Thermalisation properties of various field theories

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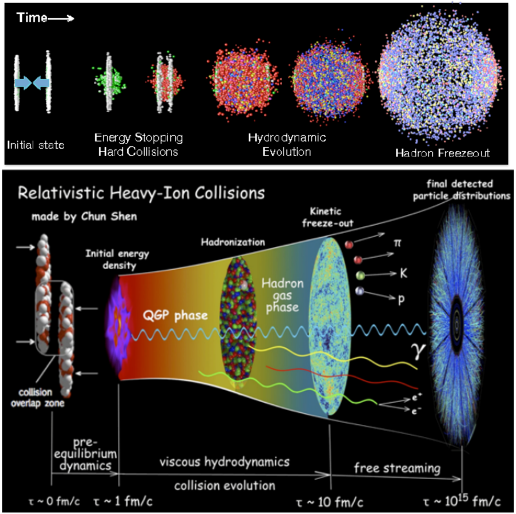
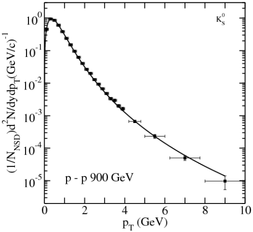
heavy-ion collision event



Tsallis distribution in p-p collisions and hadronization

Cleymans, J. and Worku, D.,
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s}$ = 0.9 TeV at the LHC,
J.Phys.G39:025006, 2012



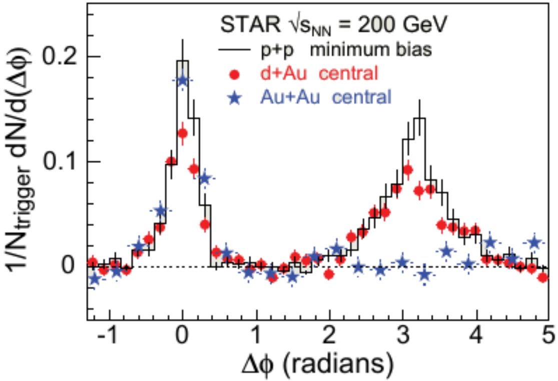
Urmossy, K. and Barnaföldi, G.G. and Harangozó, Sz. and Biró, T.S. and Xu, Z.,
A 'soft + hard' model for heavy-ion collisions, arXiv:1501.02352 [hep-ph], 2015

	$q_{2,soft}$	$q_{2,hard}$
CMS	1.058 ± 0.025	1.136 ± 0.001
ALICE	1.074 ± 0.018	1.131 ± 0.002
PHENIX	1.073 ± 0.016	1.100 ± 0.002

Au+Au $\sqrt{s} = 200\text{AGeV}$

If hadrons are created locally . . .

Jet suppression



[J. Adams et al.[STAR] (2003), Phys.Rev.Lett.91:172302] Adapted from DOE/NSF, Nuclear Science Advisory Committee, 2007, The Frontiers of Nuclear Science: A Long Range Plan

If hadrons are created locally . . .

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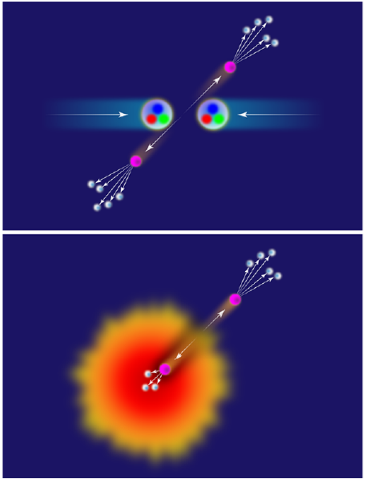

Cartoon from C.Manuel, PRL Viewpoint:
“The stopping power of hot nuclear matter”

Suggestions

- if hadrons are created locally
- creation probability depends on local energy density
- local energy density distribution can be determined by computer simulations $\rightarrow$
- it might be a tool for measuring hadron distribution function
- toy modells: classical Φ^4, quantum SU(3) pure Yang-Mills

Local energy-density distribution

- X stochastic variable
- indicator of X being in a Δx interval $\mathbb{I}_{[x,x+\Delta x]}(X)$
- $\langle \mathbb{I}_{[x,\Delta x]}(X) \rangle = \mathcal{P}(X \in [x, x + \Delta x])$
- density-function of X: $f(x) = \lim_{\Delta x \to 0+} \frac{\mathcal{P}(X \in [x,x+\Delta x])}{\Delta x}$
- $\to f(x) = \langle \delta(X - x) \rangle$

Local energy-density distribution (density-function):

$$\mathcal{P}(\epsilon) = \langle \delta(\epsilon_x - \epsilon) \rangle$$

- histogram of local energy-density
- density function over ϵ

Expectation values of local quantities

- Local quantity: $A(\Phi(t), \Pi(t))$
- Ensemble average:

$$\langle A(\Phi(t), \Pi(t)) \rangle = \int \mathcal{D}\bar{\Phi}\mathcal{D}\bar{\Pi} A(\bar{\Phi}, \bar{\Pi}) f(\bar{\Phi}, \bar{\Pi})$$

- $f(\bar{\Phi}, \bar{\Pi})$ is a histogram
- density-function over $\bar{\Phi}$ and $\bar{\Pi}$
- e.g. canonical: $e^{-\beta\mathcal{H}}$

Expectation values of local quantities

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Simulation

- canonical equations, periodic boundary conditions, leap-frog algorithm
- initial conditions: $\left\{\Pi\left(t_0+\frac{\delta t}{2}\right), \Phi(t_0)\right\}$
- uniform and $f(\Pi) \sim \operatorname{sech}\left(\frac{\pi}{2}\Pi\right) \rightarrow$ hyperbolic secant distribution
- Canonical eq. of $\dot{\Phi}$ (1st part of time step): Initial condition $\rightarrow \Phi(t_0+\delta t)$
- Canonical eq. of $\dot{\Pi}$ (2nd part of time step): $\left\{\Phi(t_0+\delta t), \Pi\left(t_0+\frac{\delta t}{2}\right)\right\} \rightarrow \Pi\left(t_0+\frac{3\delta t}{2}\right)$
- input parameters: N^3 lattice size, $a=1$ (grid), λ (interaction), m^2 Lagrangian-mass

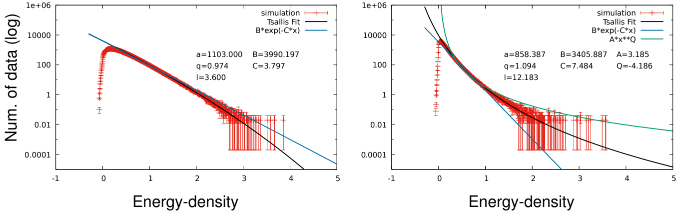
energy-density for classical Φ^4

$$\epsilon_{\mathbf{x}} = \frac{1}{2}\Pi^2_{\mathbf{x}} + \frac{1}{2}(\nabla\Phi)^2_{\mathbf{x}} + \frac{m^2}{2}\Phi^2_{\mathbf{x}} + \frac{\lambda}{24}\Phi^4_{\mathbf{x}}. \qquad (1)$$

- discretized form
- derivation connects neighbouring sites
- total energy is constant in continuum, algo. preserves this

Energy histogram

Early time energy-distribution function is not Boltzmannian



(a) uniform init. (b) sech init.

Various fits on logscale energy histogram

Tsallis distribution is an excellent fit!

$$f(x) = a\left[1 + (q-1)\beta x\right]^{\frac{1}{1-q}} \quad (2)$$

$\rightarrow$ consider the time evolution of q

Not Tsallis?

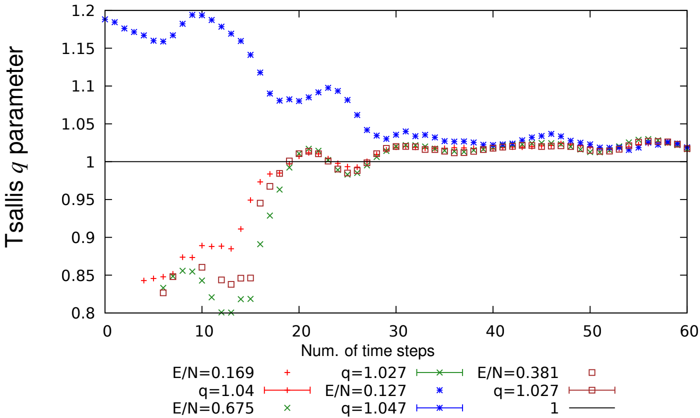


Figure: Time dependence of the Tsallis parameter

Tsallis?

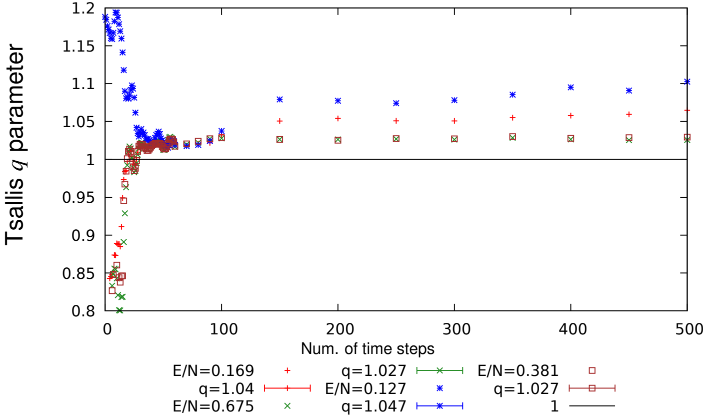


Figure: Time dependence of the Tsallis parameter

Just pre-thermal?

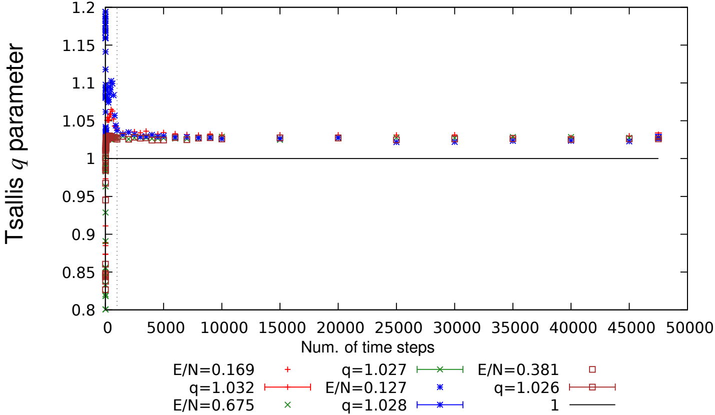


Figure: Time dependence of the Tsallis parameter

$$q = 1.028 \pm 0.003$$

$\Pi(x)$ histogram

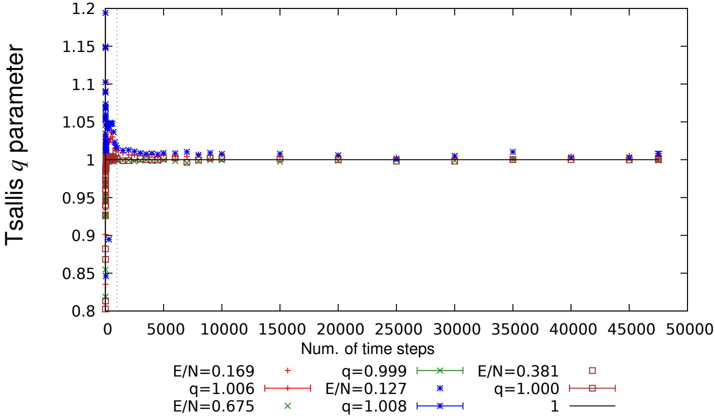
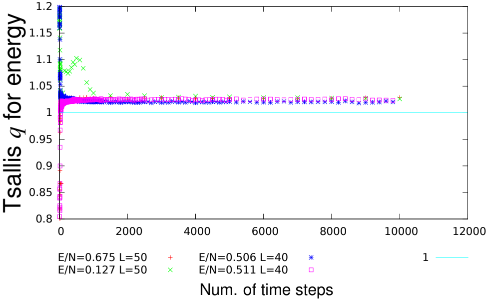
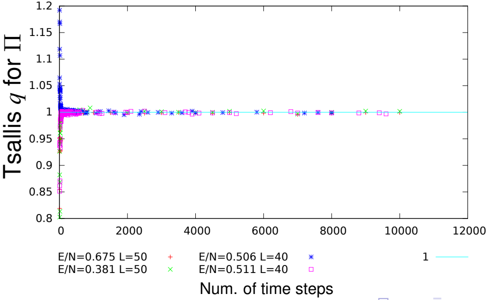


Figure: Time dependence of the Tsallis parameter for Π histogram

$$q = 1.003 \pm 0.009$$

Lattice size





SU(N) Yang - Mills theory

- pure gauge theory
- continuum quantum theory in Euclidean formalism:

$$\mathcal{S}_{YM} = \frac{1}{4}\int d^4x\, F^a_{\mu\nu}F^a_{\mu\nu} \qquad (3)$$

$$F_{\mu\nu}(x) = -\mathrm{i}gF^a_{\mu\nu}(x)T_a \qquad (4)$$

- Wilson action (lattice):

$$S[U] = \sum_p \beta\left(1 - \frac{1}{N}\mathrm{Re\,Tr}\,U_p\right), \qquad (5)$$

where $U(x,\mu) = \mathrm{e}^{-aA_\mu(x)}$, $\beta = 2N/g^2$.
- MC simulation for $N = 3$ with heat-bath algorithm

Observables

The expectation value of $\mathcal{O}$:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod_b \mathrm{d}U(b)\, \mathcal{O}\, e^{-S(U)} \qquad (6)$$

Wilson loop:

$$W(\mathcal{C}) = \langle \mathrm{Tr} U(\mathcal{C}) \rangle \qquad (7)$$

Internal energy:

$$\varepsilon = \left\langle 1 - \frac{1}{\mathrm{Tr}\mathbb{I}} \mathrm{Tr} U_p \right\rangle \qquad (8)$$

Polyakov loop (pure gauge theory: order param $|\langle L \rangle|$):

$$L_{\mathbf{x}} = \mathrm{Tr} \prod_{x_4=1}^{L_t} U_{\mathbf{x},x_4;4} \qquad (9)$$

MC Heatbath

Montvay - Münster: Quantum Fields on a Lattice

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod_b \mathrm{d}U(b)\, \mathcal{O}\, e^{-S(U)} \qquad (10)$$

if configs. generated according to the appropriate distribution:
expectation value $\rightarrow$ simple average

- updating: stoch. process with given transition prob.
- Markov process (transition prob.normalised, strong ergodicity, ensemble density normalised)
- "reasonable" initial ensemble $\rightarrow$ canonical
- sufficient: detailed balance
- Metropolis, overrelaxation, heatbath

Heatbath algorithm

- φ_x: local variables on a selected link, $\check{\varphi}_x$ fix variables on other links
- $W_c(\varphi_x|\check{\varphi}_x)$ conditional distribution for φ_x
- $W_c[\varphi] = W_c(\varphi_x|\check{\varphi}_x)\check{W}_c(\check{\varphi}_x)$
- transition probability:
  $\mathbb{P}([\varphi'] \leftarrow [\varphi]) = \mathbb{P}_x(\varphi'_x \leftarrow \varphi_x|\check{\varphi}_x)\delta(\check{\varphi}'_x - \check{\varphi}_x)$
  - $\sum_{[\varphi']} \mathbb{P}([\varphi'] \leftarrow [\varphi]) = 1$
  - Fix point: $\mathbb{P}W_c = W_c$ where $W_c = \lim_{k\to\infty} \mathbb{P}^k W_0$
- condition: local detailed balance
- $\mathbb{P}_x(\varphi'_x \leftarrow \varphi_x|\check{\varphi}_x)W_c(\varphi_x|\check{\varphi}_x) = \mathbb{P}_x(\varphi_x \leftarrow \varphi'_x|\check{\varphi}_x)W_c(\varphi'_x|\check{\varphi}_x)$

Heatbath algorithm

- local ergodicity: $\mathbb{P}_x(\varphi'_x \leftarrow \varphi_x|\check{\varphi}_x) > 0$
- ergodicity for whole config. by sweep:

$$\mathbb{P}([\varphi'] \leftarrow [\varphi]) = \prod_x \mathbb{P}_x(\varphi'_x \leftarrow \varphi_x|\check{\varphi}_x) \qquad (11)$$

- heatbath algo. corresponds to the cond. trans. prob. matrix: $\mathbb{P}_x(\varphi'_x \leftarrow \varphi_x|\check{\varphi}_x) = W_c(\varphi'_x|\check{\varphi}_x)$
- Task: generate $W_c(\varphi'_x|\check{\varphi}_x)$

Transition probability - SU(2) and SU(3)

[M. Creutz (1980), PhysRevD.21.2308],
[N. Cabibbo, E. Marinari (1982), Phys.Lett.B119 387-390]

In case of SU(3) choose a set of SU(2) subgroups:
$\{F : SU(2)_k, k = 1 \dots m\}$

Parametrization:

(2) $U_l = \alpha_{l0} + \sum_{r=1}^{3} i\sigma_r \alpha_r$, where $\alpha_0^2 + \alpha_r^2 = 1$

(3) $a_l^{(k)} = \alpha_{l0}^{(k)} + \sum_{r=1}^{3} i\sigma_r \alpha_r^{(k)}$

Variable to generate:

(2) $U_l \in SU(2)$

(3) $a_k \in SU(2)_k$ sub. for $k = 1, \dots, m$

Invar. Haar-measure:

(2) $dU = \frac{1}{2\pi^2}\delta(\alpha^2 - 1)d^4\alpha \rightarrow \frac{1}{2}d\alpha_0(1-\alpha_0^2)^{1/2}d\Omega$

(3) $d^{(k)}a_k$

Staple:

(2) $S_l = \sum U_\alpha = k\bar{U}$ where $\bar{U} \in \mathrm{SU}(2)$, $k = (\det S_l)^{-1/2}$

(3) $S_l = \sum U_\alpha$

Transition probability - SU(2) and SU(3)

Transition:

(2) $d\mathbb{P}(U_l) = W_c(U_l; \check{U}_l)dU_l \propto e^{\frac{\beta}{2}Tr(U_l S_l)}dU_l$

(3) $d\mathbb{P}(a_k) \propto e^{\frac{\beta}{N}ReTr(a_k U_l S_l)}da_k$

Parametrization:

(2) $S_l = s_{l0} - \sum_{r=1}^{3} i\sigma_r s_{lr}$ with real coeffs.

(3) $(U_l S_l)_{subk} = s_{kl0} - \sum_{r=1}^{3} i\sigma_r s_{klr}$ with complex coeffs.

Transition:

(2) $d\mathbb{P}(U_l) \propto e^{\beta \sum \alpha_{lr} s_{lr}} dU_l$

(3) $d\mathbb{P}(a_k) \propto e^{\frac{2\beta}{N} \sum \alpha_{lr}^{(k)} Re(s_{lr})} da_l^{(k)}$

$\rightarrow$ build $\mathbb{S}_l$ from $Re(s_{lr})$ coeffs.

Transition probability - SU(2) - transformed variable

- $d\mathbb{P}(U_l) = W_c(U_l; \check{U}_l)dU_l \propto e^{\frac{\beta}{2}Tr(U_l S_l)} dU_l$
- $S_l = \sum U_\alpha = k\bar{U}$
- $d\mathbb{P}(U_l) = W_c(U_l; \check{U}_l)dU_l \propto e^{\frac{\beta}{2}kTr(U_l \bar{U})} dU_l$
- $U_l^{tr} := U_l \bar{U}$
- $\int W_c(U_l; \check{U}_l)dU_l = \int W_c(U_l^{tr}; \check{U}_l^{tr})dU_l^{tr}$

New link

(2) generate α_r coeffs. $\rightarrow$ U
new link: $U' = UkS_l^{-1}$

(3) (kth step) generate α_r coeffs. $\rightarrow a_k$
new a_k: $a_k' = a_k k \mathbb{S}_l^{-1}$
new link: $U' = a_k U_{prev}$ (HB like proc. on a link)

Coefficients:

- uniform random $x \in \left[e^{-2\beta k}, 1\right]$ (Boost::uniform_rand_distribution)
- count $\alpha_0 = 1 + \frac{1}{\beta k}\log(x)$
- accept with $(1 - a_0^2)^{(1/2)}$ (Boost::bernoulli_distribution repeat till acc.)
- generate ($\hat{\mu}$) unit vector (Boost::uniform_on_sphere)
$\rightarrow \mathbf{a} = \sqrt{1 - a_0^2}\,\hat{\mu}$

SU(2) Average plaquette

[M.Creutz (1980), Phys.Rev.D21:2308-2315]

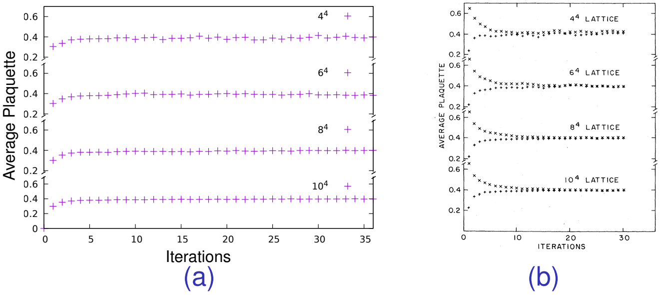


Figure: Average plaquette as a funct. of num. of iterations, $\beta = 2.3$

SU(2) Average plaquette

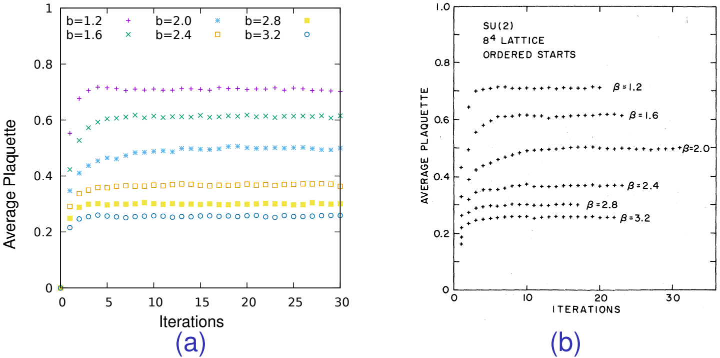


Figure: Average plauette as a funct. of num. of iterations, lattice 8^4

SU(3) Mean plaquette energy

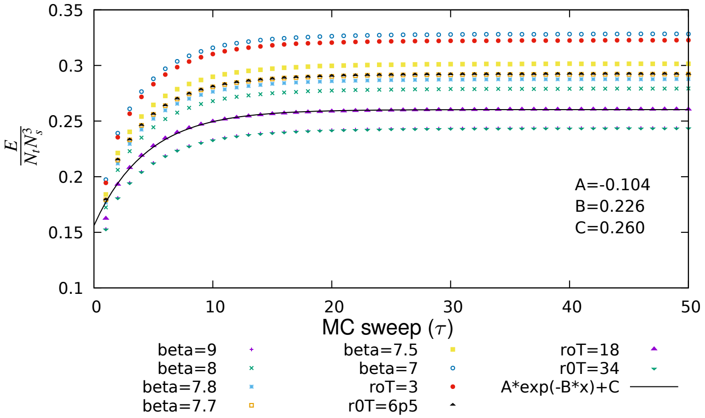
[N. Cabibbo and E. Marinari (1982), Phys.Lett.B119:387-390]

- minimal choice for SU(2) subsets (2)
- Lattice: 4^4
- $\beta = 6$
- average over the last 600 of 1000 iterations

Mean plaquette energy (every plaq. 4 times)

- reference: 0.4027 ± 0.0006
- reconstructed: 0.4021 ± 0.003 (SEM)

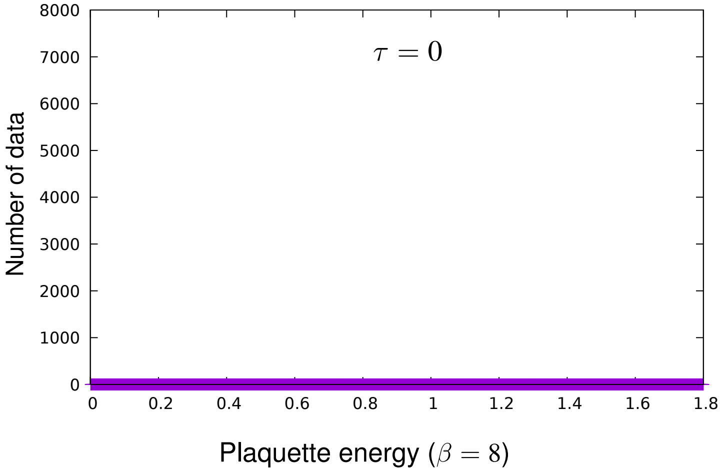
SU(3) Mean plaquette energy



- $N_t = 8,\ N_s = 60,\ \beta = 8.5$

Plaquette energy histogram

$N_t = 2,\ N_s = 50$



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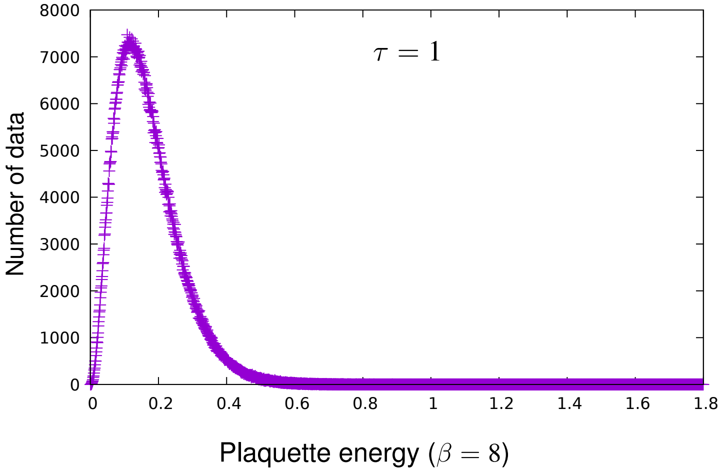

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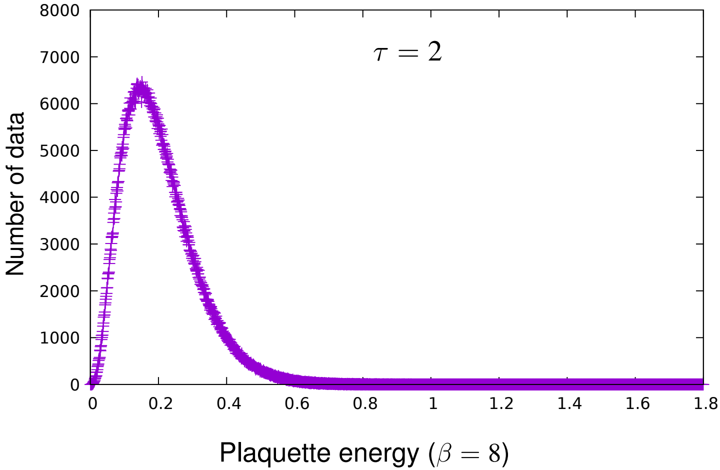

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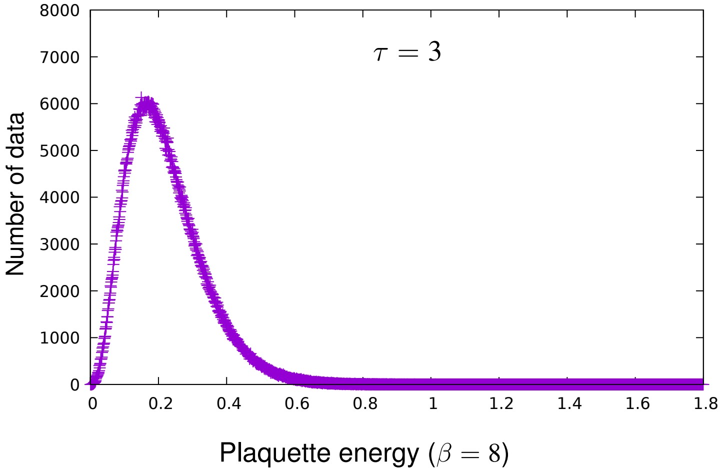

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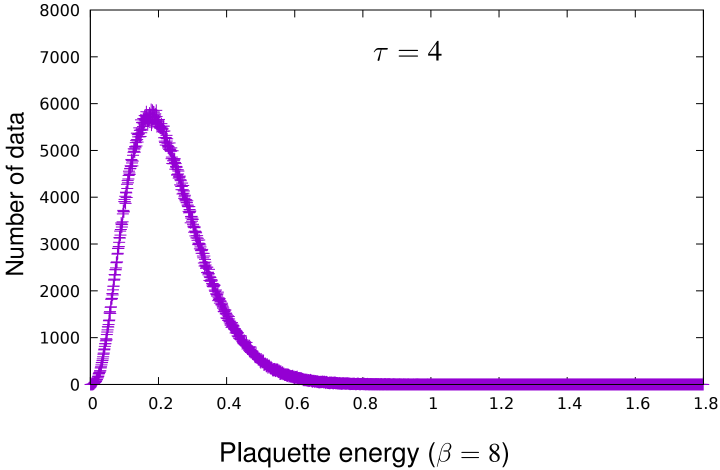

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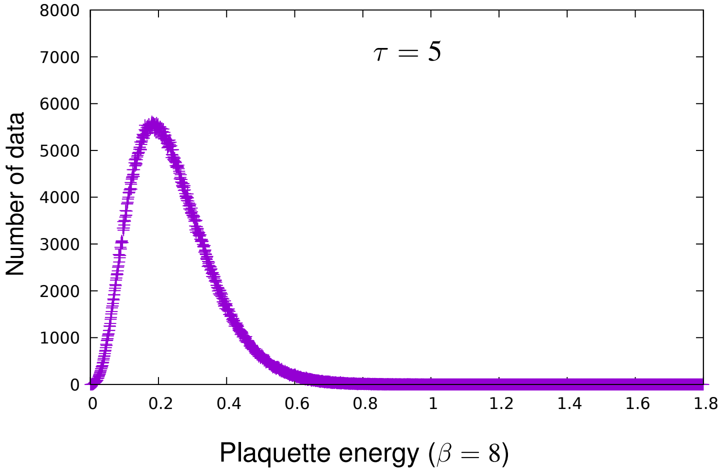
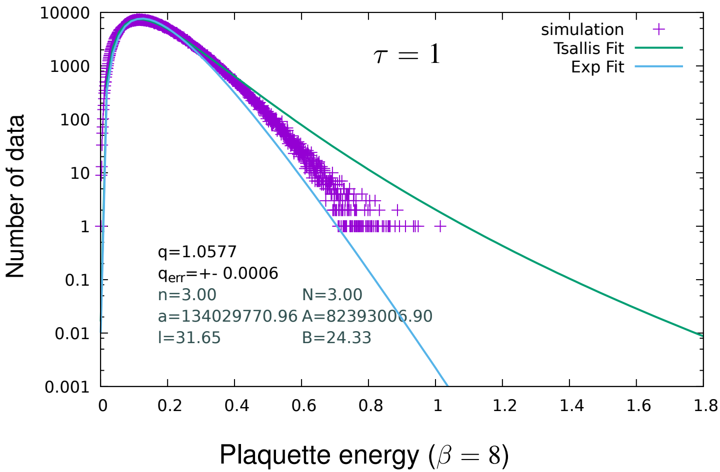


Plaquette energy histogram - semi-log

$N_t = 2,\ N_s = 50$



Tsallis: $f(x) = ax^n\left(1 + (q-1)\, l\, x\right)^{\frac{1}{1-q}}$

Exp: $g(x) = Ax^N \mathrm{e}^{-Bx}$

Plaquette energy histogram - semi-log

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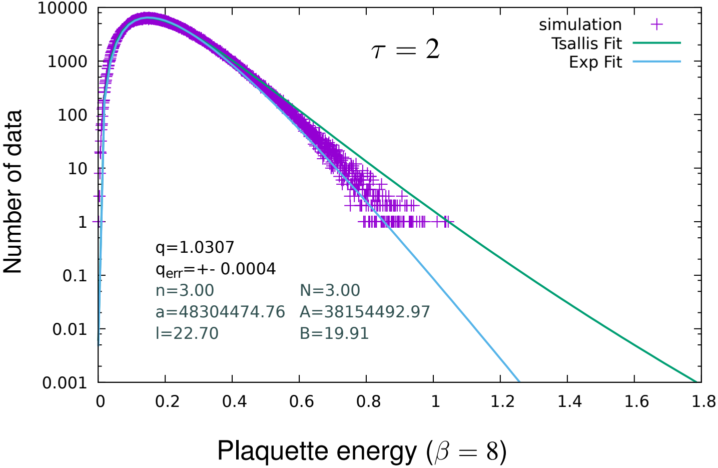

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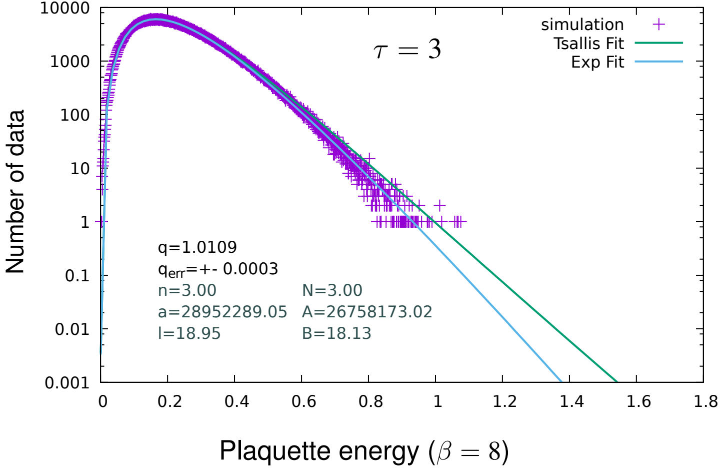

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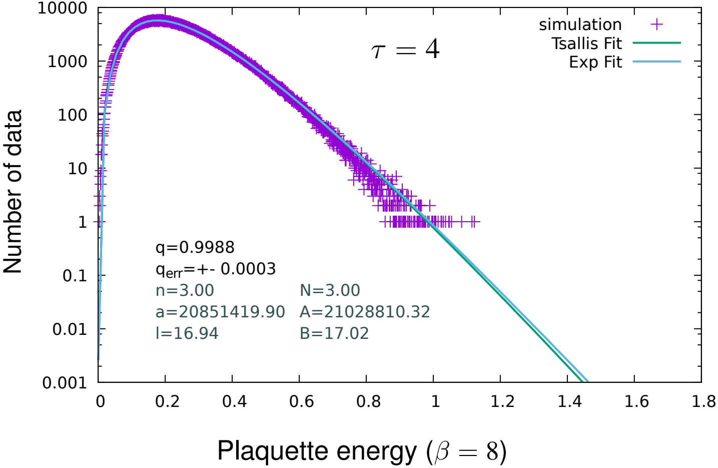

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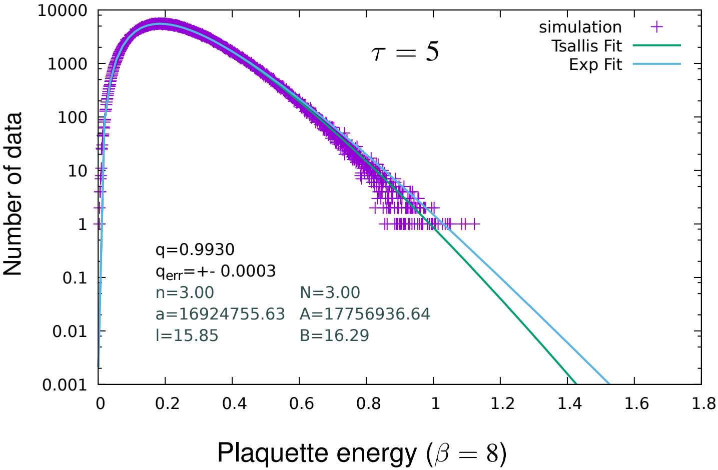

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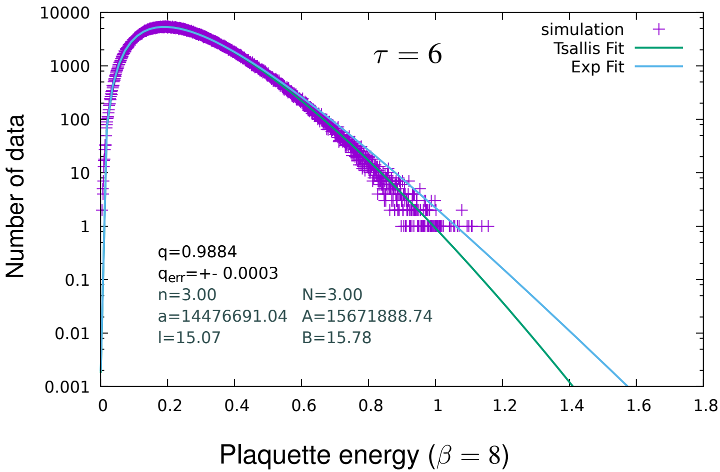

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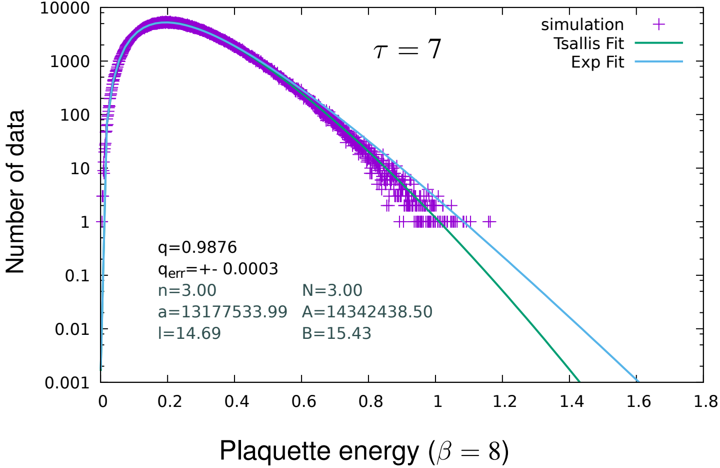

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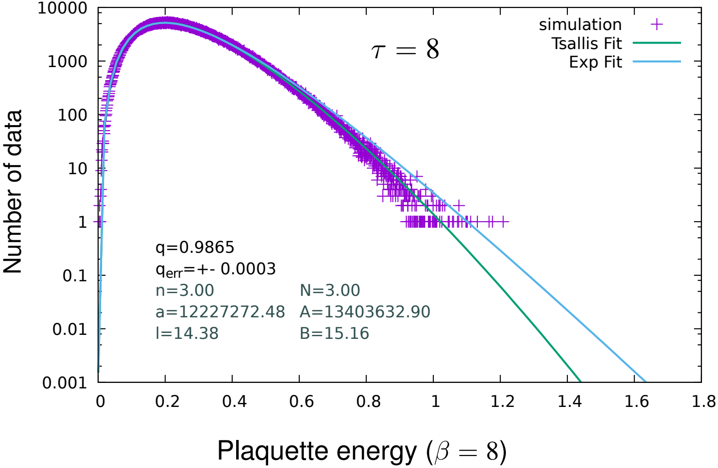

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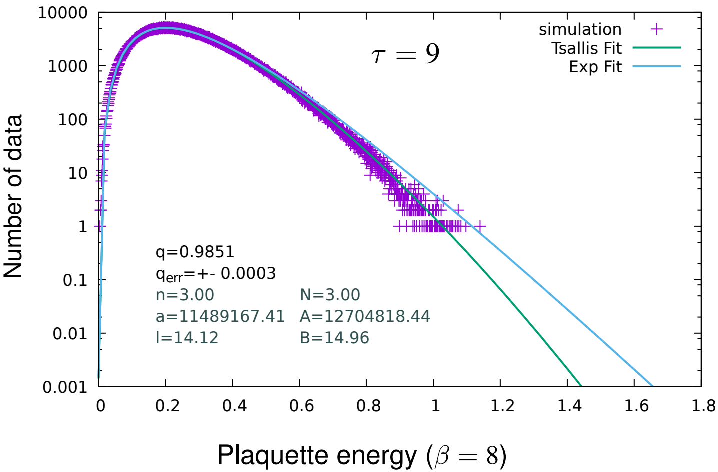

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Plaquette energy histogram - semi-log

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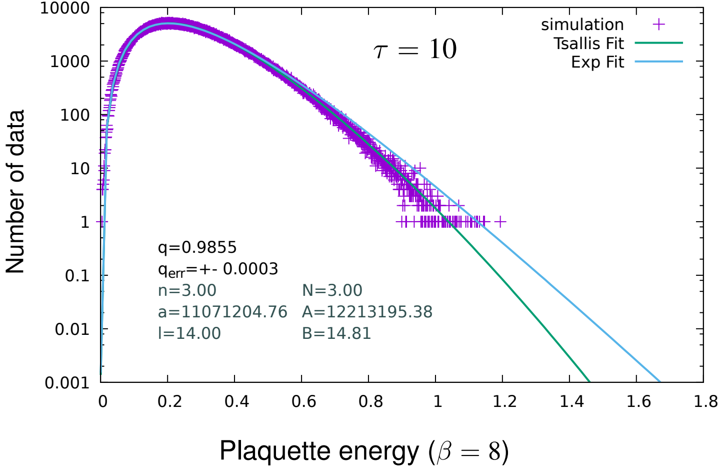

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Exp: $g(x) = Ax^N \mathrm{e}^{-Bx}$

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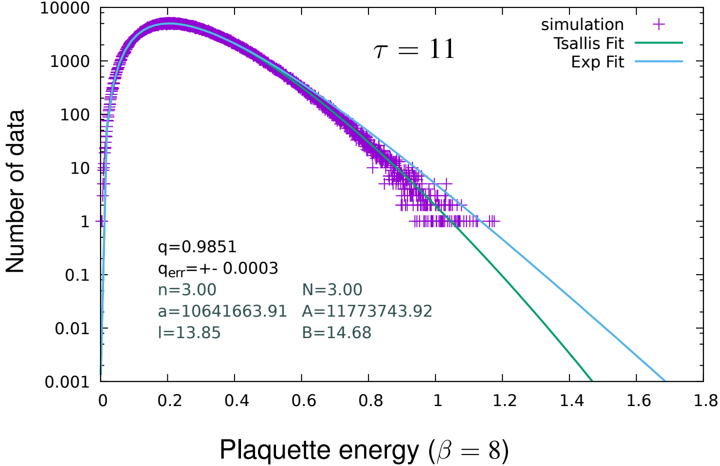

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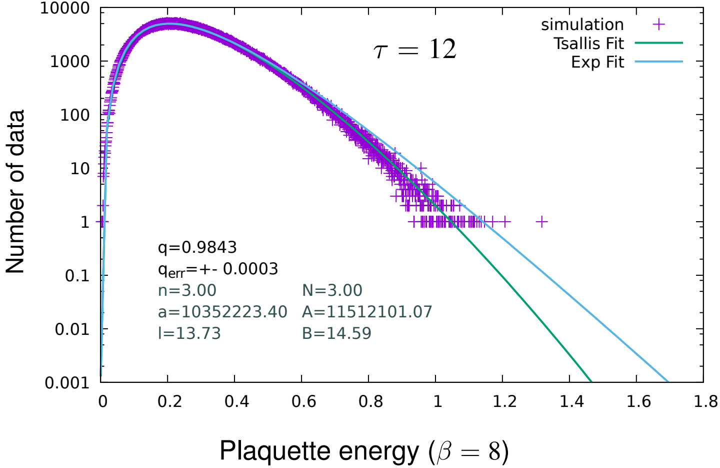

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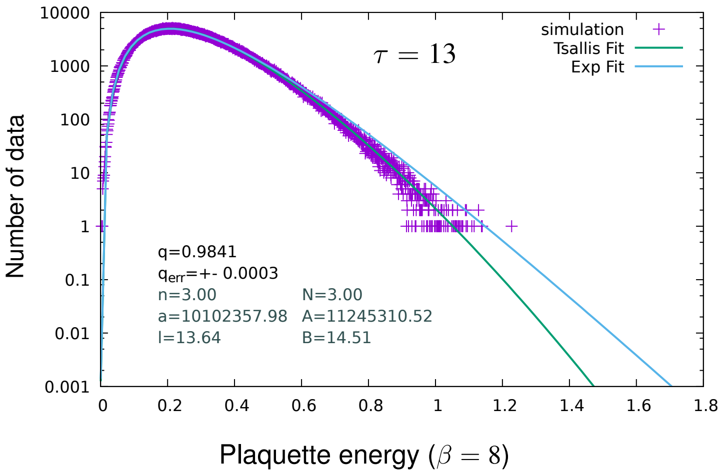

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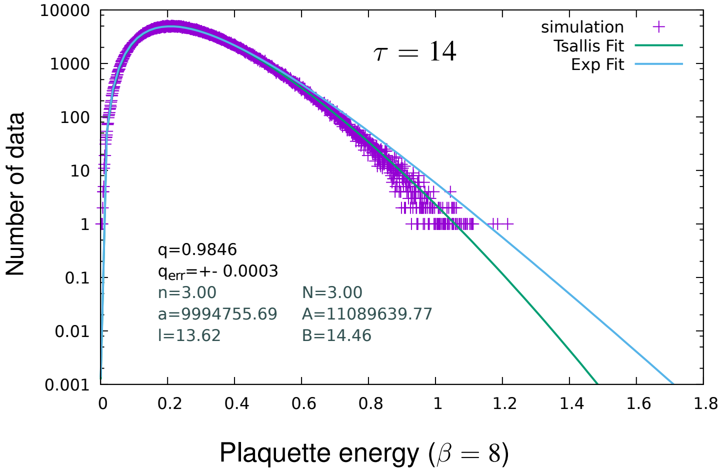

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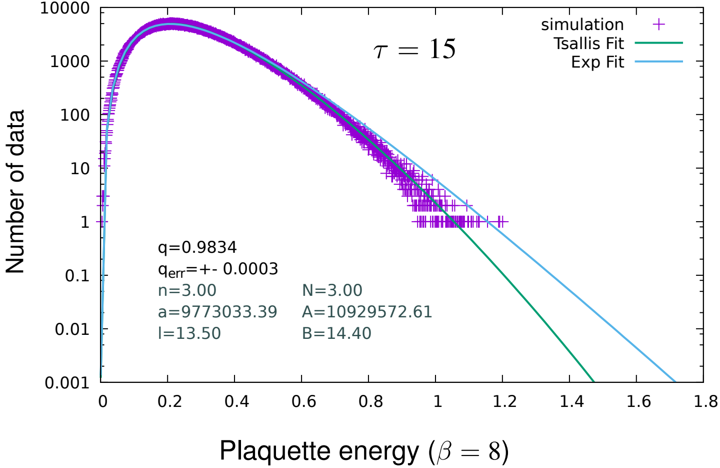

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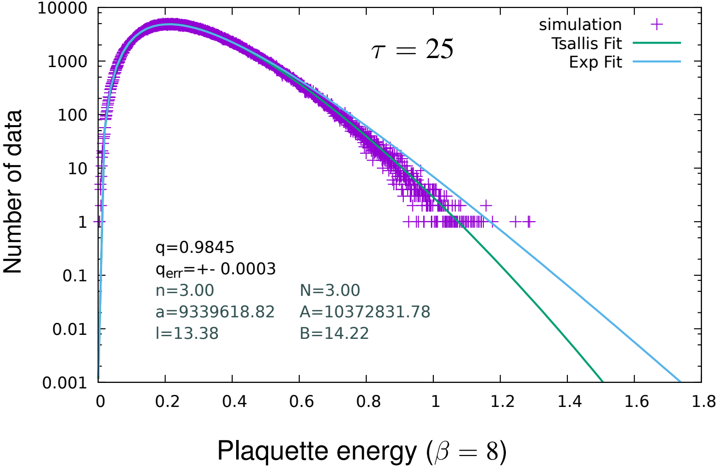

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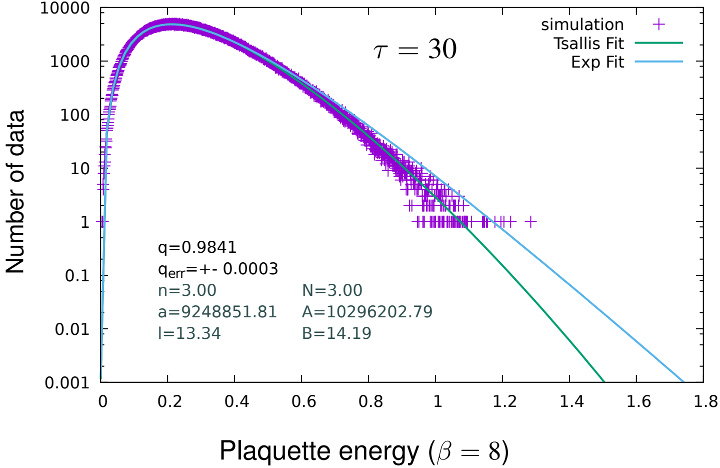

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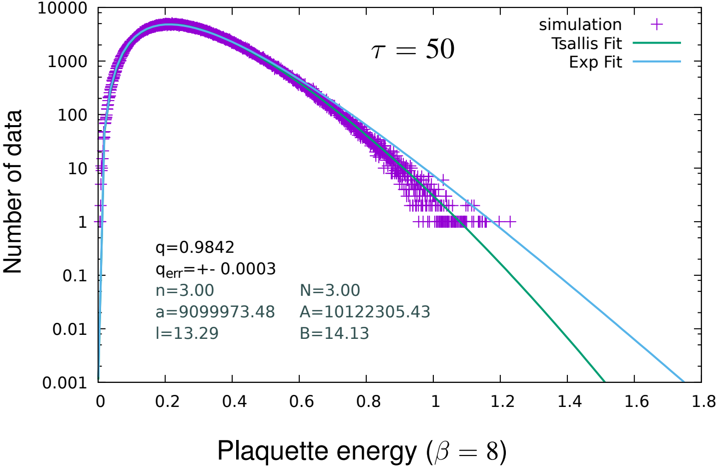

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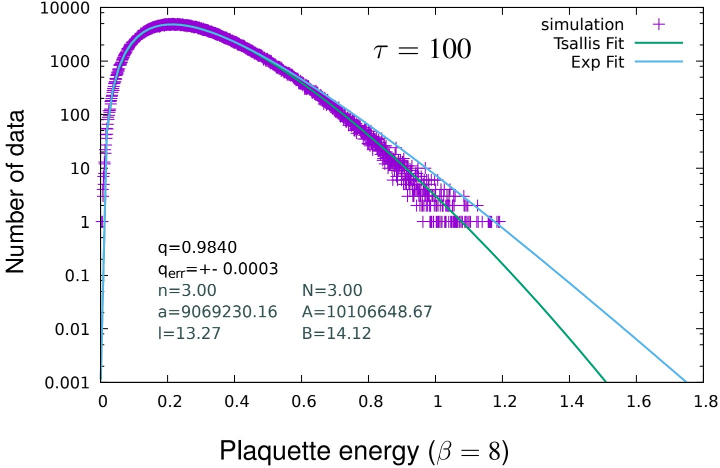

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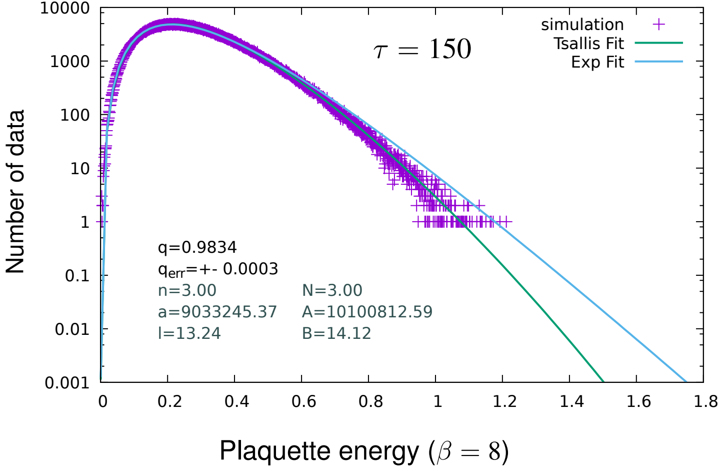

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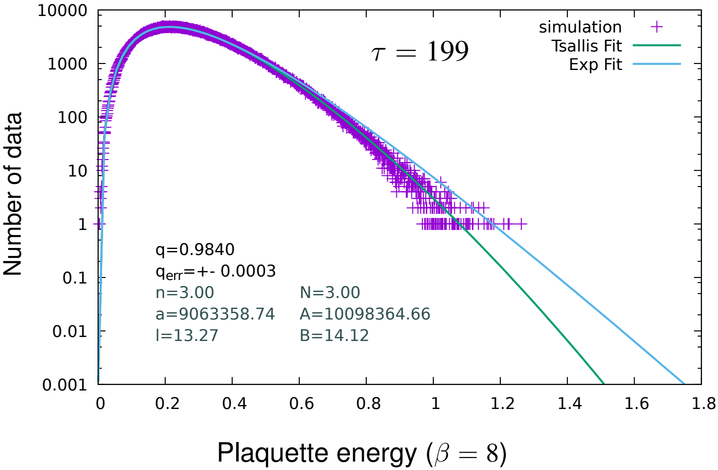

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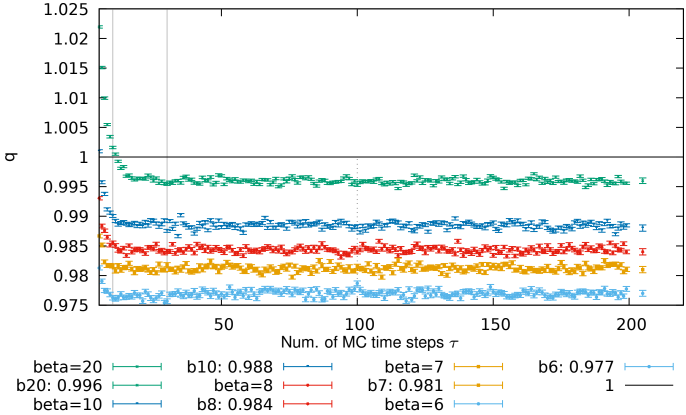
Tsallis q for various β vs. MC time



- N_t fix (not the phys. temperature)

Sommer-scale

[R. Sommer (1994), Nucl.Phys.B411:839-854]

- $F(r)$ - force between static quarks
- $$F(R(c))R(c)^2 = c \qquad (12)$$
- $c = 1.65 \rightarrow R_0 \equiv R(1.65) = 0.49\,\mathrm{fm}$ (fenom. models)
- $R(c)$ - hadronic length scale

Sommer-scale

- calculate the potential $V(\mathbf{r})$
- $F_{\mathbf{d}}(r_I) = |\mathbf{d}|^{-1}[V(\mathbf{r}) - V(\mathbf{r}-\mathbf{d})]$
  $r_I = [4\pi|\mathbf{d}|^{-1}(G(\mathbf{r}) - G(\mathbf{r}-\mathbf{d}))]^{-1/2}$
  $G(\mathbf{r}) = a^{-1}\int_{-\pi}^{\pi} \frac{d^3k}{(2\pi)^3} \frac{\prod_{j=1}^{3}\cos(r_j k_j/a)}{4\sum_{j=1}^{3}\sin^2(k_j/2)}$
- interpolate from neighbouring points:
  $F(r) = f_1 + f_2 r^{-2}$

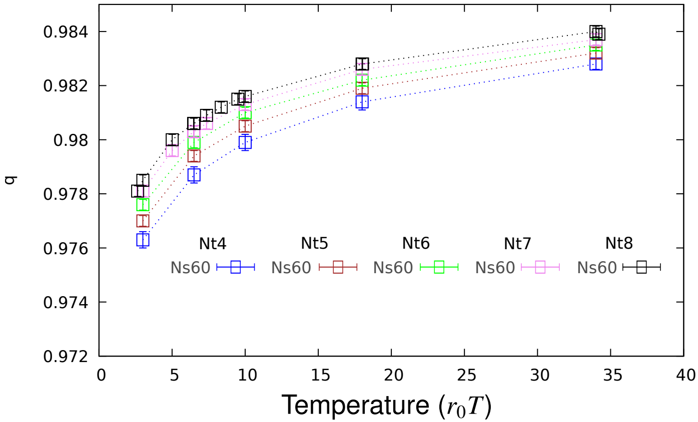
Literature

β	r_0/a [24]	r_0/a [25]	r_0/a [our value]	$N_\tau \times N_s^3$	N_{conf}
5.7	2.922(9)				
5.8	3.673(5)				
5.95	4.898(12)				
6.07	6.033(17)				
6.2	7.380(26)				
6.3			8.52(4)	32×32^3	216
6.3			8.51(2)	32×48^3	211
6.3			8.52(2)*	32×64^3	202
6.336			8.95(3)	64×32^3	220
6.4	9.74(5)		9.80(3)	36×36^3	206
6.5			11.16(2)	44×44^3	202
6.57	12.38(7)	12.18(10)**			
6.69		14.20(12)**			
6.81		16.54(12)**			
6.92		19.13(15)**			

[A. Francisa et al. (2015), Phys.Rev.D91.096002],
[S. Necco and R. Sommer (2002), Nucl.Phys.B622:328],
[M. Guagnelli et al.[ALPHA] (1998), Nucl.Phys.B535:389]

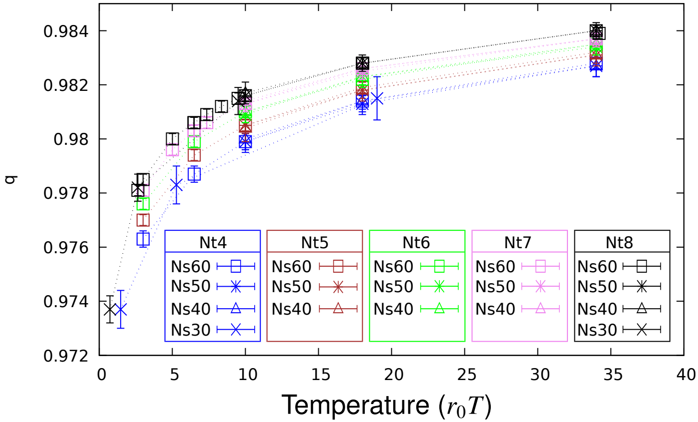
My try: $\beta = 5.5 \rightarrow r_0/a = 2.427$, $(20 \times 10^3$ smear lvl. 2)

Tsallis q for various β and N_t



- $r_0 T = \frac{1}{N_t} \frac{r_0}{a}$

Tsallis q for various β and N_t - TDL Check



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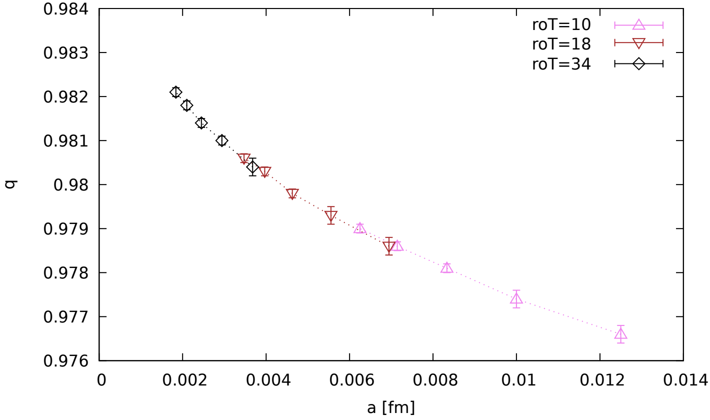
Tsallis q vs a - fix N_t



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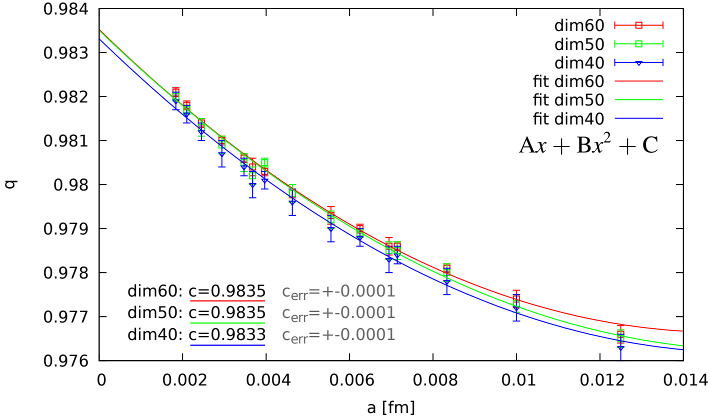
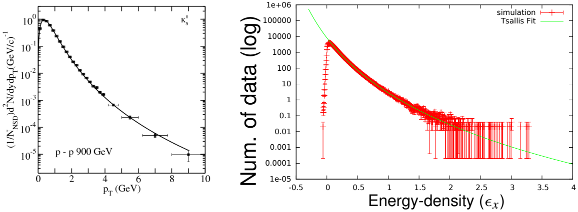


Conclusions



Experiment vs. Φ^4 simulation

Classical Φ^4: Tsallis fits well

$q \approx 1.028 > 1$ is in the order of the exp. values

SU(3) YM: Tsallis fits well

$q \approx 0.984 < 1 \rightarrow$ asymptotic freedom (?)

Future?

- full QCD

Thank you for your attention!

Order parameter

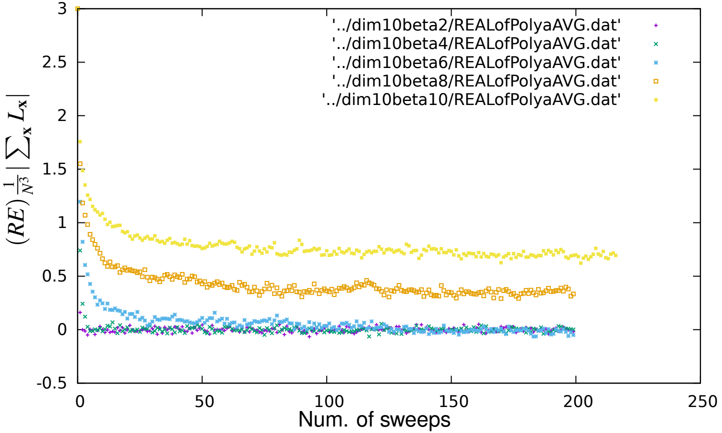


Figure: order parameter