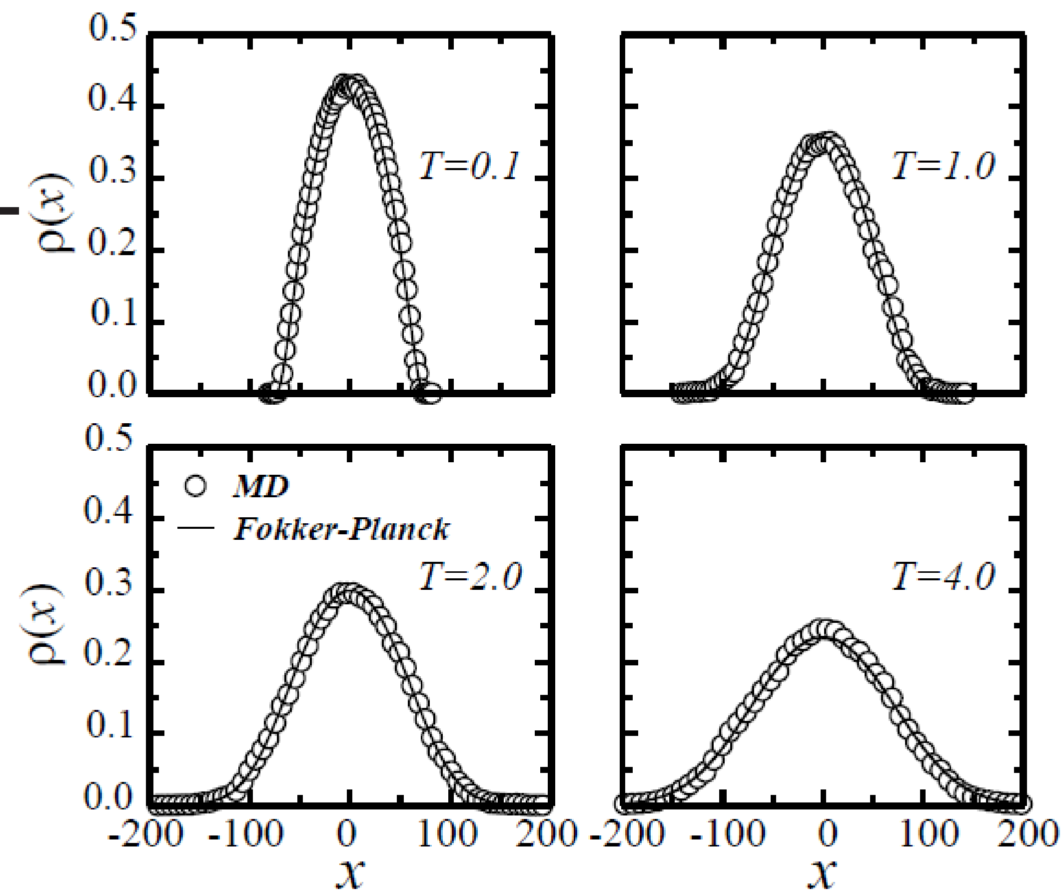
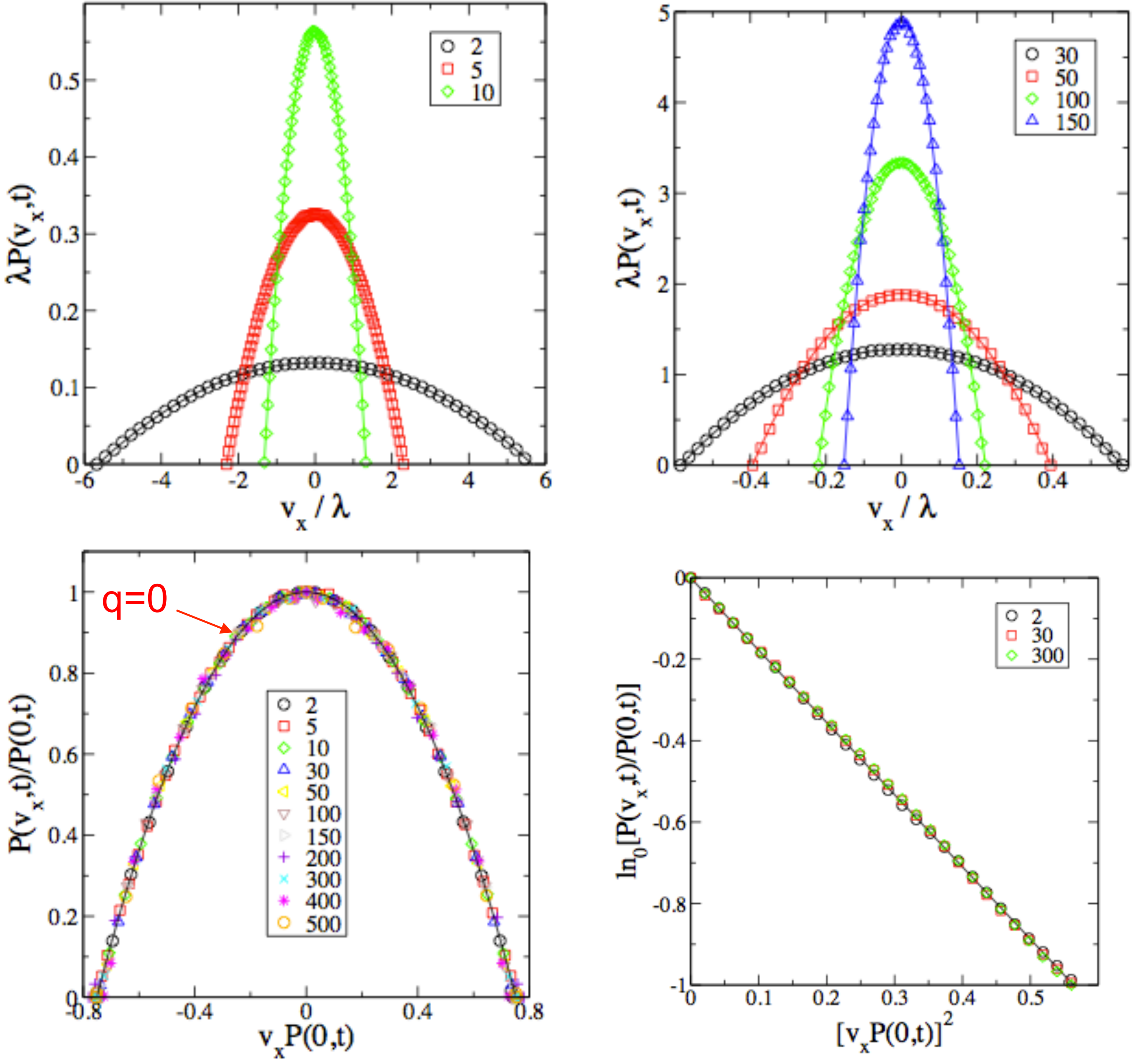
We show through a nonlinear Fokker-Planck formalism, and confirm by molecular dynamics simulations, that the overdamped motion of interacting particles at $T = 0$, where T is the temperature of a thermal bath connected to the system, can be directly associated with Tsallis thermostatistics. For sufficiently high values of T, the distribution of particles becomes Gaussian, so that the classical Boltzmann-Gibbs behavior is recovered. For intermediate temperatures of the thermal bath, the system displays a mixed behavior that follows a novel type of thermostatistics, where the entropy is given by a linear combination of Tsallis and Boltzmann-Gibbs entropies.





See also:
Levin and Pakter, PRL **107**, 088901 (2011)
Andrade, Silva, Moreira, Nobre and Curado, PRL **107**, 088902 (2011)

Andrade, Silva, Moreira, Nobre and Curado, Phys Rev Lett **105**, 260601 (2010)



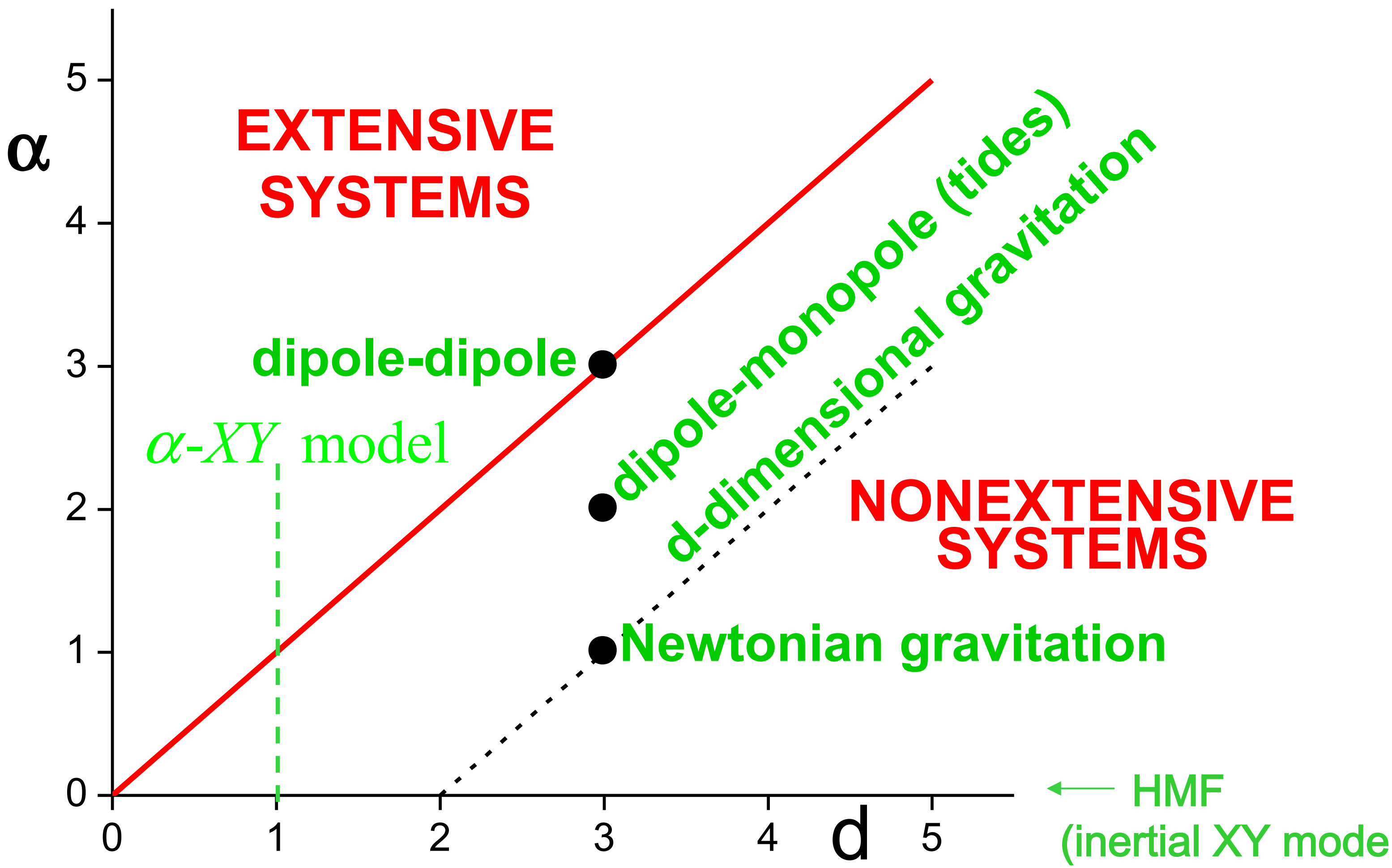
MS Ribeiro, FD Nobre and EMF Curado, PRE **85**, 121046 (2012)

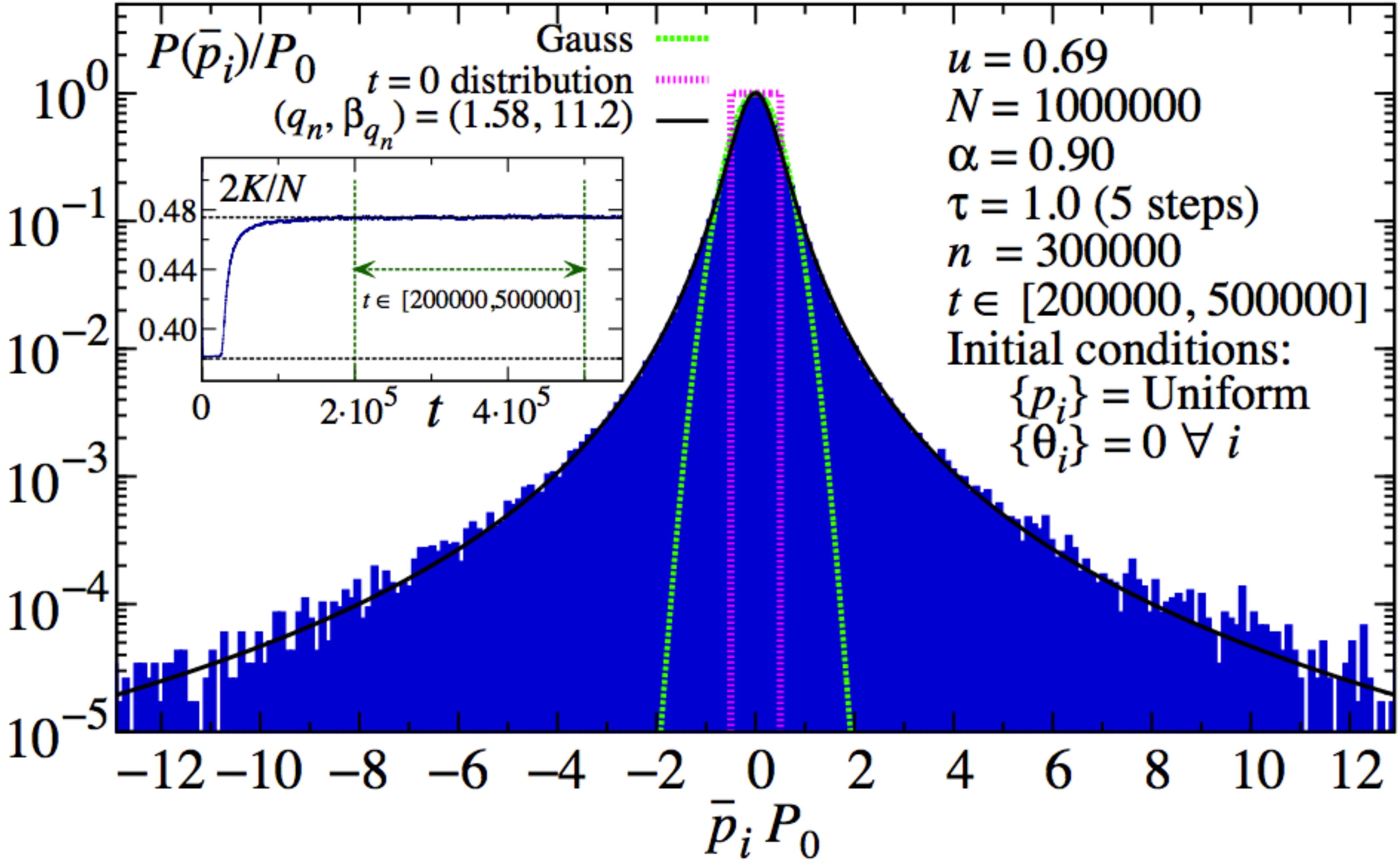
CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

$$V(r) \sim -\frac{A}{r^{\alpha}} \qquad (r \to \infty) \qquad (A > 0,\ \ \alpha \geq 0)$$

$$\textit{integrable if} \quad \alpha / d > 1 \qquad (\textit{short-ranged})$$

$$\textit{non-integrable if} \quad 0 \leq \alpha / d \leq 1 \qquad (\textit{long-ranged})$$





L.J.L. Cirto, V.R.V. Assis and C. T. (2012),1206.6133 [cond-mat.stat-mech]

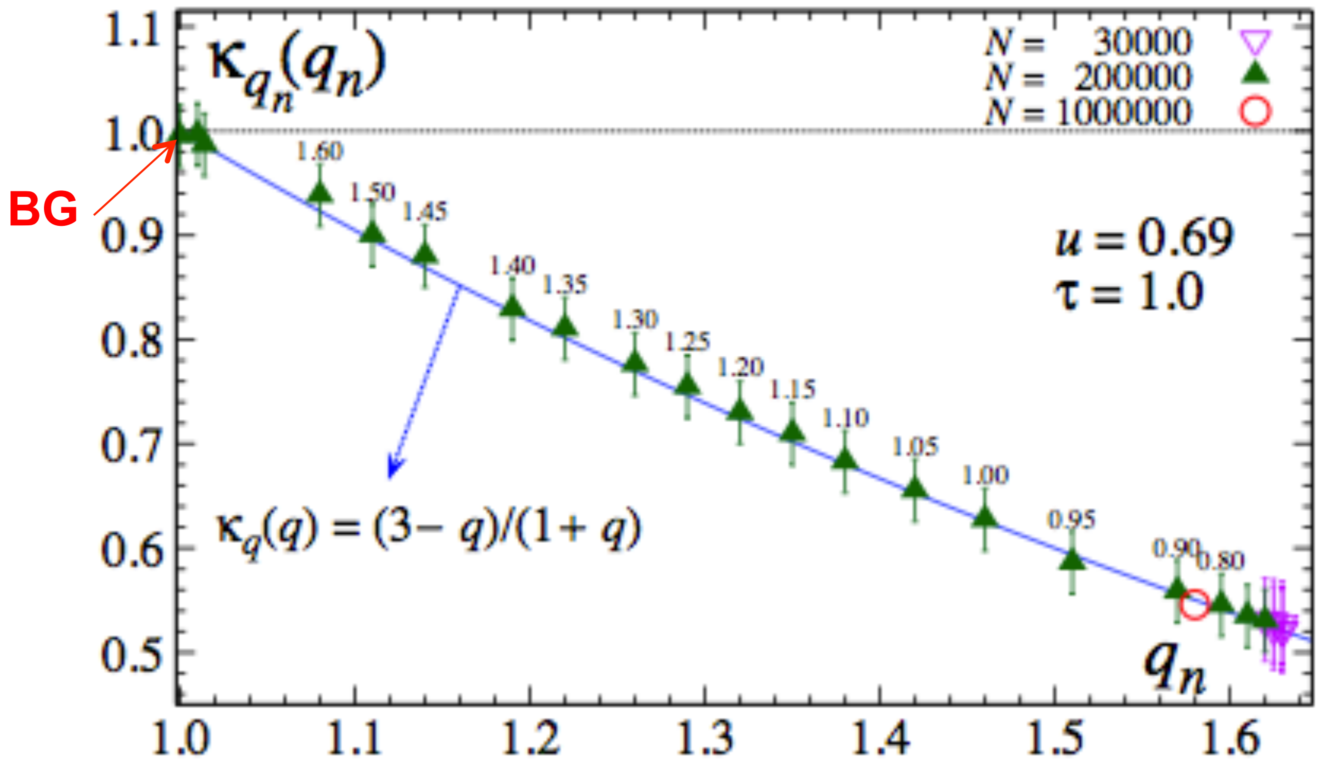
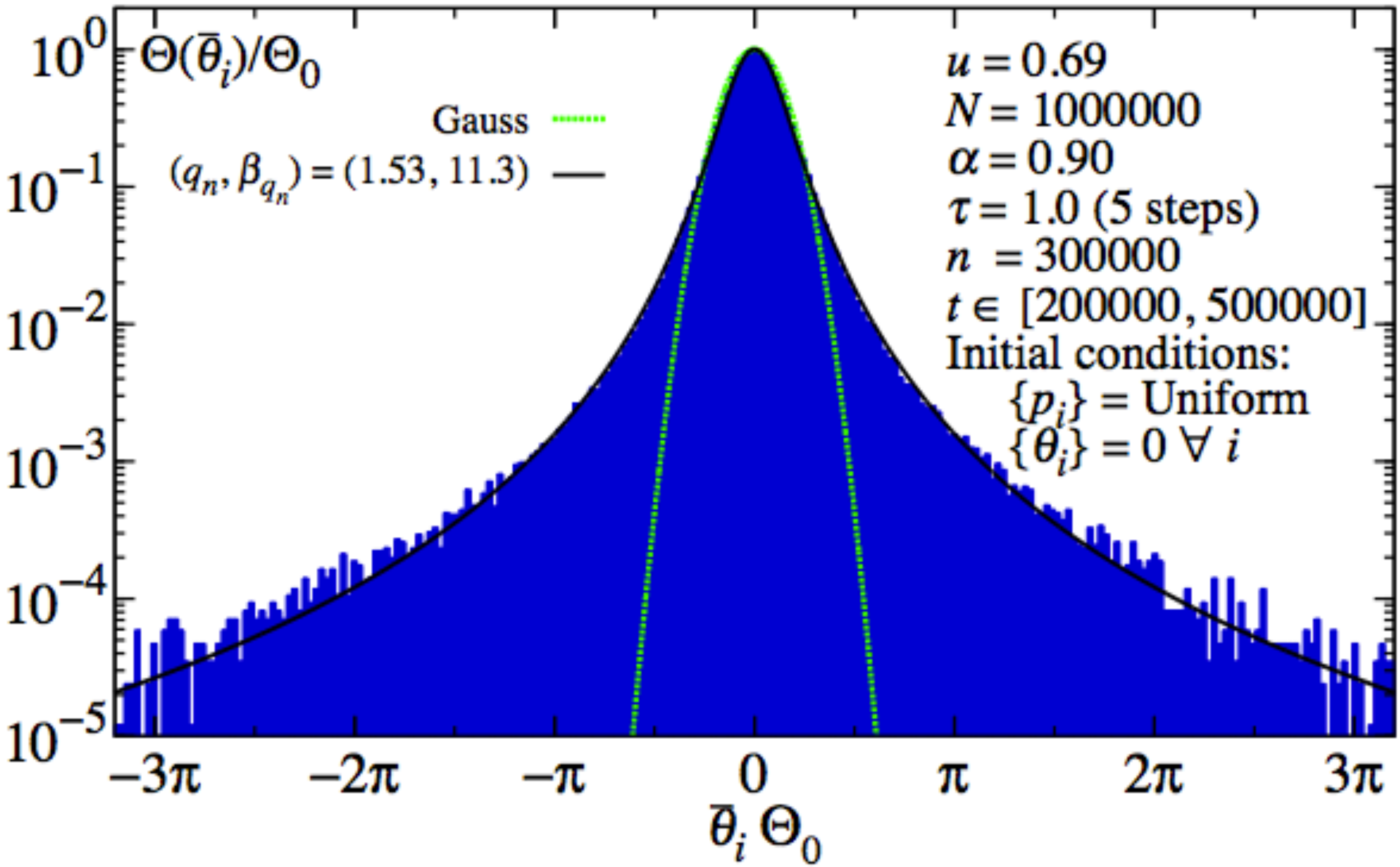


FIG. 3: q_n and q-kurtosis κ_{q_n} that have been obtained from the histograms corresponding to typical values of α (numbers indicated on top of the points). The red circle corresponds to Fig. 2. The continuous curve $\kappa_q = (3-q)/(1+q)$ is the analytical one obtained with q-Gaussians. Notice that κ_q is finite up to $q = 3$ (maximal admissible value for a q-Gaussian to be normalizable), and that it does not depend on β_{q_n}. The visible departure from the dotted line at $\kappa_q = 1$ corresponding to a Maxwellian distribution, neatly reflects the departure from BG thermostatistics.

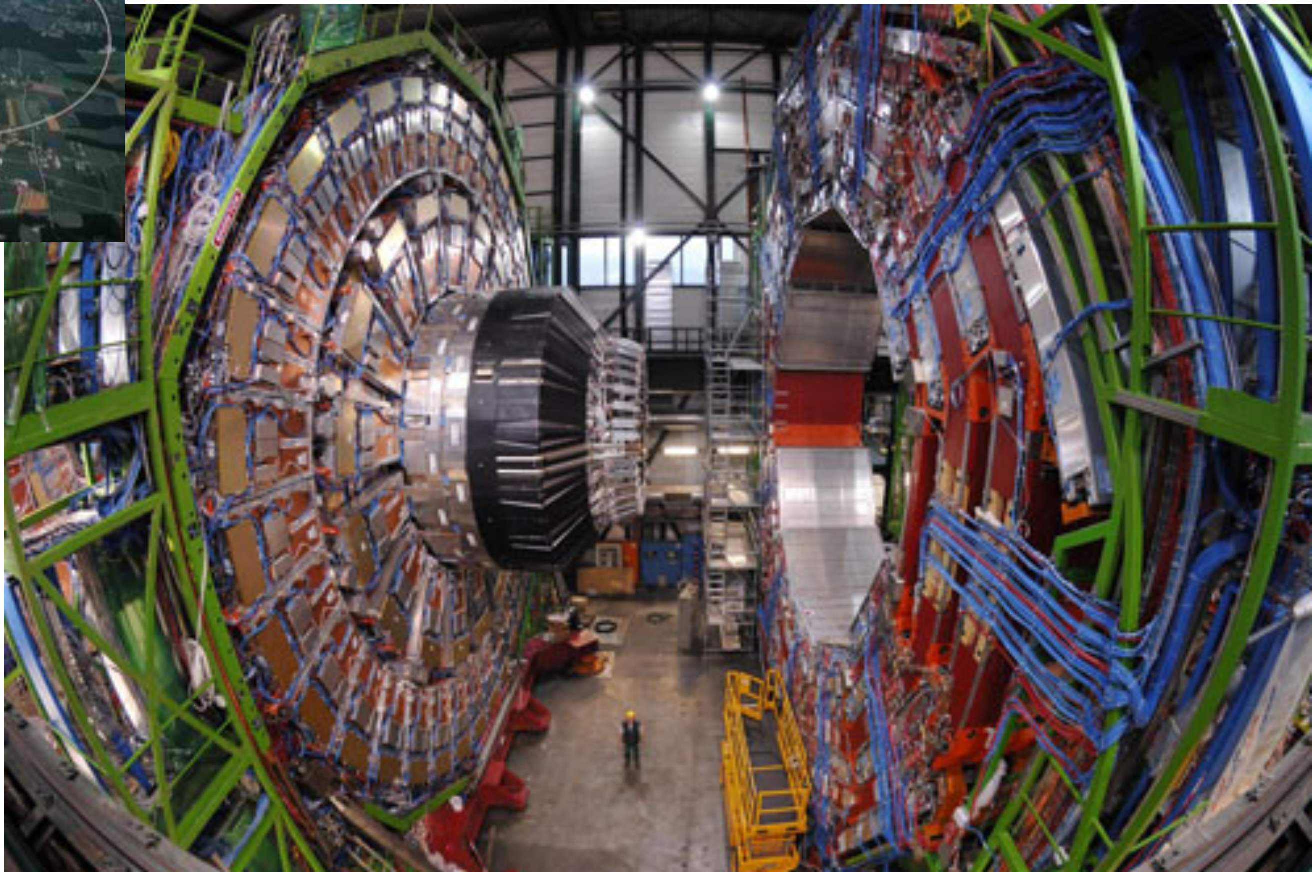


L.J.L. Cirto, V.R.V. Assis and C. T. (2012),1206.6133 [cond-mat.stat-mech]

LHC (Large Hadron Collider)

CMS (Compact Muon Solenoid) detector

~ 2500 scientists/engineers from 183 institutions of 38 countries

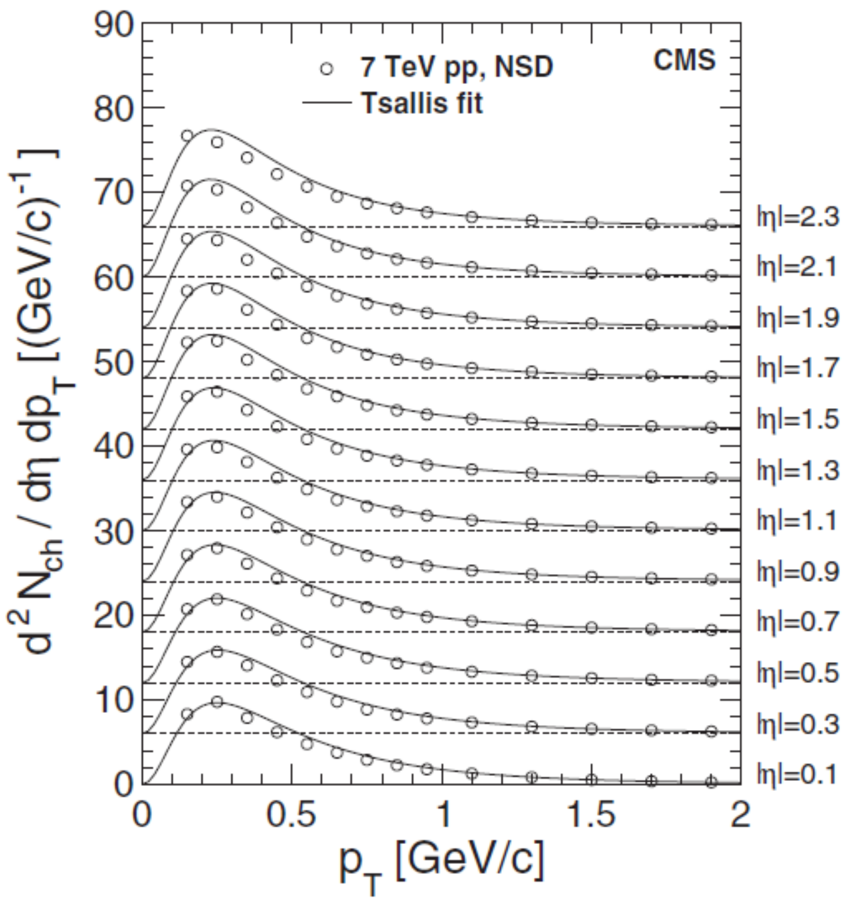


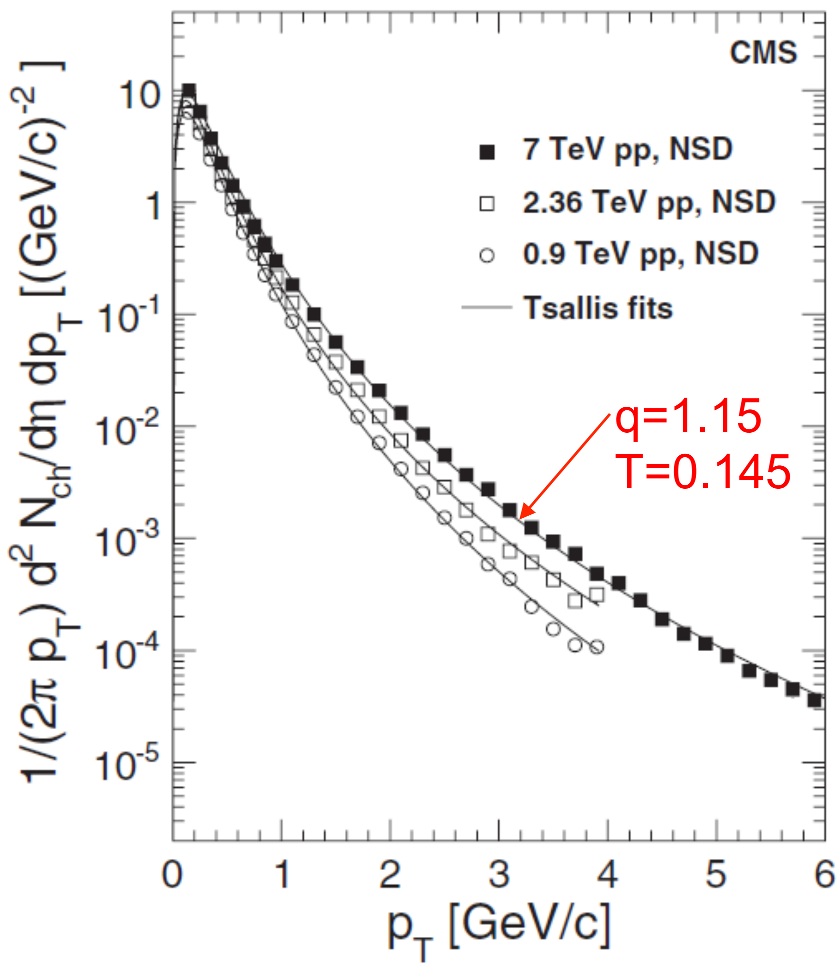
PRL 105, 022002 (2010) PHYSICAL REVIEW LETTERS week ending 9 JULY 2010

Transverse-Momentum and Pseudorapidity Distributions of Charged Hadrons in pp Collisions at $\sqrt{s} = 7$ TeV

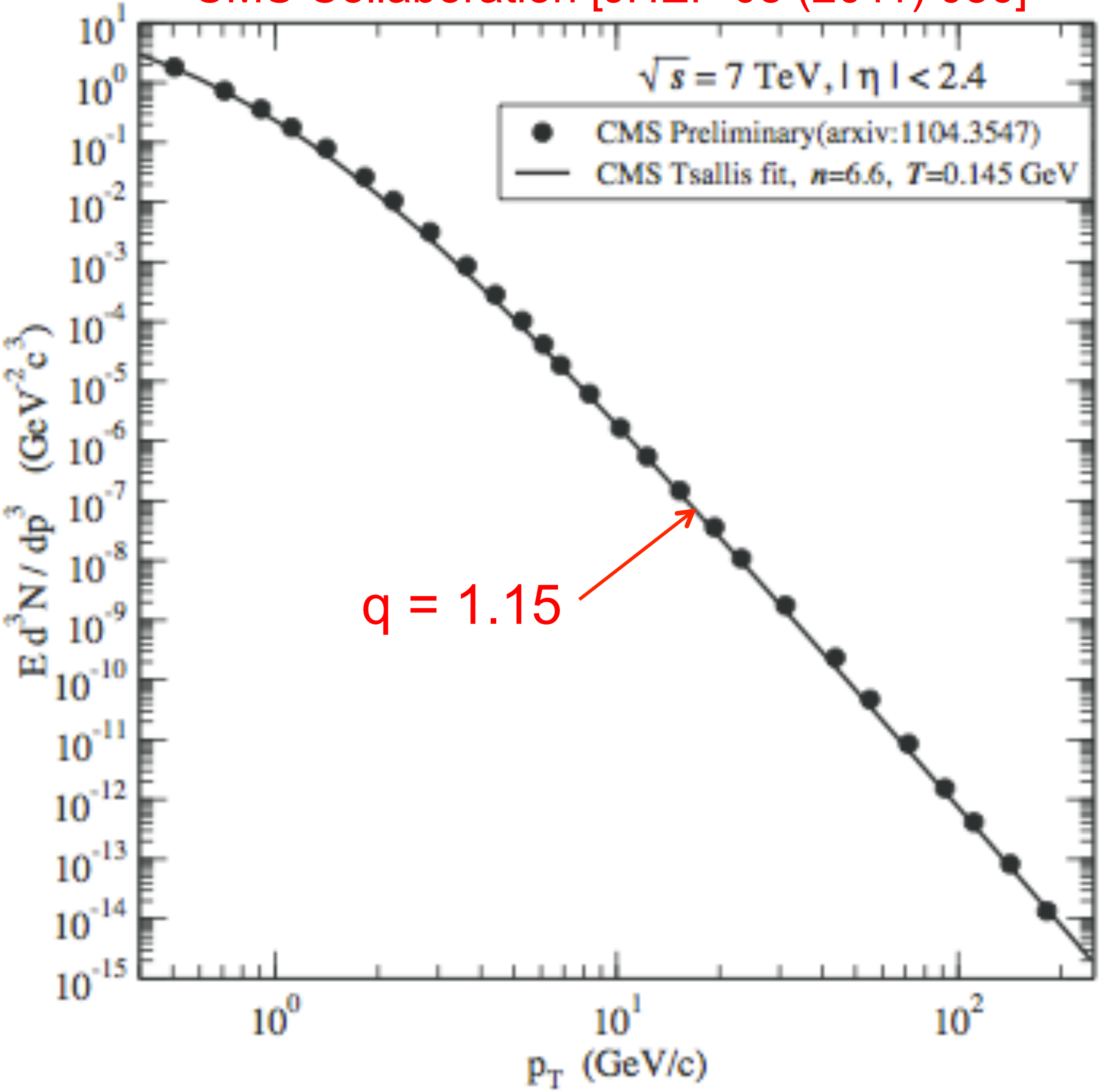
V. Khachatryan *et al.**
(CMS Collaboration)

(Received 18 May 2010; published 6 July 2010)





CMS Collaboration [JHEP 08 (2011) 086]



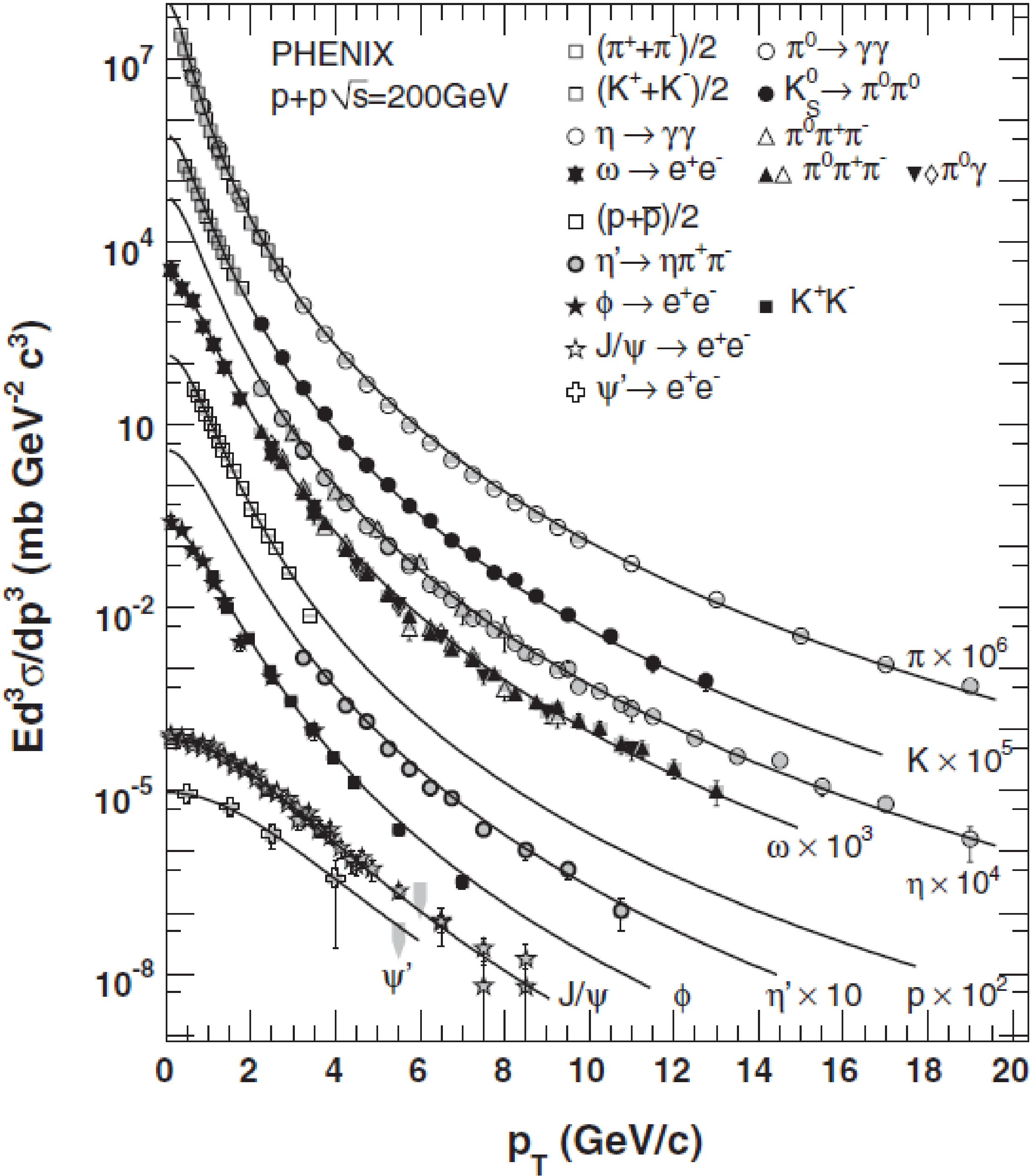
Fitting: C.Y. Wong and G. Wilk (2011, private comm.)

PHYSICAL REVIEW D **83,** 052004 (2011)

Measurement of neutral mesons in $p + p$ collisions at $\sqrt{s} = 200$ GeV and scaling properties of hadron production

PHENIX @ RHIC

A. Adare,[11] S. Afanasiev,[25] C. Aidala,[12,36] N. N. Ajitanand,[53] Y. Akiba,[47,48] H. Al-Bataineh,[42] J. Alexander,[53] K. Aoki,[30,47] L. Aphecetche,[55] R. Armendariz,[42] S. H. Aronson,[6] J. Asai,[47,48] E. T. Atomssa,[31] R. Averbeck,[54] T. C. Awes,[43] B. Azmoun,[6] V. Babintsev,[21] M. Bai,[5] G. Baksay,[17] L. Baksay,[17] A. Baldisseri,[14] K. N. Barish,[7] P. D. Barnes,[33] B. Bassalleck,[41] A. T. Basye,[1] S. Bathe,[7] S. Batsouli,[43] V. Baublis,[46] C. Baumann,[37] A. Bazilevsky,[6] S. Belikov,[6,*] R. Bennett,[54] A. Berdnikov,[50] Y. Berdnikov,[50] A. A. Bickley,[11] J. G. Boissevain,[33] H. Borel,[14] K. Boyle,[54] M. L. Brooks,[33] H. Buesching,[6] V. Bumazhnov,[21] G. Bunce,[6,48] S. Butsyk,[33,54] C. M. Camacho,[33] S. Campbell,[54] B. S. Chang,[62] W. C. Chang,[2] J.-L. Charvet,[14] S. Chernichenko,[21] J. Chiba,[26] C. Y. Chi,[12] M. Chiu,[22] I. J. Choi,[62] R. K. Choudhury,[4] T. Chujo,[58,59] P. Chung,[53] A. Churyn,[21] V. Cianciolo,[43] Z. Citron,[54] C. R. Cleven,[19] B. A. Cole,[12] M. P. Comets,[44] P. Constantin,[33] M. Csanád,[16] T. Csörgő,[27] T. Dahms,[54] S. Dairaku,[30,47] K. Das,[18] G. David,[6] M. B. Deaton,[1] K. Dehmelt,[17] H. Delagrange,[55] A. Denisov,[21] D. d'Enterria,[12,31] A. Deshpande,[48,54] E. J. Desmond,[6] O. Dietzsch,[51] A. Dion,[54] M. Donadelli,[51] O. Drapier,[31] A. Drees,[54] K. A. Drees,[5] A. K. Dubey,[61] A. Durum,[21] D. Dutta,[4] V. Dzhordzhadze,[7] Y. V. Efremenko,[43] J. Egdemir,[54] F. Ellinghaus,[11] W. S. Emam,[7] T. Engelmore,[12] A. Enokizono,[32] H. En'yo,[47,48] S. Esumi,[58] K. O. Eyser,[7] B. Fadem,[38] D. E. Fields,[41,48] M. Finger, Jr.,[8,25] M. Finger,[8,25] F. Fleuret,[31] S. L. Fokin,[29] Z. Fraenkel,[61,*] J. E. Frantz,[54] A. Franz,[6] A. D. Frawley,[18] K. Fujiwara,[47] Y. Fukao,[30,47] T. Fusayasu,[40] S. Gadrat,[34] I. Garishvili,[56] A. Glenn,[11] H. Gong,[54] M. Gonin,[31] J. Gosset,[14] Y. Goto,[47,48] R. Granier de Cassagnac,[31] N. Grau,[12,24] S. V. Greene,[59] M. Grosse Perdekamp,[22,48] T. Gunji,[10] H. -Å. Gustafsson,[35,*] T. Hachiya,[20] A. Hadj Henni,[55] C. Haegemann,[41] J. S. Haggerty,[6] H. Hamagaki,[10] R. Han,[45] H. Harada,[20] E. P. Hartouni,[32] K. Haruna,[20] E. Haslum,[35] R. Hayano,[10] M. Heffner,[32] T. K. Hemmick,[54] T. Hester,[7] X. He,[19] H. Hiejima,[22] J. C. Hill,[24] R. Hobbs,[41] M. Hohlmann,[17] W. Holzmann,[53] K. Homma,[20] B. Hong,[28] T. Horaguchi,[10,47,57] D. Hornback,[56] S. Huang,[59] T. Ichihara,[47,48] R. Ichimiya,[47] H. Iinuma,[30,47] Y. Ikeda,[58] K. Imai,[30,47] J. Imrek,[15] M. Inaba,[58] Y. Inoue,[49,47] D. Isenhower,[1] L. Isenhower,[1] M. Ishihara,[47] T. Isobe,[10] M. Issah,[53] A. Isupov,[25] D. Ivanischev,[46] B. V. Jacak,[54,†] J. Jia,[12] J. Jin,[12] O. Jinnouchi,[48] B. M. Johnson,[6] K. S. Joo,[39] D. Jouan,[44] F. Kajihara,[10] S. Kametani,[10,47,60] N. Kamihara,[47,48] J. Kamin,[54] M. Kaneta,[48] J. H. Kang,[62] H. Kanou,[47,57] J. Kapustinsky,[33] D. Kawall,[36,48] A. V. Kazantsev,[29] T. Kempel,[24] A. Khanzadeev,[46] K. M. Kijima,[20] J. Kikuchi,[60] B. I. Kim,[28] D. H. Kim,[39] D. J. Kim,[62] E. Kim,[52] S. H. Kim,[62] E. Kinney,[11] K. Kiriluk,[11] Á. Kiss,[16] E. Kistenev,[6] A. Kiyomichi,[47] J. Klay,[32] C. Klein-Boesing,[37] L. Kochenda,[46] V. Kochetkov,[21] B. Komkov,[46] M. Konno,[58] J. Koster,[22] D. Kotchetkov,[7] A. Kozlov,[61] A. Král,[13] A. Kravitz,[12] J. Kubart,[8,23] G. J. Kunde,[33] N. Kurihara,[10]


PHENIX
p+p √s=200GeV
(π⁺+π⁻)/2
(K⁺+K⁻)/2
η → γγ
ω → e⁺e⁻
(p+p̄)/2
η'→ ηπ⁺π⁻
φ → e⁺e⁻
J/ψ → e⁺e⁻
ψ'→ e⁺e⁻
π⁰→ γγ
K⁰S→ π⁰π⁰
π⁰π⁺π⁻
π⁰π⁺π⁻
π⁰γ
K⁺K⁻
π × 10⁶
K × 10⁵
ω × 10³
η × 10⁴
ψ'
J/ψ
φ
η' × 10
p × 10²
Ed³σ/dp³ (mb GeV⁻² c³)
p_T (GeV/c)

$q \simeq 1.10$

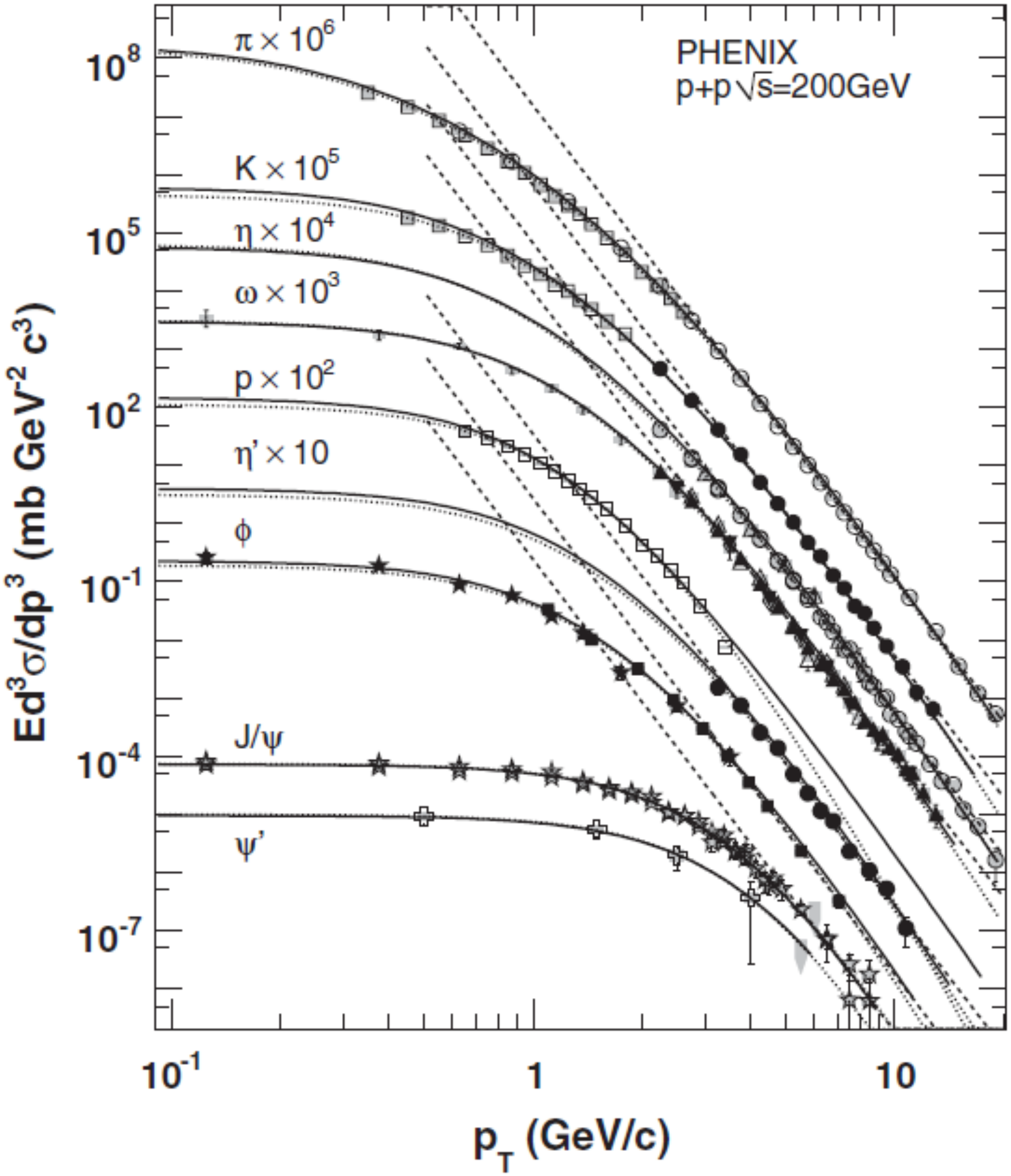


FIG. 13. The p_T spectra of various hadrons measured by PHENIX fitted to the power law fit (dashed lines) and Tsallis fit (solid lines). See text for more details.

$q \simeq 1.10$

IOP PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **37** (2010) 094027 (7pp) doi:10.1088/0954-3899/37/9/094027

Thermodynamics of composition rules

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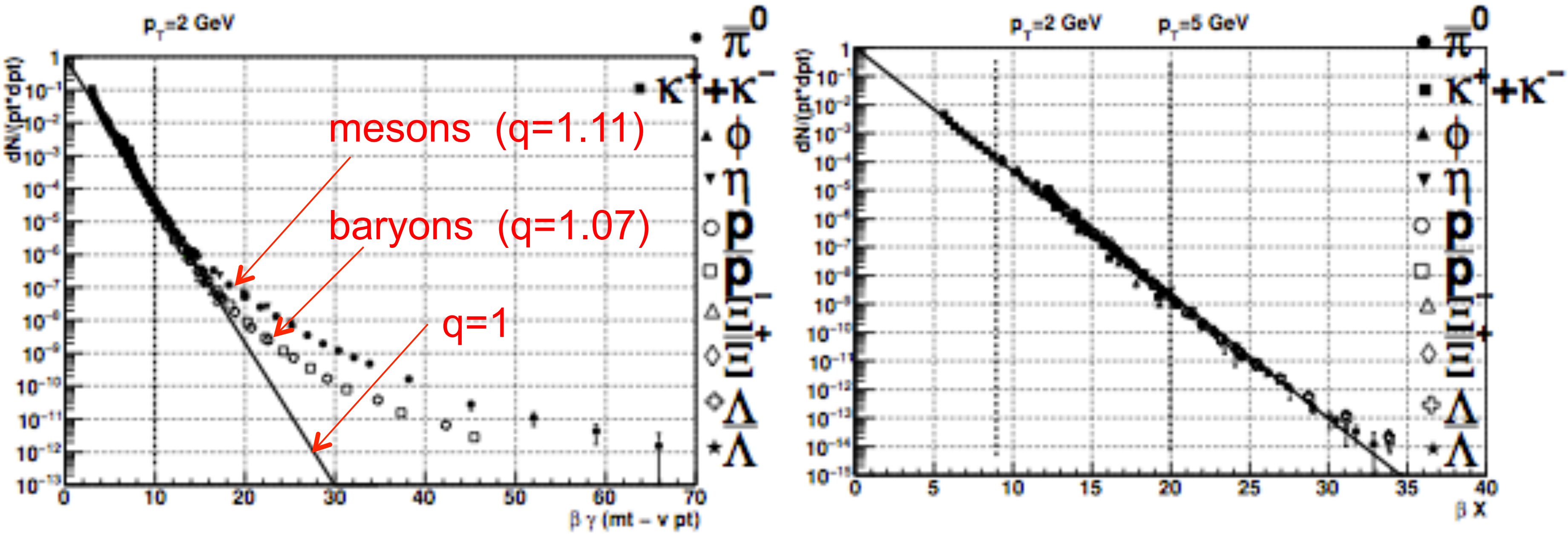
Abstract

We review our recent approach to non-extensive thermodynamics based on general composition rules. We discuss arguments in favor of using such an approach for quark matter due to momentum-dependent interaction and present first results of attempts to compute the equation of state of $SU(2)$ gluon matter on a lattice in this non-extensive, generalized canonical framework.

STAR + PHENIX @ RHIC



$$q_{quark} - 1 = 2(q_{meson} - 1) = 3(q_{baryon} - 1) \approx \frac{2}{\pi^2} = 0.202\ldots$$

(quark coalescence)

T.S. Biro, K. Urmossy and Z. Schram, J Phys G **37**, 094027 (2010)

Computers and Mathematics with Applications 60 (2010) 2426–2432

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A novel automatic microcalcification detection technique using Tsallis entropy & a type II fuzzy index

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Mammograms
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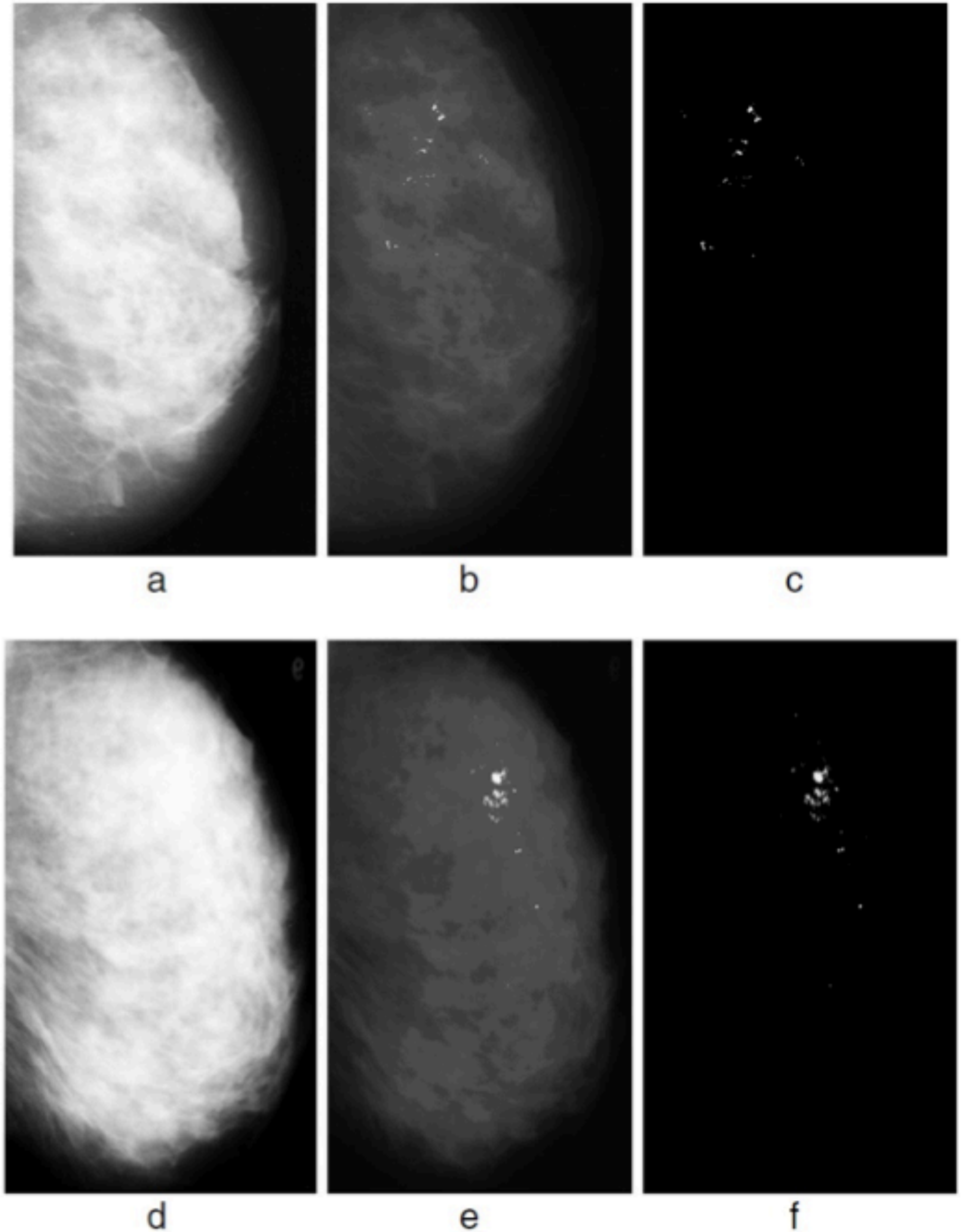
ABSTRACT

This article investigates a novel automatic microcalcification detection method using a type II fuzzy index. The thresholding is performed using the Tsallis entropy characterized by another parameter '*q*', which depends on the non-extensiveness of a mammogram. In previous studies, '*q*' was calculated using the histogram distribution, which can lead to erroneous results when pectoral muscles are included. In this study, we have used a type II fuzzy index to find the optimal value of '*q*'. The proposed approach has been tested on several mammograms. The results suggest that the proposed Tsallis entropy approach outperforms the two-dimensional non-fuzzy approach and the conventional Shannon entropy partition approach. Moreover, our thresholding technique is completely automatic, unlike the methods of previous related works. Without Tsallis entropy enhancement, detection of microcalcifications is meager: 80.21% Tps (true positives) with 8.1 Fps (false positives), whereas upon introduction of the Tsallis entropy, the results surge to 96.55% Tps with 0.4 Fps.


a
b
c
d
e
f

Weili SHI, Yanfang LI, Yu MIAO, Yinlong HU

Changchun University of Science and Technology

Research on the Key Technology of Image Guided Surgery

***Abstract**. It research on the key technology on IGS (image-guided surgery). It proposes medical image segmentation based on PCNN and the virtual endoscopic scenes real-time rendering method based on GPU parallel computing technology, which improves the display quality of IGS's virtual scene and real-time rendering speed. These methods are very important for IGS's applications.*

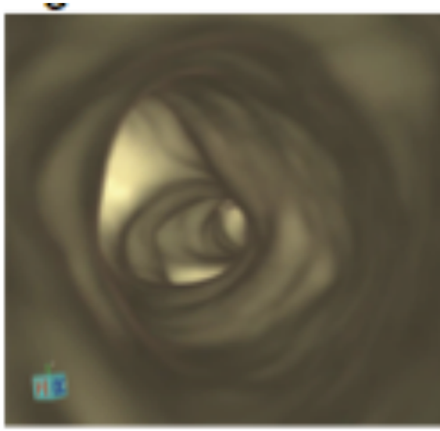


Fig.12. Bronchus

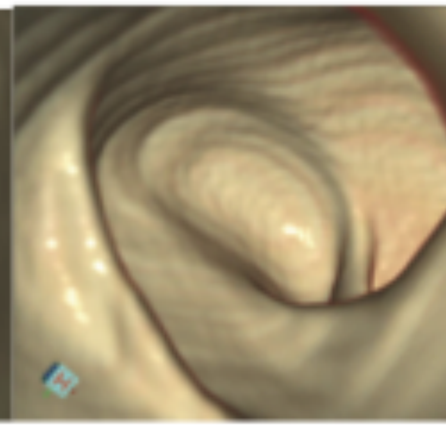


Fig.13.Colon

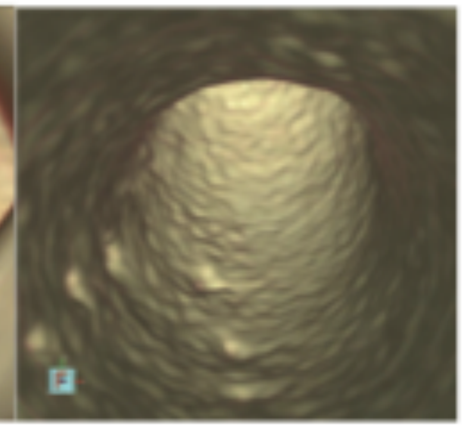


Fig.14.blood vessel

Table 1. Speed Comparison between Traditional Algorithms and Present Algorithm(uint: fps)

CT Image	Bronchus	Colon	blood vessel
Image Extent	512*51*217	512*512*252	512*512*355
Ray casting	9.8	6.4	3.5
Our algorithm	36.2	35.7	32.1

Tissue Radiation Response with Maximum Tsallis Entropy

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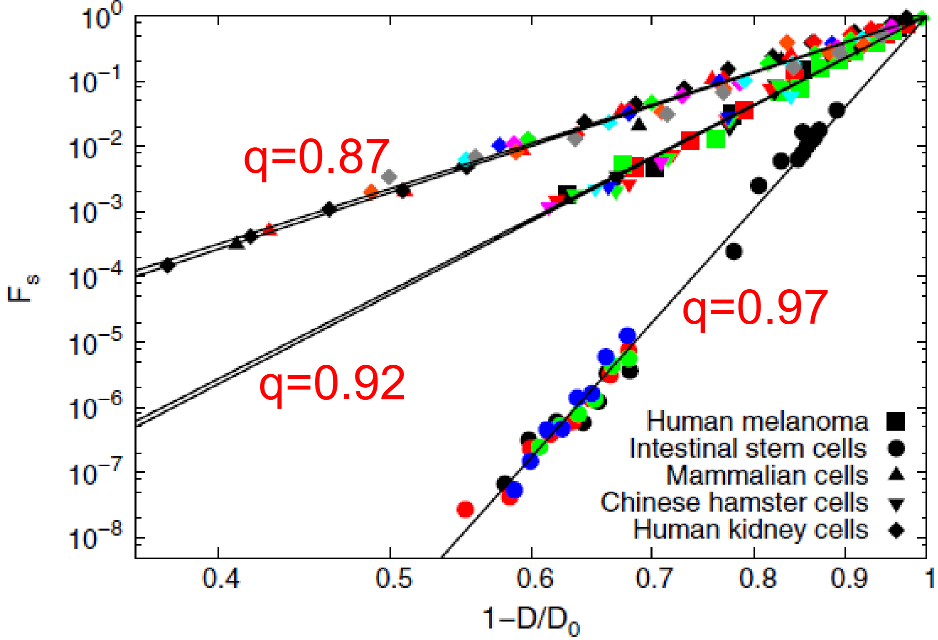
J. C. Antoranz
UNED, Departamento de Física Matemática y de Fluidos, 28040 Madrid, Spain,
and University of Havana, Cátedra de Sistemas Complejos Henri Poincaré, Havana 10400, Cuba

Oscar Sotolongo-Costa
University of Havana, Cátedra de Sistemas Complejos Henri Poincaré, Havana 10400, Cuba
(Received 22 June 2010; published 7 October 2010)

The expression of survival factors for radiation damaged cells is currently based on probabilistic assumptions and experimentally fitted for each tumor, radiation, and conditions. Here, we show how the simplest of these radiobiological models can be derived from the maximum entropy principle of the classical Boltzmann-Gibbs expression. We extend this derivation using the Tsallis entropy and a cutoff hypothesis, motivated by clinical observations. The obtained expression shows a remarkable agreement with the experimental data found in the literature.



$$\gamma = \frac{2-q}{1-q}$$

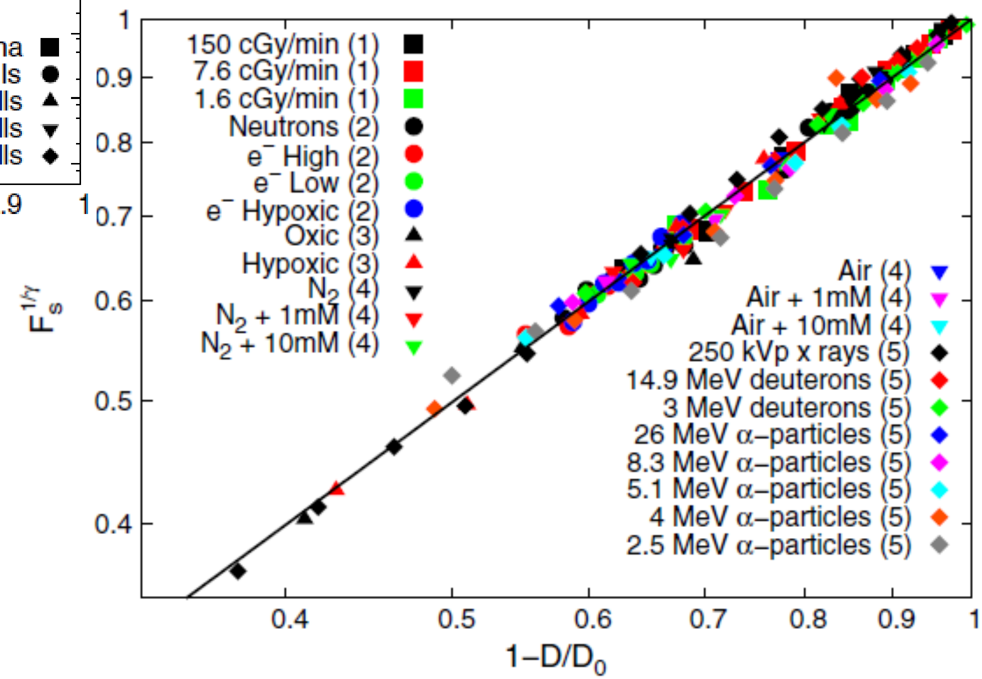


FIG. 2 (color online). Normalized survival fractions $(F_s)^{1/\gamma}$ as a function of the rescaled radiation dose, $1 - D/D_0$ for different tissues: intestinal stem cells (■), chinese hamster cells (●), human melanoma (▲), human kidney cells (▼), and cultured mammalian cells (◆) under different irradiation conditions detailed in [17–21] and grouped in [23]. The straight line shown is $y = x$.

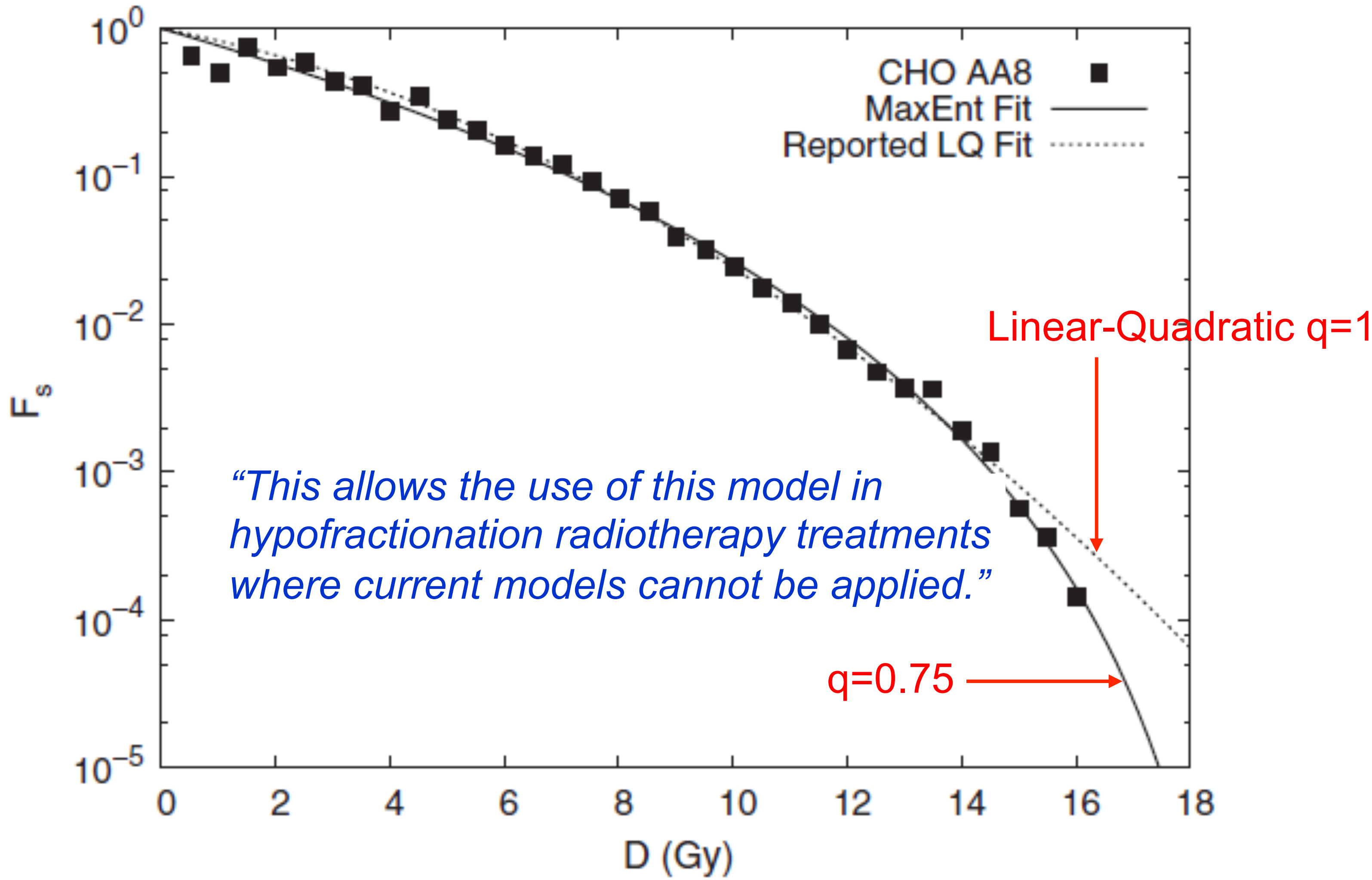


FIG. 3. Comparison between the LQ model best fit ($\alpha = 0.167 \pm 0.015$ Gy^{-1} and $\beta = 0.0205 \pm 0.0015$ Gy^{-2}) reported in [24] and our model fitted to $\gamma = 5.0 \pm 0.4$ and $D_0 = 19.4 \pm 0.4$ Gy for the cell line CHO AA8 under 250 k-Vp x rays.

Limoges - France

Journal of
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Crystallography
ISSN 0021-8898

Strain-profile determination in ion-implanted single crystals using generalized simulated annealing

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Accepted 29 July 2010

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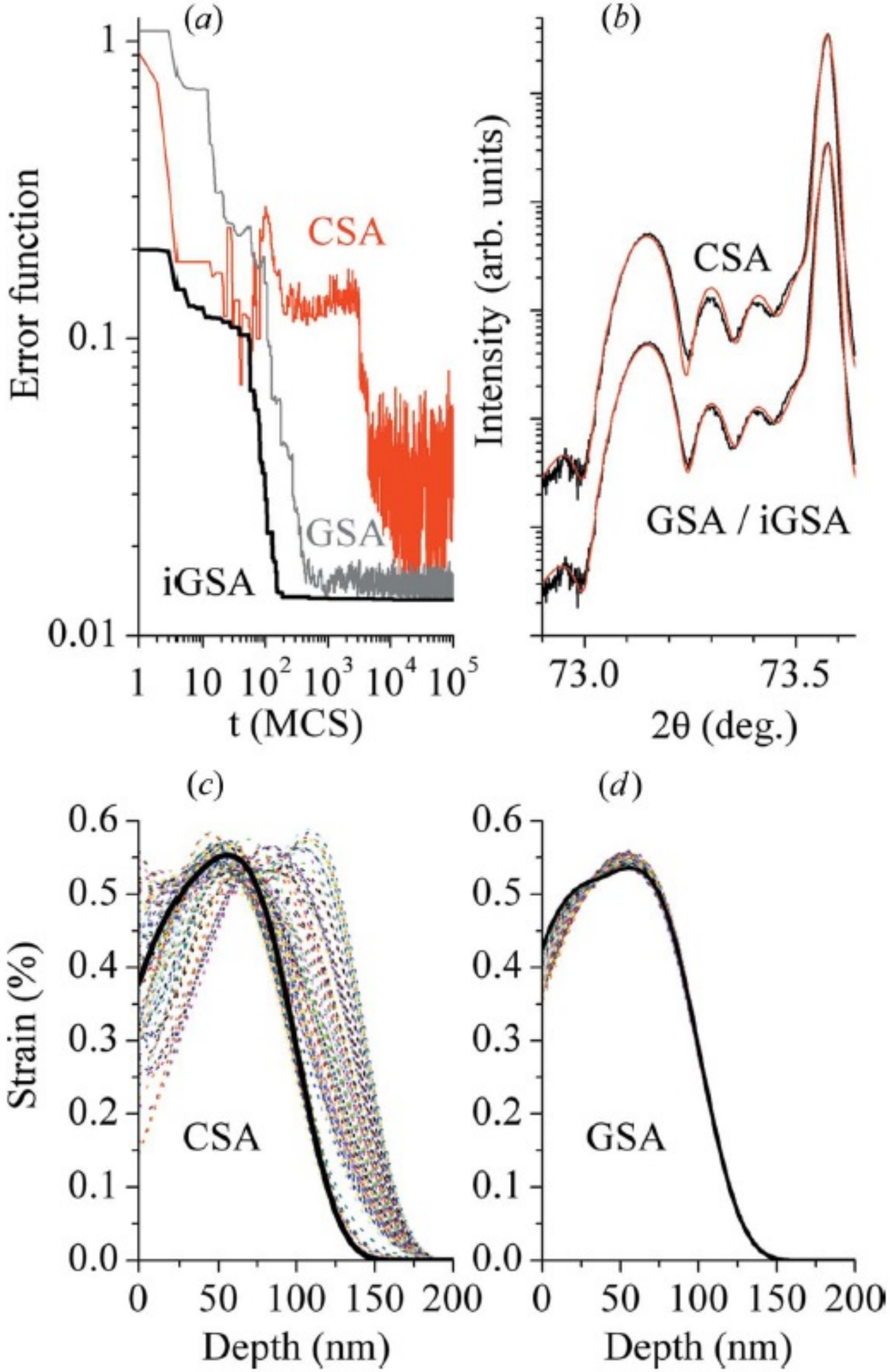
A novel least-squares fitting procedure is presented that allows the retrieval of strain profiles in ion-implanted single crystals using high-resolution X-ray diffraction. The model is based on the dynamical theory of diffraction, including a B-spline-based description of the lattice strain. The fitting procedure relies on the generalized simulated annealing algorithm which, contrarily to most common least-squares fitting-based methods, allows the global minimum of the error function (the difference between the experimental and the calculated curves) to be found extremely quickly. It is shown that convergence can be achieved in a few hundred Monte Carlo steps, *i.e.* a few seconds. The method is model-independent and allows determination of the strain profile even without any 'guess' regarding its shape. This procedure is applied to the determination of strain profiles in Cs-implanted yttria-stabilized zirconia (YSZ). The strain and damage profiles of YSZ single crystals implanted at different ion fluences are analyzed and discussed.


(a)
Error function
1
0.1
0.01
CSA
GSA
iGSA
1
10
10^2
10^3
10^4
10^5
t (MCS)
(b)
Intensity (arb. units)
CSA
GSA / iGSA
73.0
73.5
2θ (deg.)
(c)
Strain (%)
0.6
0.5
0.4
0.3
0.2
0.1
0.0
CSA
0
50
100
150
200
Depth (nm)
(d)
GSA
Depth (nm)

PRL **106,** 140601 (2011) PHYSICAL REVIEW LETTERS week ending 8 APRIL 2011

Nonlinear Relativistic and Quantum Equations with a Common Type of Solution

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Generalizations of the three main equations of quantum physics, namely, the Schrödinger, Klein-Gordon, and Dirac equations, are proposed. Nonlinear terms, characterized by exponents depending on an index q, are considered in such a way that the standard linear equations are recovered in the limit $q \to 1$. Interestingly, these equations present a common, solitonlike, traveling solution, which is written in terms of the q-exponential function that naturally emerges within nonextensive statistical mechanics. In all cases, the well-known Einstein energy-momentum relation is preserved for arbitrary values of q.

$q-$ generalized Schroedinger equation

(quantum non-relativistic spinless free particle)

$$i\hbar \frac{\partial}{\partial t}\left[\frac{\Phi\left(\vec{x},t\right)}{\Phi_0}\right] = -\frac{1}{2-q}\frac{\hbar^2}{2m}\nabla^2\left[\frac{\Phi\left(\vec{x},t\right)}{\Phi_0}\right]^{2-q} \qquad (q \in R)$$

Its exact solution is given by

$$\Phi\left(\vec{x},t\right) = \Phi_0 \; e_q^{i\left(\vec{p}\,.\,\vec{x}-Et\right)/\hbar} = \Phi_0 \; e_q^{i\left(\vec{k}\,.\,\vec{x}-\omega t\right)}$$

with

$E = \frac{p^2}{2m}$ (Newtonian relation!)

$E = \hbar\omega$ (Planck relation!)

$p = \hbar k$ (de Broglie relation!)

$\forall q$

F.D. Nobre, M.A. Rego-Monteiro and C. T., Phys Rev Lett **106**, 140601 (2011)

q-generalized Klein-Gordon equation:

(quantum relativistic spinless free particle: e.g., mesons π)

$$\nabla^2 \Phi(\vec{x},t) = \frac{1}{c^2}\frac{\partial^2 \Phi(\vec{x},t)}{\partial t^2} + q\,\frac{m^2c^2}{\hbar^2}\Phi(\vec{x},t)\left[\frac{\Phi(\vec{x},t)}{\Phi_0}\right]^{2(q-1)} \qquad (q \in R)$$

Its exact solution is given by

$$\Phi(\vec{x},t) = \Phi_0\; e_q^{i(\vec{p}\,.\,\vec{x}-Et)/\hbar} = \Phi_0\; e_q^{i(\vec{k}\,.\,\vec{x}-\omega\, t)}$$

with

$$E^2 = p^2c^2 + m^2c^4 \qquad (\forall q) \qquad \text{(Einstein relation!)}$$

Particular case: $m = 0 \;\Rightarrow\; q$-plane waves

F.D. Nobre, M.A. Rego-Monteiro and C. T., Phys Rev Lett **106**, 140601 (2011)

q-generalized Dirac equation:

(quantum relativistic spin $1/2$ matter and anti-matter free particles:

e.g., electron and positron)

$$i\hbar \frac{\partial\, \Phi\left(\vec{x},t\right)}{\partial t} + i\hbar c\left(\vec{\alpha}.\vec{\nabla}\right)\Phi\left(\vec{x},t\right) = \beta m c^2 A^{(q)}\left(\vec{x},t\right)\, \Phi\left(\vec{x},t\right) \qquad (q \in R)$$

with

$$\vec{\alpha} \equiv \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}; \quad \beta \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (4 \times 4 \text{ matrices})$$

$$A^{(q)}_{i\,j}\left(\vec{x},t\right) \equiv \delta_{ij}\left[\frac{\Phi_j\left(\vec{x},t\right)}{a_j}\right]^{q-1} \qquad \left(A^{(1)}_{i\,j}\left(\vec{x},t\right) = \delta_{ij}\right) \qquad (4 \times 4 \text{ matrix})$$

where $\left\{a_j\right\}$ are complex constants.

F.D. Nobre, M.A. Rego-Monteiro and C. T., Phys Rev Lett **106**, 140601 (2011)

Its exact solution is given by

$$\Phi(\vec{x},t) \equiv \begin{pmatrix} \Phi_1(\vec{x},t) \\ \Phi_2(\vec{x},t) \\ \Phi_3(\vec{x},t) \\ \Phi_4(\vec{x},t) \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} e_q^{i(\vec{p}\,.\,\vec{x}-Et)/\hbar} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} e_q^{i(\vec{k}\,.\,\vec{x}-\omega t)}$$

with $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$ being the same $\forall q$

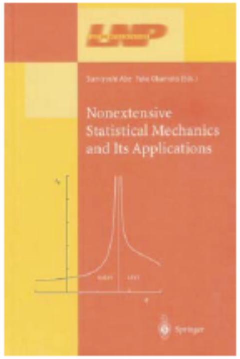
hence

$$E^2 = p^2c^2 + m^2c^4 \quad (q \in R)$$ (Einstein relation!)

F.D. Nobre, M.A. Rego-Monteiro and C. T., Phys Rev Lett **106**, 140601 (2011)

BOOKS AND SPECIAL ISSUES ON NONEXTENSIVE STATISTICAL MECHANICS

1999



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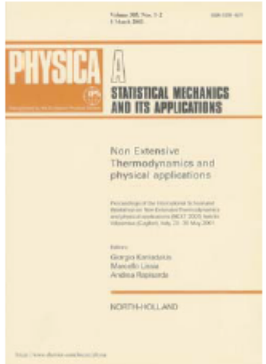

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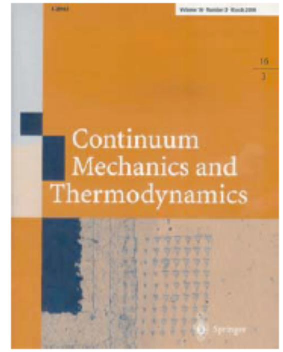

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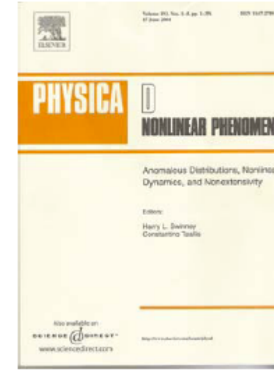

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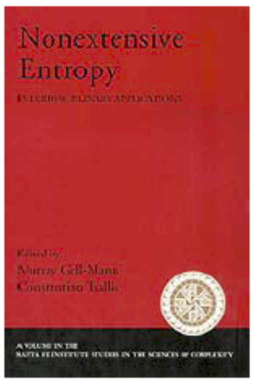

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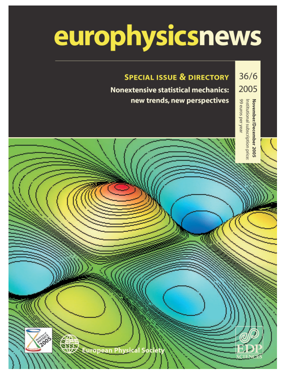

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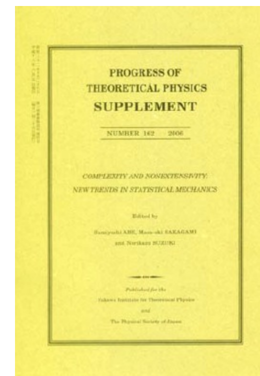

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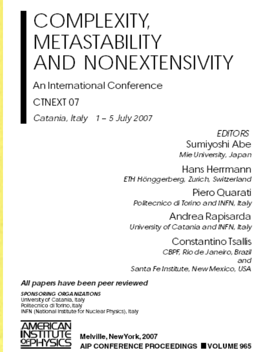

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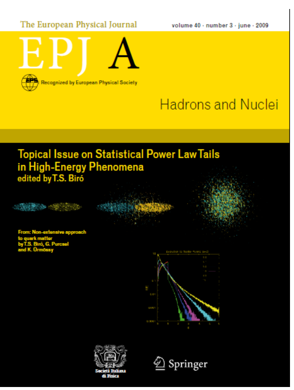

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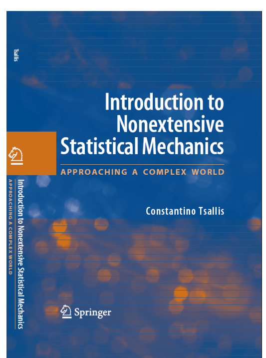
2009



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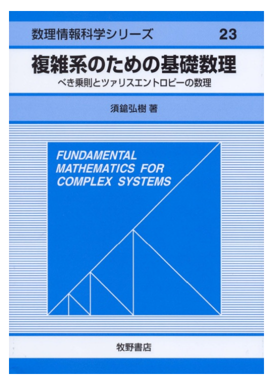

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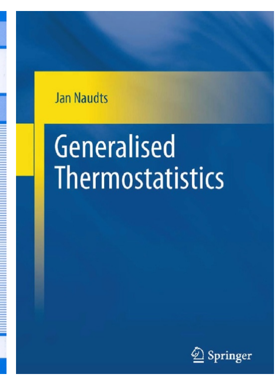

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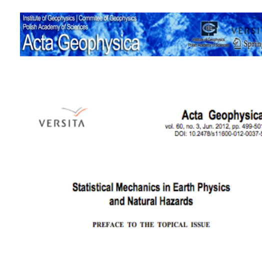

2012

Introduction to Nonextensive Statistical Mechanics

APPROACHING A COMPLEX WORLD

Constantino Tsallis

Springer

Full bibliography (regularly updated):

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3777 articles by 5633 scientists from 74 countries

[2 September 2012]

337
520
417
334
306
283
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204
200
190
187
166
100
99
85
81
81
68
60
55
43

[Updated 2 September 2012]

TAIWAN	43	VENEZUELA	11	PHILIPINES	2
MEXICO	40	PUERTO RICO	10	ECUADOR	1
IRAN	34	NORWAY	8	GEORGIA	1
ISRAEL	33	SERBIA	8	INDONESIA	1
UKRAINE	33	SLOVAK	8	JORDAN	1
AUSTRALIA	31	CHILE	7	PERU	1
CZECK	30	ESTONIA	7	QATAR	1
FINLAND	24	IRELAND	7	SRI LANKA	1
BELGIUM	21	PAKISTAN	7	THAILAND	1
BULGARIA	19	NEW ZEALAND	6	UNITED ARAB EMIRATES	1
NETHERLANDS	19	SWEDEN	6	UZBEKISTAN	1
CROATIA	16	SAUDI ARABIA	5		
DENMARK	12	SINGAPORE	5		
EGYPT	12	ARMENIA	4		
ROMENIA	12	BOLIVIA	4		
SOUTH AFRICA	12	MALAYSIA	4		
ALGERIA	11	SLOVENIA	4		
COLOMBIA	11	URUGUAY	4		
CUBA	11	BELARUS	3		
CYPRUS	11	KAZAKSTAN	2		
LITHUANIA	11	MOLDOVA	2		

74 COUNTRIES **5633 SCIENTISTS**