Time evolution of the entanglement entropy after quantum quenches

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Outline

- Global quenches
- Local quenches
- Long range interactions
- Disordered systems, localization
- Finite time ramps, Kibble-Zurek scaling: next time

Ballistic Spreading of Entanglement in a Diffusive Nonintegrable System

Hyungwon Kim and David A. Huse

“ Entanglement is not a conserved quantity like energy, that is transported. Instead, it is more like an infection or epidemic [3] that multiplies and spreads. An initial state that is a product state has the information about the initial state of each local degree of freedom (spins in our model below) initially localized on that degree of freedom. Under the system’s unitary time evolution, quantum information about each spin’s initial state can spread with time to other spins, due to the spin-spin interactions. This can make those spins that share this information entangled. ”

Global quenches

Evolution of entanglement entropy in one-dimensional systems

Pasquale Calabrese[1] **and John Cardy**[1,2]

J. Stat. Mech. (2005) P04010

$$S_A(t) \sim \begin{cases} \dfrac{\pi c t}{6\epsilon} & (t < \ell/2)\,, \\[2ex] \dfrac{\pi c\,\ell}{12\epsilon} & (t > \ell/2)\,, \end{cases}$$

Quasiparticle picture

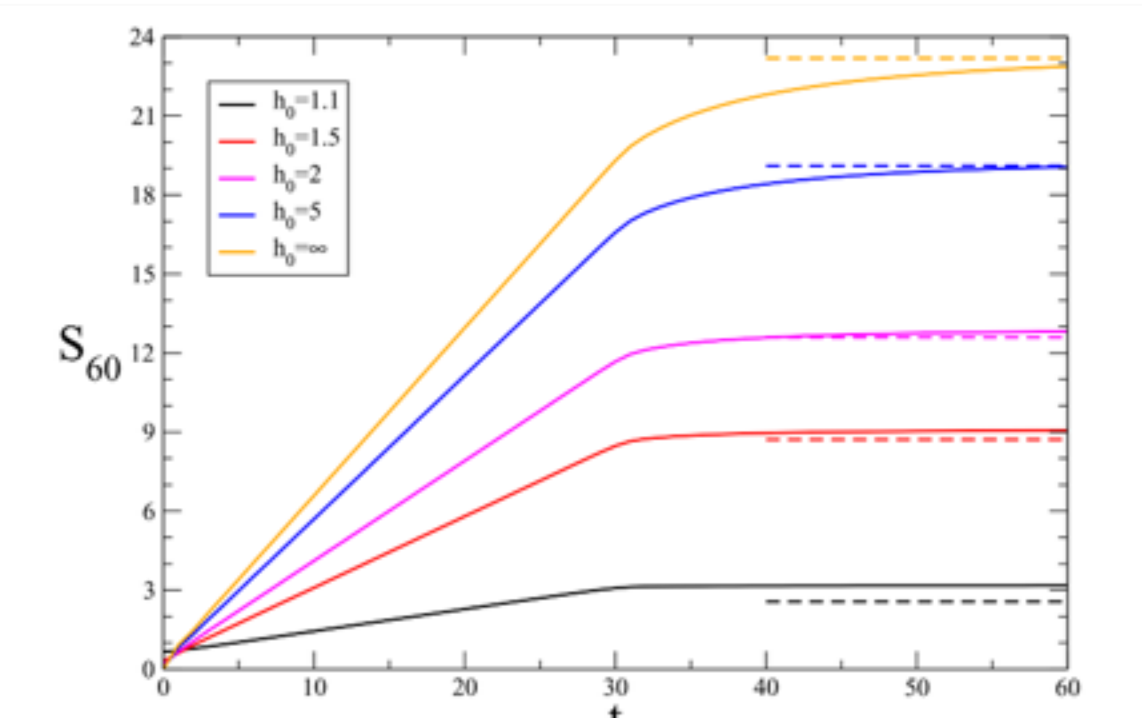
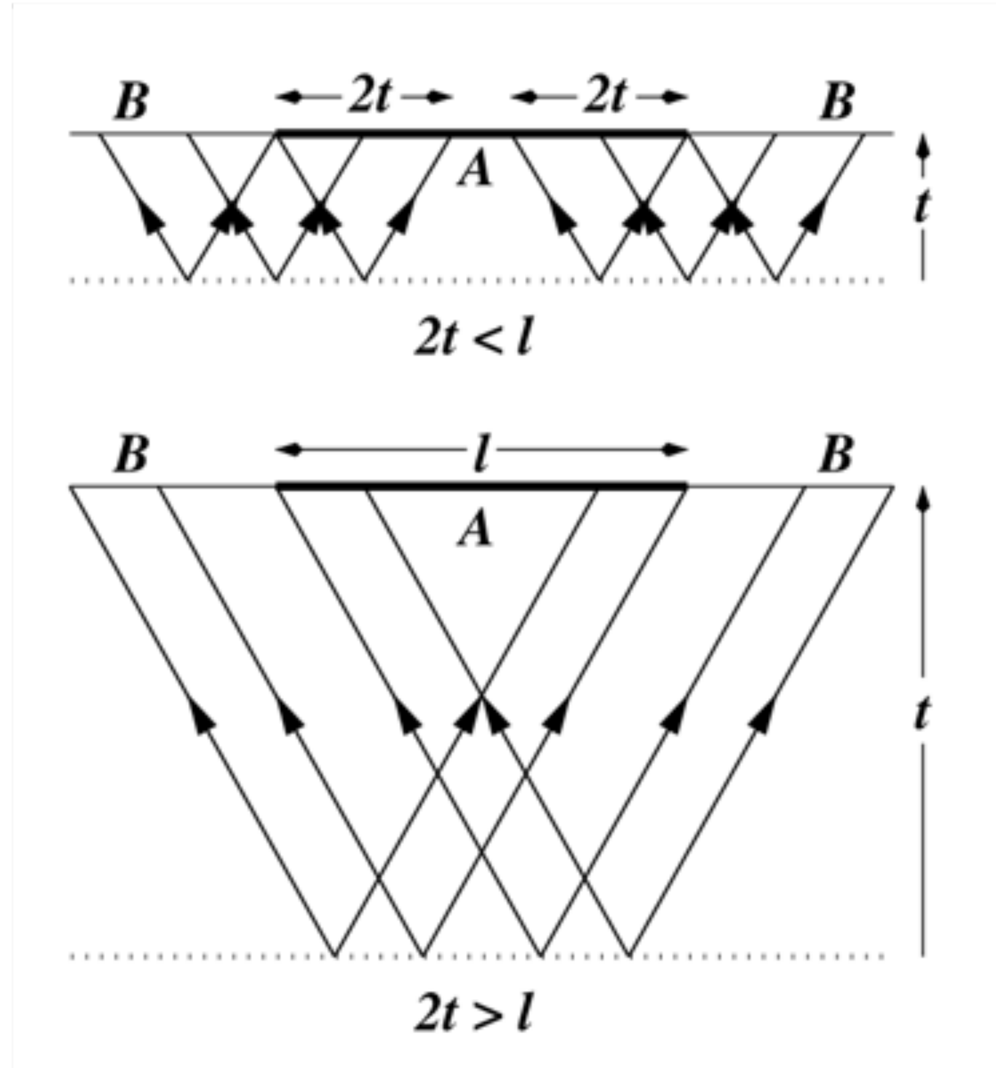


FIG. 4. $S_{60}(t)$ for the quench from $h_0 = \infty$, 5, 2, 1.5, 1.1 to $h = 1$. The dashed lines are the leading asymptotic results for large ℓ Eq. (3.19).



Evolution of entanglement entropy in one-dimensional systems

Pasquale Calabrese[1] and John Cardy[1,2]

J. Stat. Mech. (2005) P04010

Transverse field Ising chain

$$H_I(h) = -\frac{1}{2}\sum_j [\sigma_j^x \sigma_{j+1}^x + h\sigma_j^z]\,,$$

$$S_\ell(\infty) = \frac{\ell}{2\pi}\int_0^{2\pi} d\varphi H\left(\frac{1-\cos\varphi(h+h_0)+hh_0}{\sqrt{(h^2+1-2h\cos\varphi)(h_0^2+1-2h_0\cos\varphi)}}\right) \qquad H(x) = -\frac{1+x}{2}\log\frac{1+x}{2} - \frac{1-x}{2}\log\frac{1-x}{2}$$

Quench from $h_0 = \infty$ *to* $h = 1$

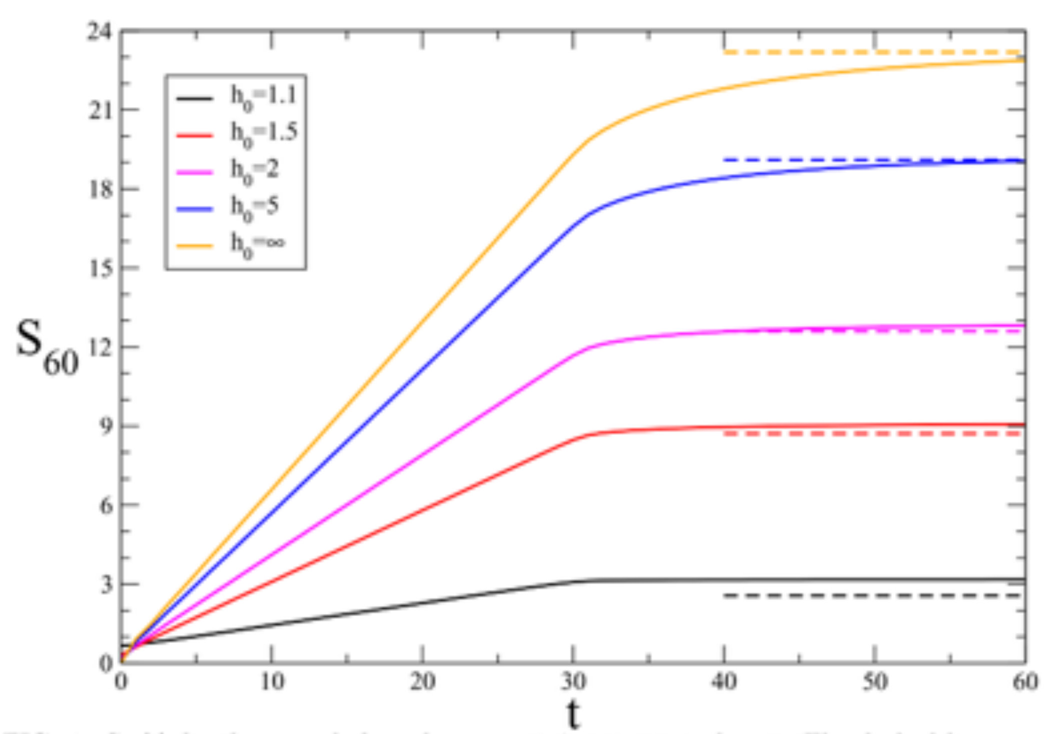


FIG. 4. $S_{60}(t)$ for the quench from $h_0 = \infty, 5, 2, 1.5, 1.1$ to $h = 1$. The dashed lines are the leading asymptotic results for large ℓ Eq. (3.19).

Quench from $h_0 > 1$ *to* $h = 1$

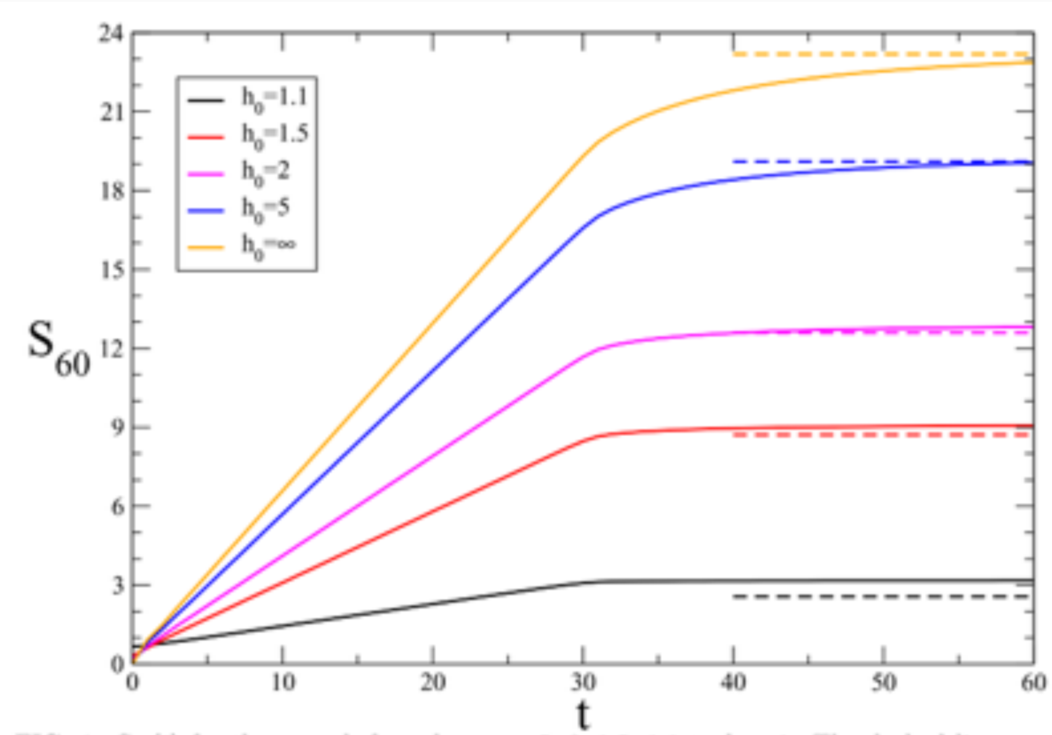


FIG. 4. $S_{60}(t)$ for the quench from $h_0 = \infty, 5, 2, 1.5, 1.1$ to $h = 1$. The dashed lines are the leading asymptotic results for large ℓ Eq. (3.19).

Evolution of entanglement entropy in one-dimensional systems

Pasquale Calabrese[1] and John Cardy[1,2]

J. Stat. Mech. (2005) P04010

quench to the gapped phase

Quench from $h_0 = \infty$ *to* $h > 1$

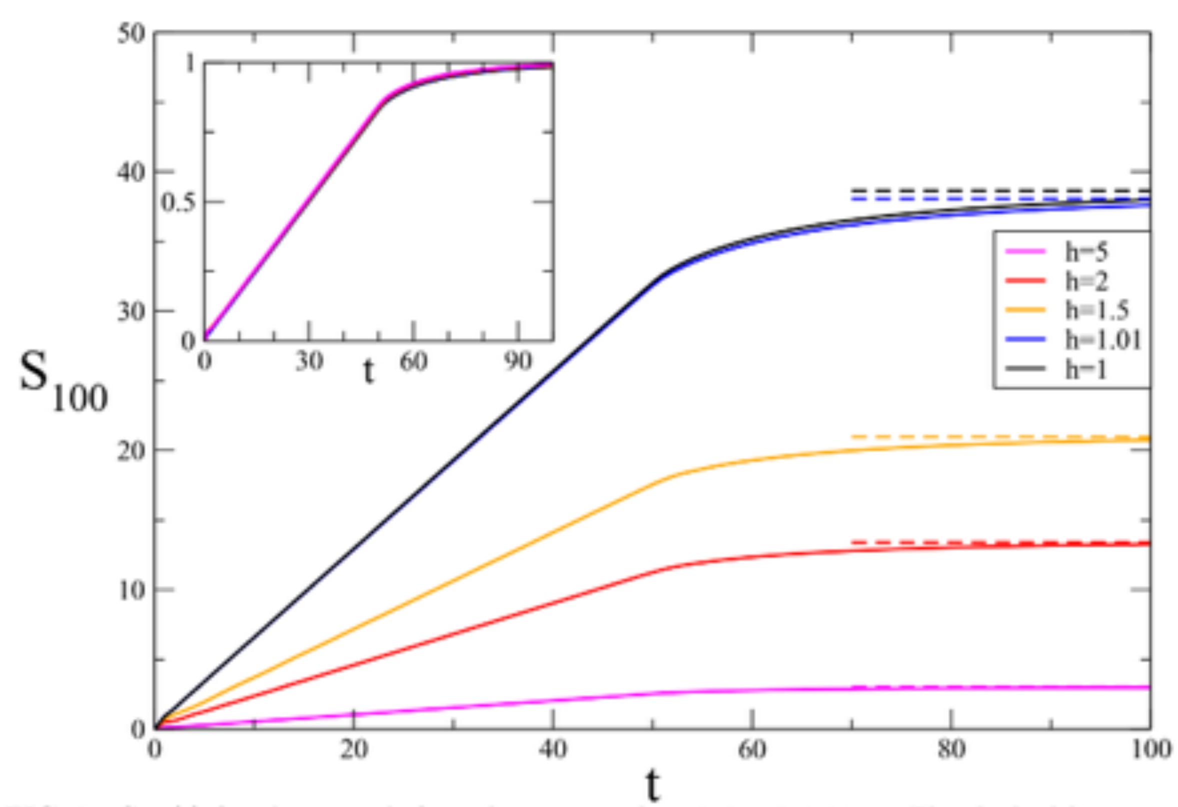


FIG. 5. $S_{100}(t)$ for the quench from $h_0 = \infty$ to $h = 5, 2, 1.5, 1.01, 1$. The dashed lines are the leading asymptotic results for large ℓ Eq. (3.19). The inset shows the rescaling of the curves, according to the asymptotic value $S_{100}(\infty)$.

PHYSICAL REVIEW X **3**, 031015 (2013)

Entanglement Growth in Quench Dynamics with Variable Range Interactions

J. Schachenmayer,[1] B. P. Lanyon,[2] C. F. Roos,[2] and A. J. Daley[1]

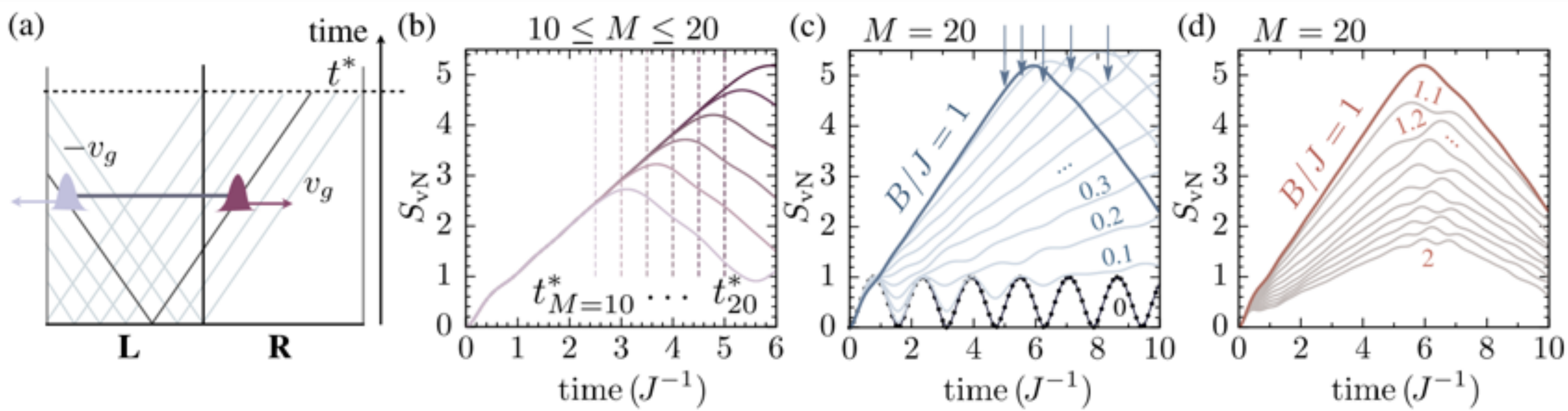


FIG. 2. Entanglement growth after a quantum quench in the transverse Ising model in which nearest-neighbor interactions are introduced suddenly. (a) Illustration of entanglement distribution, via entangled quasiparticle pair excitations that move within a Lieb-Robinson light cone. Boundary effects for this system with open boundary conditions stop the linear increase at a critical time t^*. (b–d) Time evolution of half-chain entropies for $M = 10$, 12, 14, 16, 18, and 20 spins (ED calculation). (b) Boundary effects as a breakdown of the linear growth. Respective critical times calculated for the free fermion model are shown as vertical lines. (c) The crossover from the oscillatory behavior for $B = 0$ (dots: analytical result) to a linear increase ($M = 20$). With decreasing B, boundary effects shift to later times; critical times are indicated as vertical arrows. (d) The half-chain entropy growth, which is fastest for $B = 1$ and decreases again for $B > 1$ ($M = 20$).

Entanglement entropy dynamics of Heisenberg chains

Gabriele De Chiara[1], Simone Montangero[1,2],
Pasquale Calabrese[3] and Rosario Fazio[1,4]

J. Stat. Mech. (2006) P03001

$$H = \sum_{i=1}^{N-1} J_i(\sigma_x^i \sigma_x^{i+1} + \sigma_y^i \sigma_y^{i+1} + \Delta \sigma_z^i \sigma_z^{i+1})$$

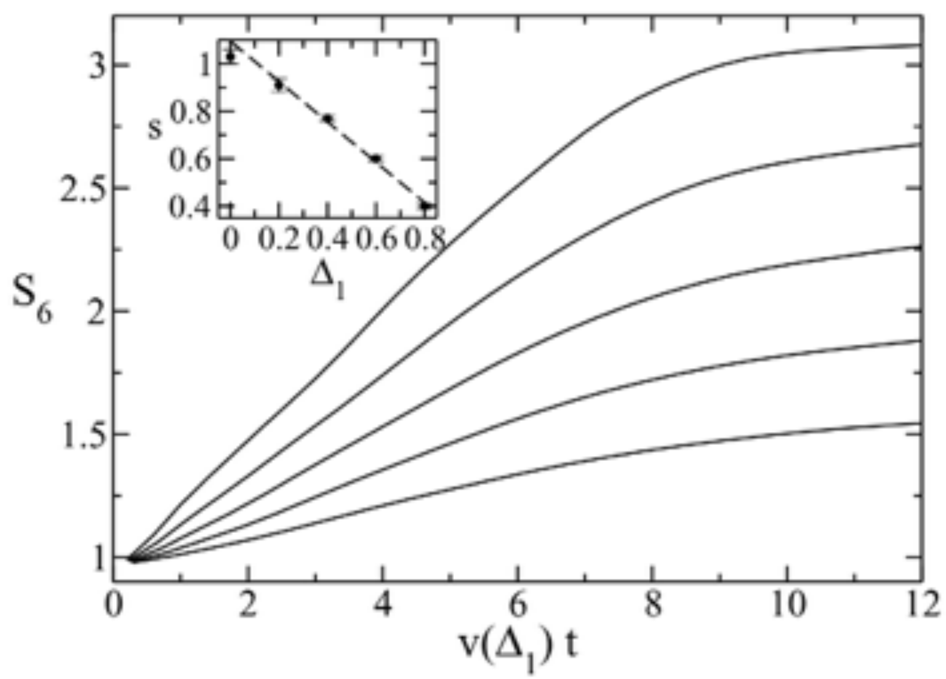


Figure 2. Evolution of the entropy S_6 with various quenches. $\Delta_0 = 1.5$ while $\Delta_1 = 0.0, 0.2, 0.4, 0.6, 0.8$ as a function of $v(\Delta_1)t$. Inset: initial slope value of S_6 as a function of Δ_1 and comparison to a linear fit with slope -0.85 ± 0.02 (dashed line).

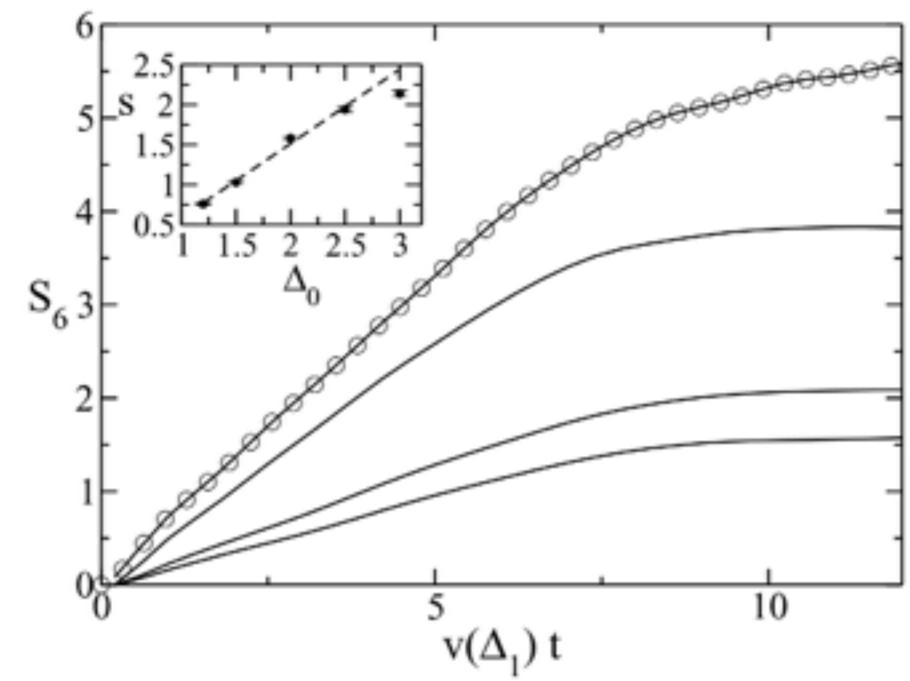


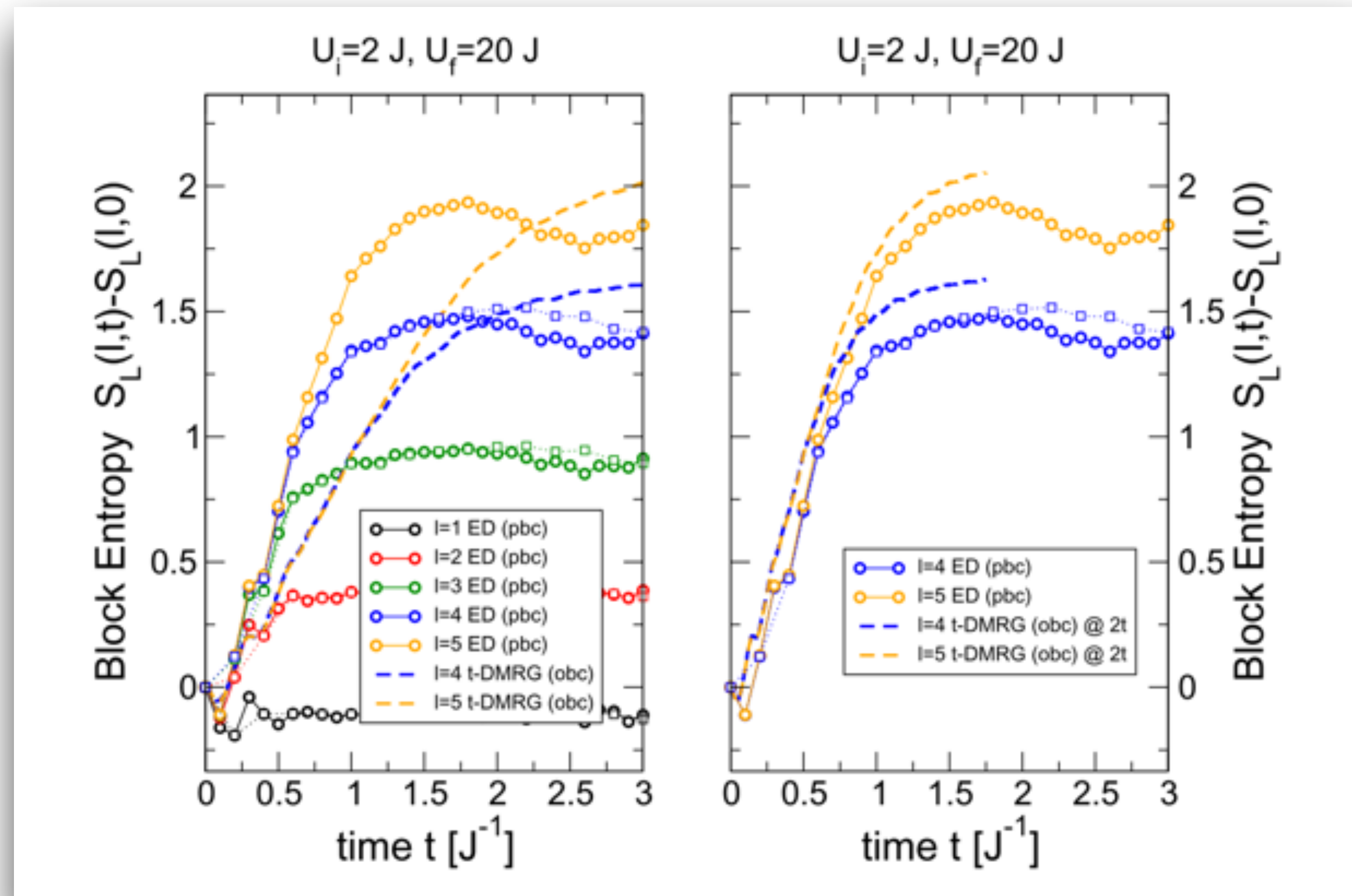
Figure 3. Evolution of the entropy S_6 with various quenches. $\Delta_0 = 1.2, 1.5, 3.0, \infty$ while $\Delta_1 = 0.0$ as a function of $v(\Delta_1)t$ and shifted so to coincide in $t = 0$. For $\Delta_0 = \infty$ we show also the exact result obtained by diagonalization (circles). Inset: initial slope value of S_6 as a function of Δ_1.

Spreading of correlations and entanglement after a quench in the one-dimensional Bose–Hubbard model

Andreas M Läuchli[1] and Corinna Kollath[2]

J. Stat. Mech. (2008) P05018

$$H(J,U) = -J\sum_j (b_j^\dagger b_{j+1} + \mathrm{h.c.}) + \frac{U}{2}\sum_j n_j(n_j-1)$$



PHYSICAL REVIEW A **78**, 010306(R) (2008)

Evolution of entanglement entropy following a quantum quench: Analytic results for the *XY* chain in a transverse magnetic field

XY chain with transverse field
arbitrary quench

$$H(h,\gamma) = -\sum_{j=1}^{N}\left[\frac{1+\gamma}{4}\sigma_j^x\sigma_{j+1}^x + \frac{1-\gamma}{4}\sigma_j^y\sigma_{j+1}^y + \frac{h}{2}\sigma_j^z\right]$$

$$S_\ell(t) = t\int_{2|\epsilon'|t<\ell}\frac{d\varphi}{2\pi}2|\epsilon'|H(\cos\Delta_\varphi) + \ell\int_{2|\epsilon'|t>\ell}\frac{d\varphi}{2\pi}H(\cos\Delta_\varphi), \qquad (2)$$

where $\epsilon' = d\epsilon/d\varphi$ is the derivative of the dispersion relation $\epsilon^2 = (h-\cos\varphi)^2 + \gamma^2\sin^2\varphi$ and represents the momentum dependent sound velocity (that because of locality has a maximum we indicate as $v_M \equiv \max_\varphi|\epsilon'|$), $\cos\Delta_\varphi = [hh_0 - \cos\varphi(h+h_0) + \cos^2\varphi + \gamma\gamma_0\sin^2\varphi]/\epsilon\epsilon_0$ contains all the quench information [9] and $H(x) = -[(1+x)/2\ln(1+x)/2 + (1-x)/2\times\ln(1-x)/2]$.

PHYSICAL REVIEW A **78**, 010306(R) (2008)

Evolution of entanglement entropy following a quantum quench: Analytic results for the *XY* chain in a transverse magnetic field

Finite ℓ

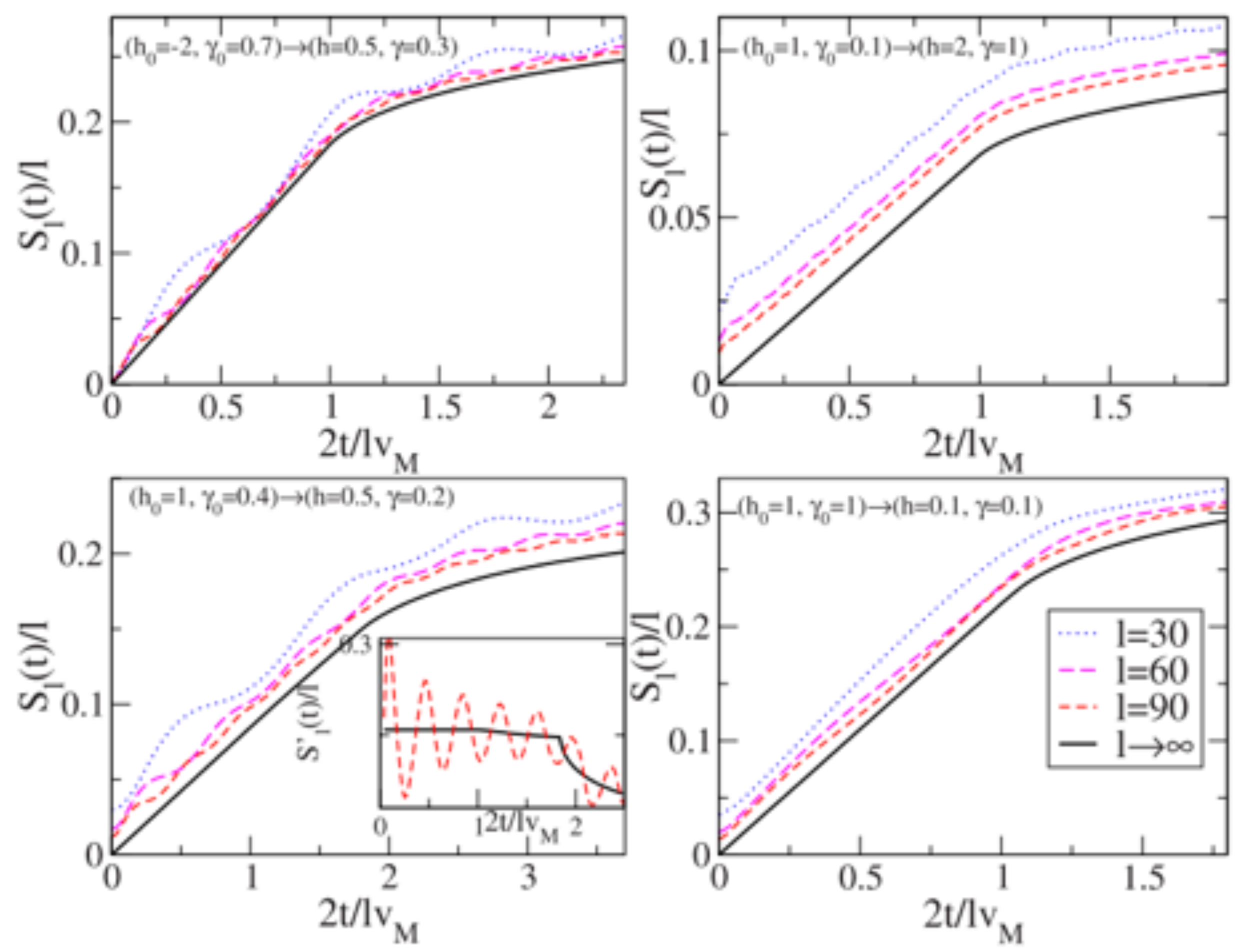


FIG. 2. (Color online) Time evolution of the entanglement entropy $S_\ell(t)/\ell$ for several quenches and ℓ. The straight line is the leading asymptotic result for large ℓ. The inset in the bottom-left graph shows the derivative with respect to time of $S_\ell(t)$ for $\ell \to \infty$ and the numerical derivative for $\ell = 90$.

Ballistic Spreading of Entanglement in a Diffusive Nonintegrable System

Hyungwon Kim and David A. Huse

$$H = \sum_{i=1}^{L} g\sigma_i^x + \sum_{i=2}^{L-1} h\sigma_i^z + (h-J)(\sigma_1^z + \sigma_L^z) + \sum_{i=1}^{L-1} J\sigma_i^z\sigma_{i+1}^z$$

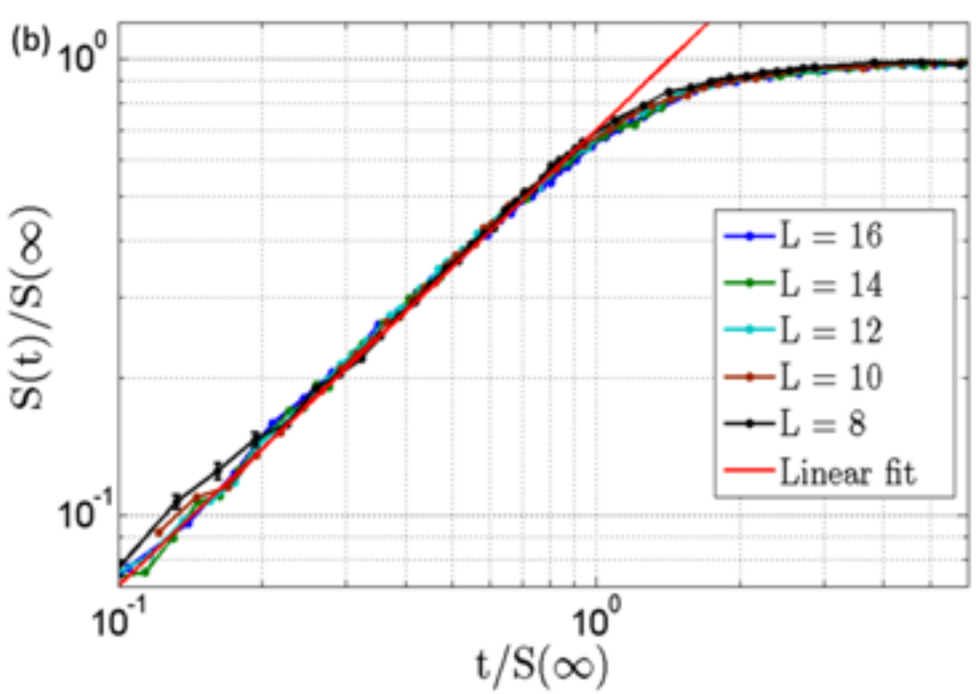


FIG. 1 (color online). (a) Spreading of entanglement entropy $S(t)$ for chains of length L. Initially, the entanglement grows linearly with time for all cases, with the same speed $v \cong 0.70$. Then, the entanglement saturates at long time. This saturation

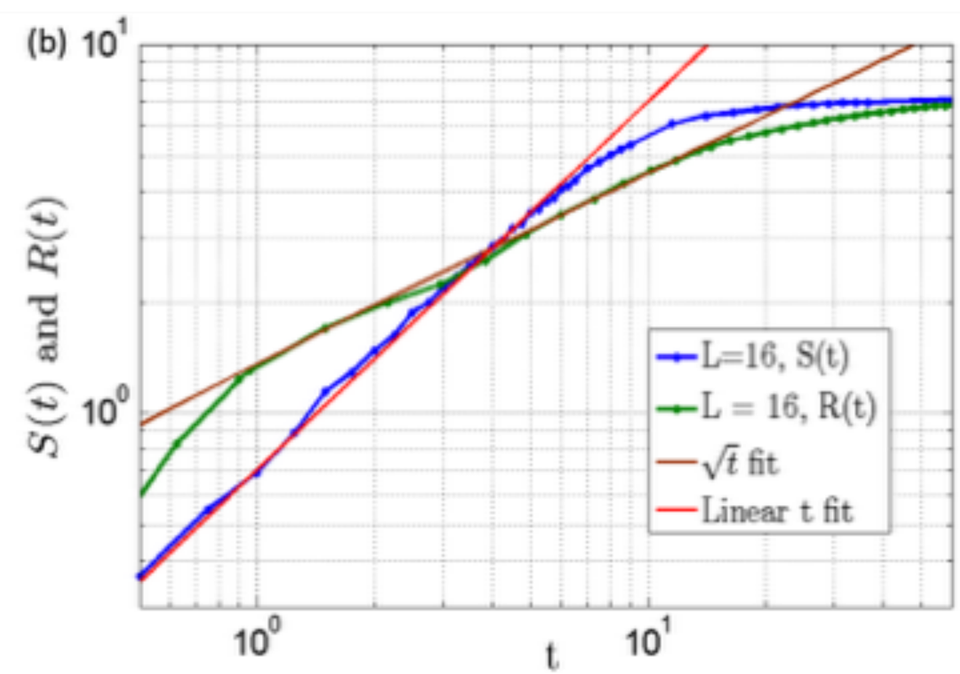


FIG. 2 (color online). (a) The average energy spreading $R(t)$ (defined in the main text) vs time. Before saturation, its behavior does not depend on the system size. As we increase the system size, diffusive $\sqrt{t}$ behavior becomes more apparent. (b) Direct comparison of $S(t)$ and $R(t)$ for $L = 16$. It is clear that the entanglement spreads faster than energy diffuses in the scaling regime before saturation.

Local quenches

Evolution of entanglement after a local quench

Viktor Eisler and Ingo Peschel

J. Stat. Mech. (2007) P06005

$$H_0 = -\tfrac{1}{2}\sum_{n=-\infty}^{\infty} t_n (c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n)$$

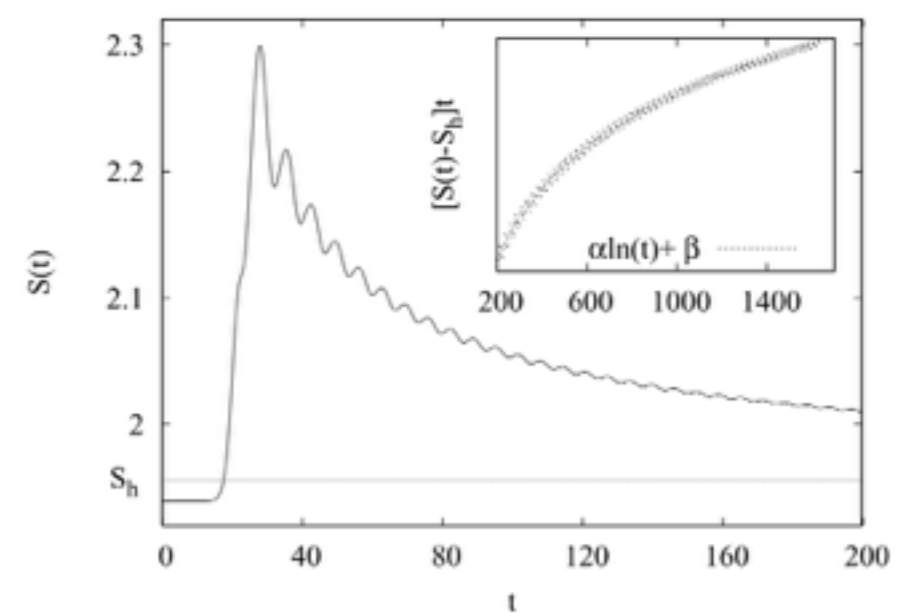


Figure 3. Time evolution of the entanglement entropy for a subsystem of $L = 40$ sites with a central defect $t' = 0$. A sudden jump is followed by a slow relaxation towards the homogeneous value S_h. The inset shows the logarithmic correction to the $1/t$ decay.

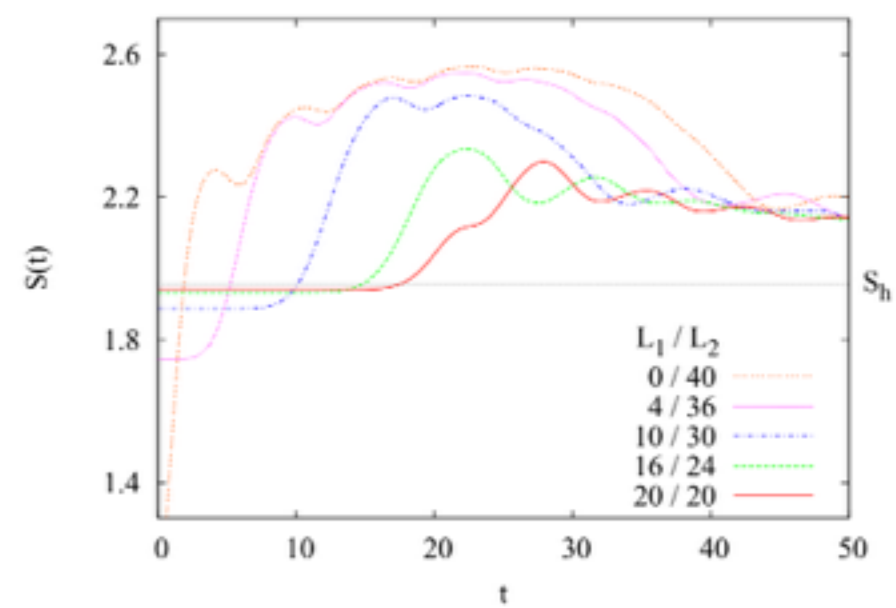


Figure 4. Time evolution of the entropy for a subsystem of $L = 40$ sites for several defect positions L_1/L_2, indicating the number of sites to the left/right of the defect with $t' = 0$.

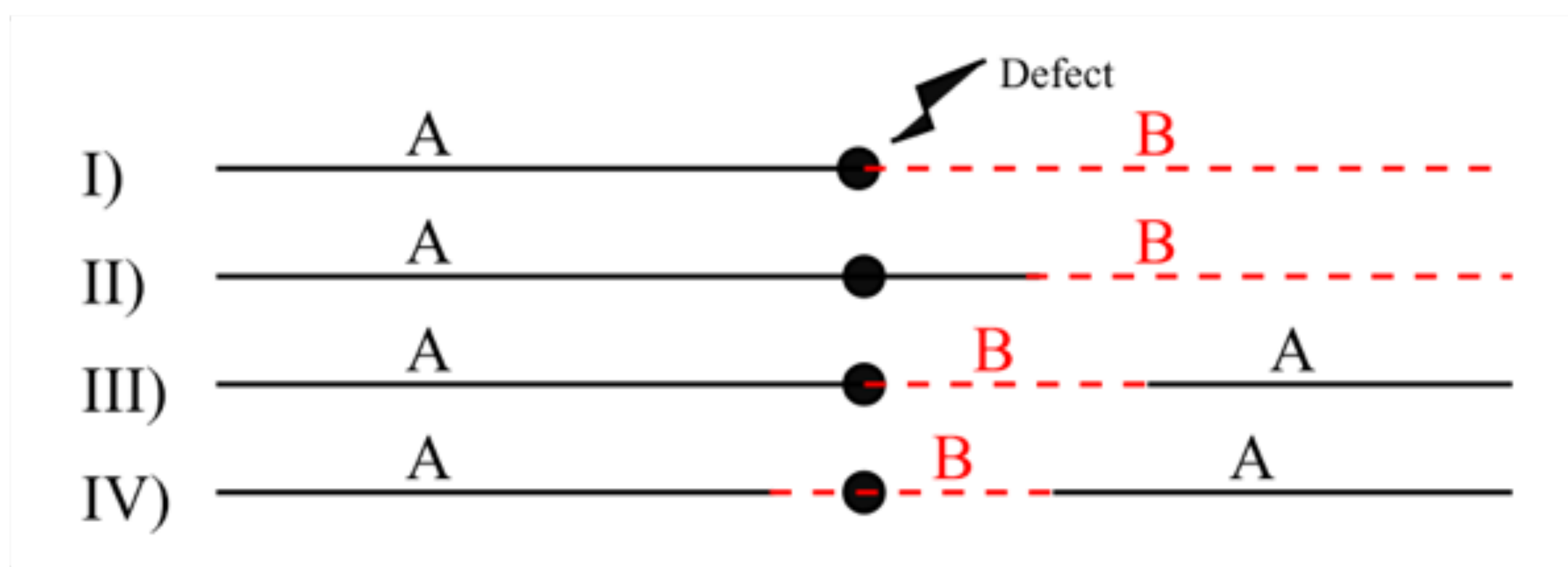
$$S(t) = S_h + \frac{\alpha \ln(t) + \beta}{t}.$$

$$f(t, L) = \frac{c_1}{3}\ln(1+t) + \frac{c_2}{3}\ln(1+L-t) + k,$$

Entanglement and correlation functions following a local quench: a conformal field theory approach

Pasquale Calabrese[1] and John Cardy[2,3]

J. Stat. Mech. (2007) P10004



I) $$S_A(t \gg \epsilon) = \frac{c}{3} \log \frac{t}{a} + k_0 \,,$$

II) $$S_A = \begin{cases} \frac{c}{6} \log \frac{2\ell}{a} + \tilde{c}_1' & t < \ell \,, \\ \frac{c}{6} \log \frac{t^2 - \ell^2}{a^2} + k_0 & t > \ell \,, \end{cases}$$

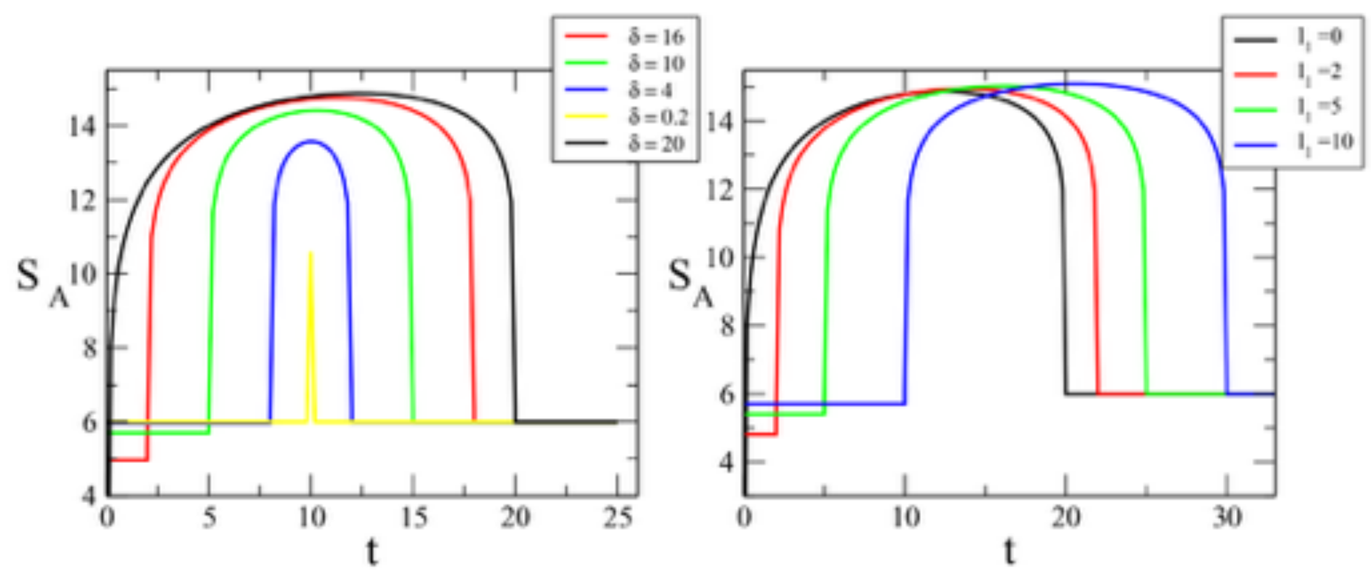
III) $$S_A = \begin{cases} \frac{c}{3} \ln \frac{t}{a} + \frac{c}{6} \ln \frac{\ell}{\epsilon} + \frac{c}{6} \ln 4 \frac{\ell - t}{\ell + t} + 2\tilde{c}_1' & t < \ell \,, \\ \frac{c}{3} \ln \frac{\ell}{a} + 2\tilde{c}_1' & t > \ell \,. \end{cases}$$

Entanglement and correlation functions following a local quench: a conformal field theory approach

Pasquale Calabrese[1] and John Cardy[2,3]

J. Stat. Mech. (2007) P10004

Figure 3. $S_A(t)$ for a general slit $B = [\ell_1, \ell_2]$. The plots are done for finite $\epsilon = 10^{-3}$, and they are indistinguishable from the asymptotic result. In all the plots $\ell_1 - \ell_2 = 20$. Left: $\ell_2 < 0$, for several $\delta = \ell_1 + \ell_2$. Right: $\ell_2 > 0$, for several ℓ_1.



IV)

$$S_A(t < |\ell_2|) = \begin{cases} \frac{c}{6} \log \frac{4\ell_1|\ell_2|}{a^2} + 2\tilde{c}_1' & \ell_2 < 0\,, \\ \frac{c}{6} \log \frac{(\ell_1-\ell_2)^2}{(\ell_1+\ell_2)^2} \frac{4\ell_1\ell_2}{a^2} + 2\tilde{c}_1' & \ell_2 > 0\,. \end{cases}$$

$$S_A(t > \ell_1) = \frac{c}{3} \ln \frac{\ell_1 - \ell_2}{a} + 2\tilde{c}_1'\,.$$

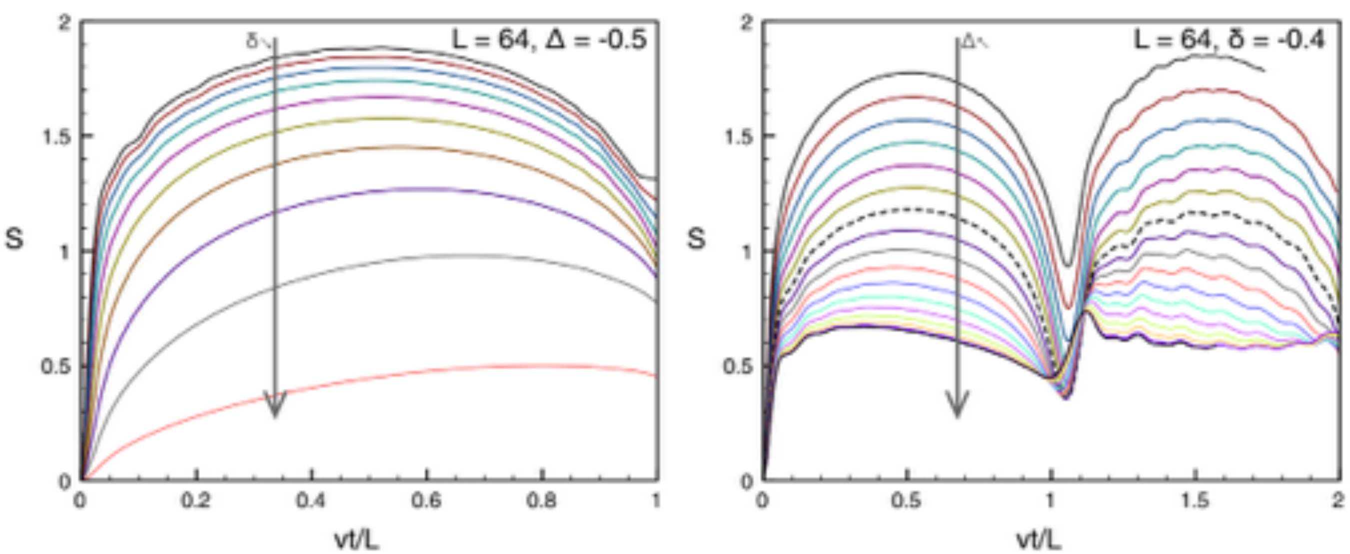
$$S_A(|\ell_2| < t < \ell_1) = \frac{c}{6} \log \frac{(\ell_1-\ell_2)(\ell_1-t)}{(\ell_1+\ell_2)(\ell_1+t)} \frac{4\ell_1(t^2-\ell_2^2)}{\epsilon a^2} + 2\tilde{c}_1'\,.$$

J. Phys. A: Math. Theor. **46** (2013) 175001 (14pp) doi:10.1088/1751-8113/46/17/175001

Entanglement evolution across defects in critical anisotropic Heisenberg chains

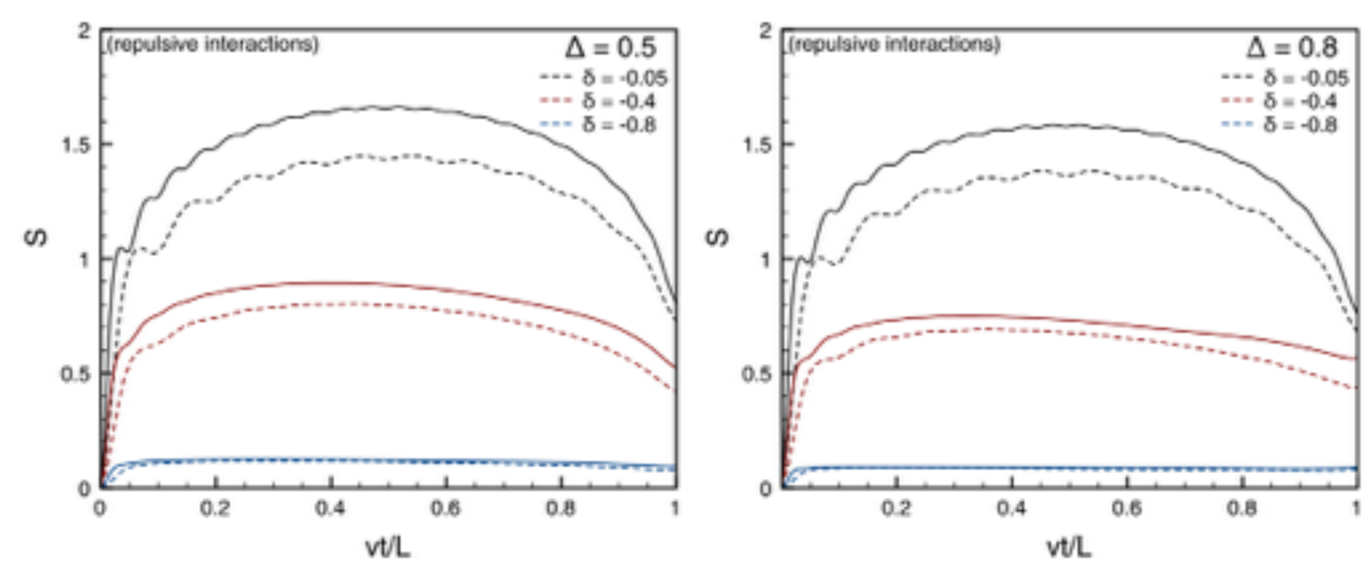
Mario Collura and Pasquale Calabrese

$$H = \sum_{i=1}^{L-1} J_i \left[S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right] \qquad J_i = \begin{cases} 1, & i \neq L/2, \\ 1+\delta, & i = L/2, \end{cases}$$



defect is irrelevant

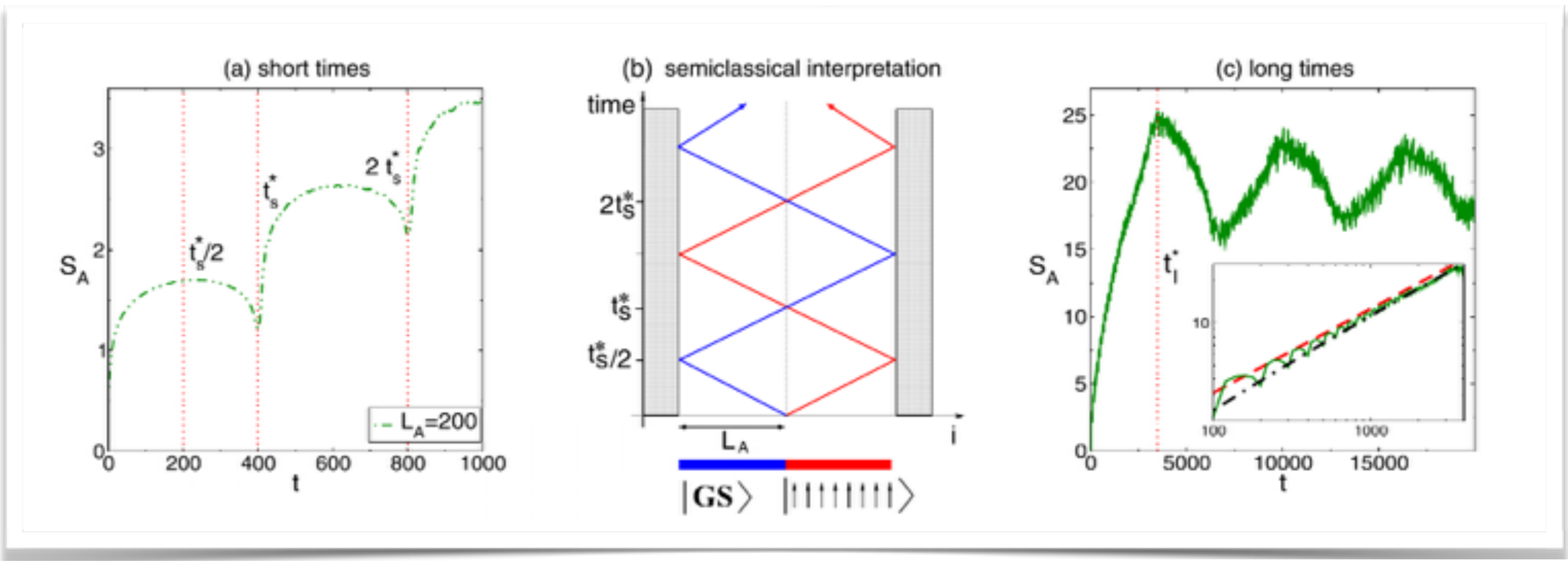
defect is relevant



PHYSICAL REVIEW B **90**, 075144 (2014)

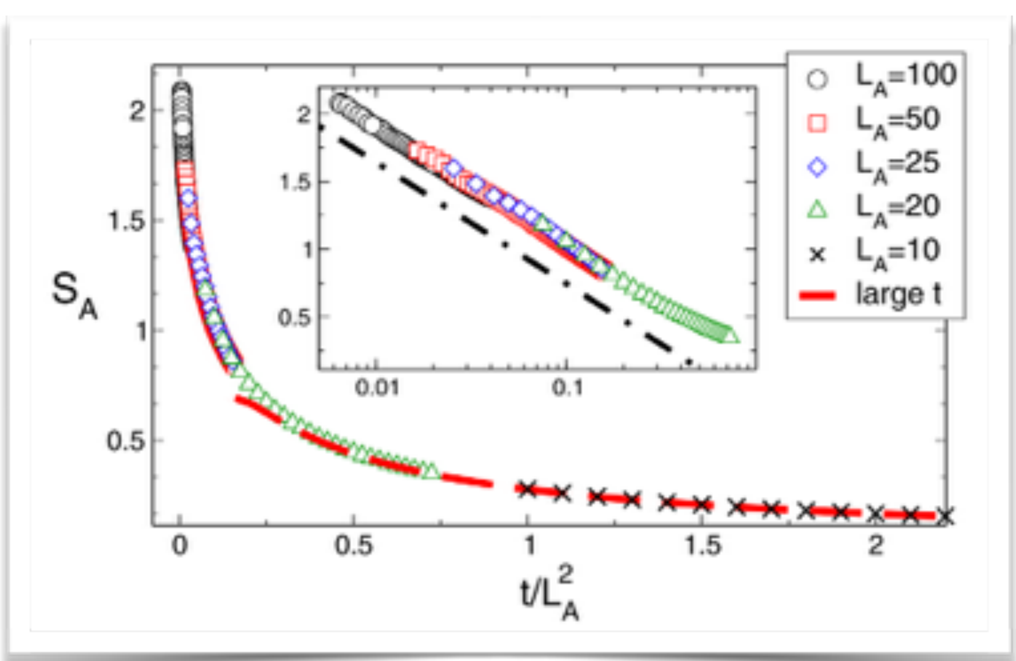
Entanglement spreading after a geometric quench in quantum spin chains

Vincenzo Alba and Fabian Heidrich-Meisner



$$S_{\mathrm{ansatz}}(t) = \alpha_\nu \log(t) + \beta_\nu \log\left[L_A \sin\frac{\nu\pi\, v_s t}{2L_A}\right] + \gamma_\nu.$$

$$\frac{S_A}{L_A} = -\frac{1}{2}[\omega\ \log\,\omega + (1-\omega)\log(1-\omega)].$$



$$S_A(t) = -\alpha\ \log(t/L_A^2) + \beta$$

$$S_A(t) \approx \frac{L_A^2}{2\pi t}\left[1 - \log\frac{L_A^2}{2\pi t}\right]$$

Long range interactions

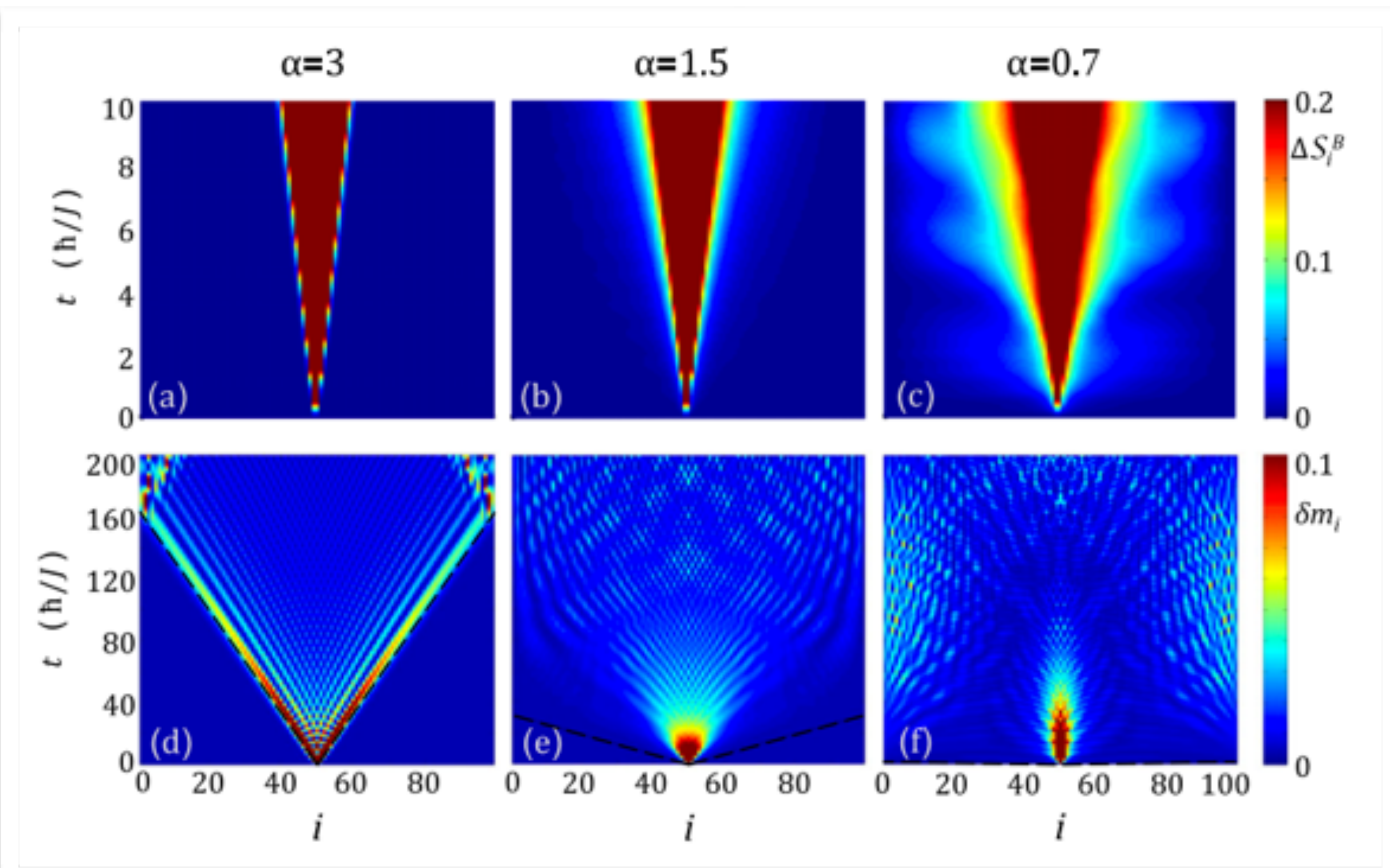
PRL 111, 207202 (2013) PHYSICAL REVIEW LETTERS week ending 15 NOVEMBER 2013

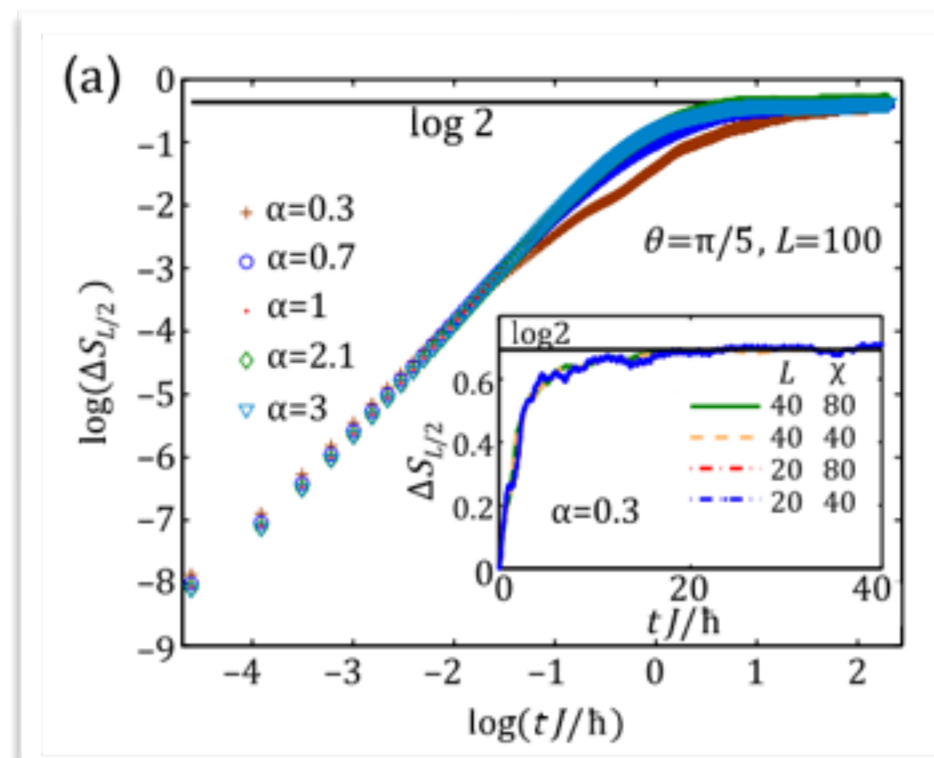
Spread of Correlations in Long-Range Interacting Quantum Systems

P. Hauke[1,2,*] and L. Tagliacozzo[3,†]

$$H = \sin(\theta)\sum_{\langle i,j\rangle}\frac{\sigma_i^x\sigma_j^x}{|i-j|^\alpha} + \cos(\theta)\sum_i \sigma_i^z$$

local perturbation



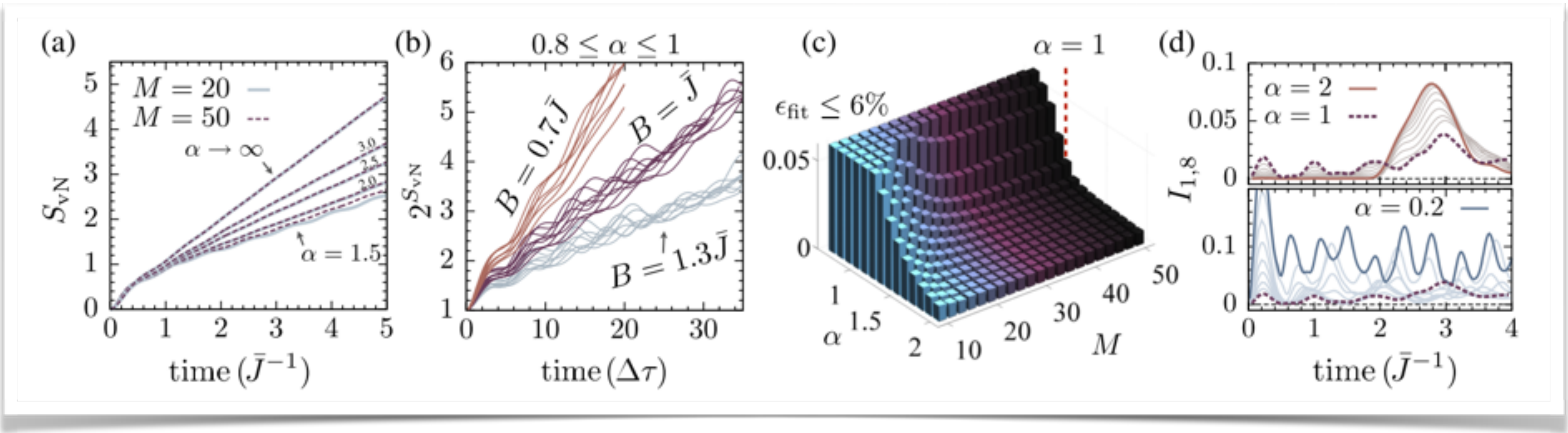


PHYSICAL REVIEW X **3**, 031015 (2013)

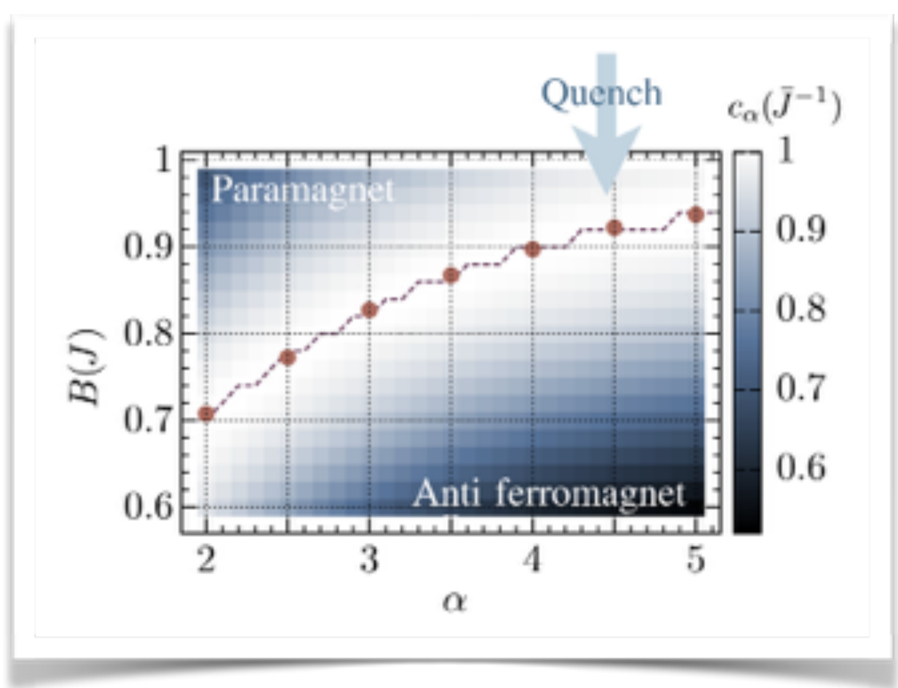
Entanglement Growth in Quench Dynamics with Variable Range Interactions

J. Schachenmayer,[1] B. P. Lanyon,[2] C. F. Roos,[2] and A. J. Daley[1]

crossover @ $\alpha \sim 1$ mutual information



growth rate is largest at the QCP:



Entanglement dynamics in short and long-range harmonic oscillators

M. Ghasemi Nezhadhaghighi[1] and M. A. Rajabpour[2,3,*]

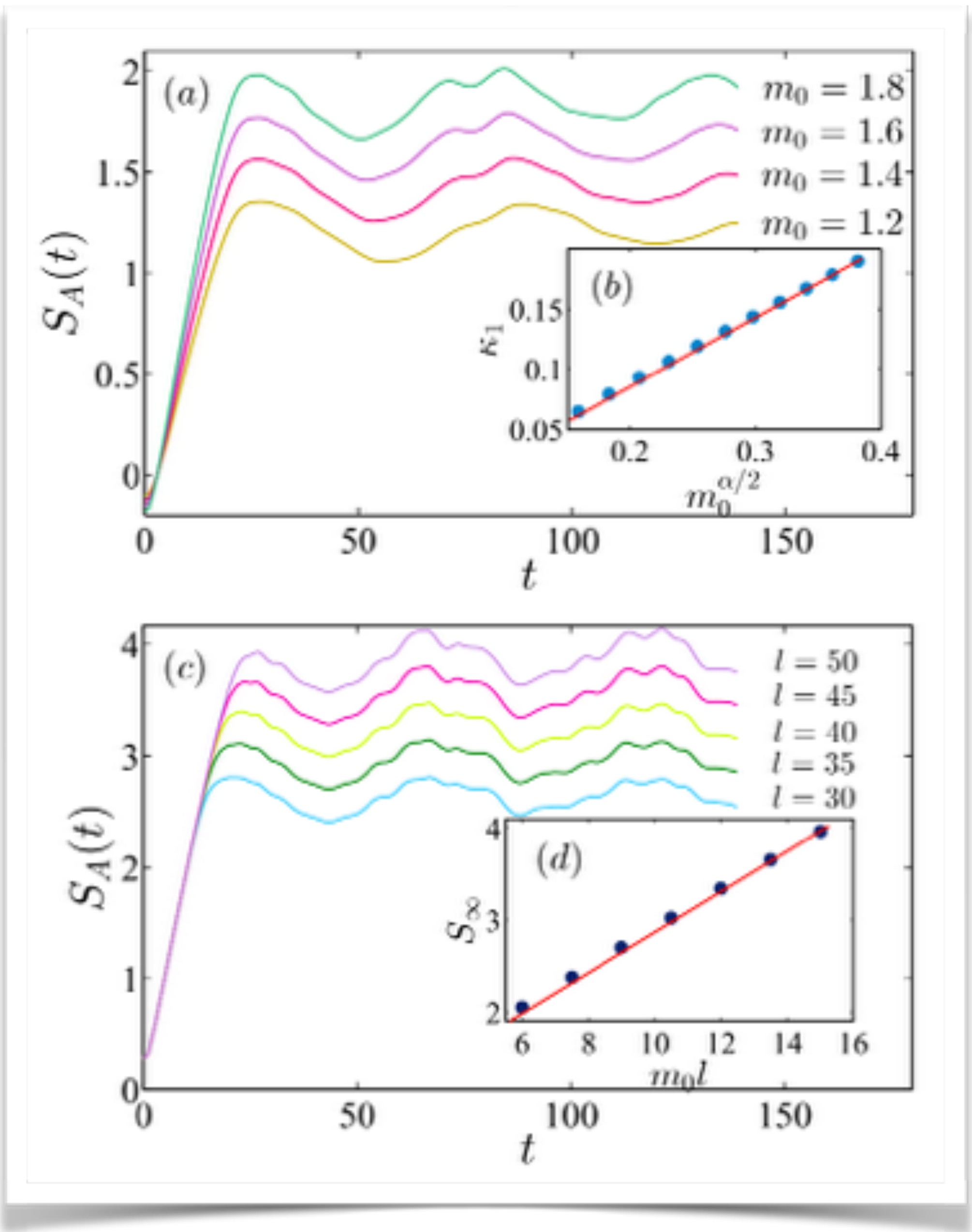
arXiv:1408.3744

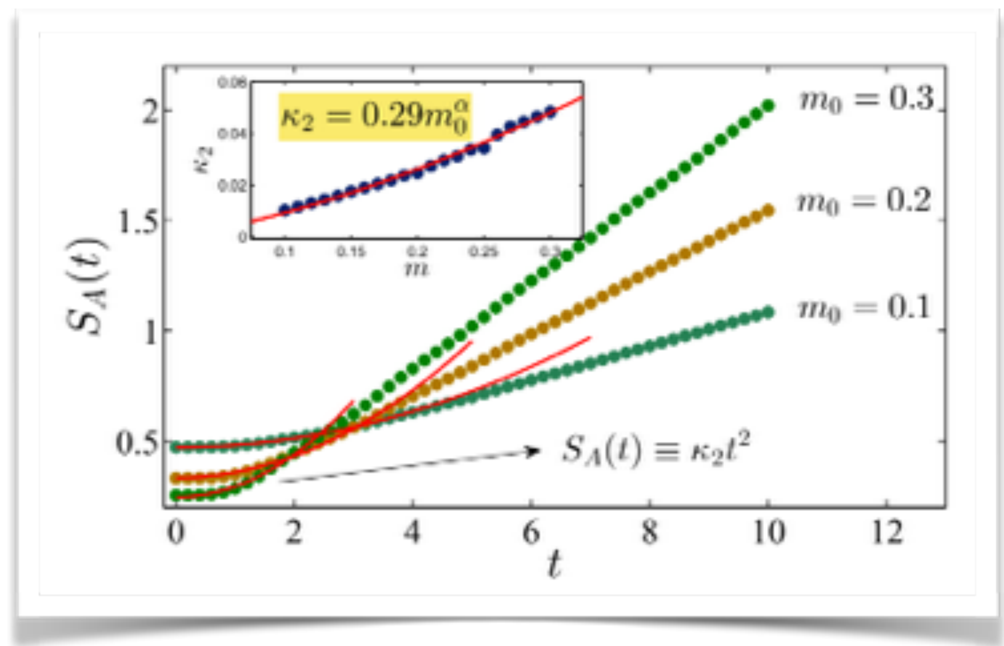
$$\mathcal{H}=\frac{1}{2}\sum_{n=1}^{N}\pi_n^2+\frac{1}{2}\sum_{n,n'=1}^{N}\phi_n K_{nn'}\phi_{n'}$$

$$\frac{1}{2}\sum_{i,j=1}^{N}\phi_i K_{ij}\phi_j \rightarrow \int[-\frac{1}{2}\phi(x)(-\bigtriangledown)^{\alpha/2}\phi(x)+\frac{1}{2}m^{\alpha}\phi^2]dx.$$

$$K_{i,j}\sim 1/|i-j|^{1+\alpha}$$

$$1<\alpha<2$$





$$S_A(t)=-\frac{c^g(\alpha)}{3}\log m_0+\begin{cases}\mathcal{A}m_0^{\alpha/2}t & t<t^*\\ \mathcal{B}m_0 l & t\gg t^*\end{cases}$$

$t^*=\lambda(l/2)^{\alpha/2}$ the dynamical exponent is $z=\alpha/2$.

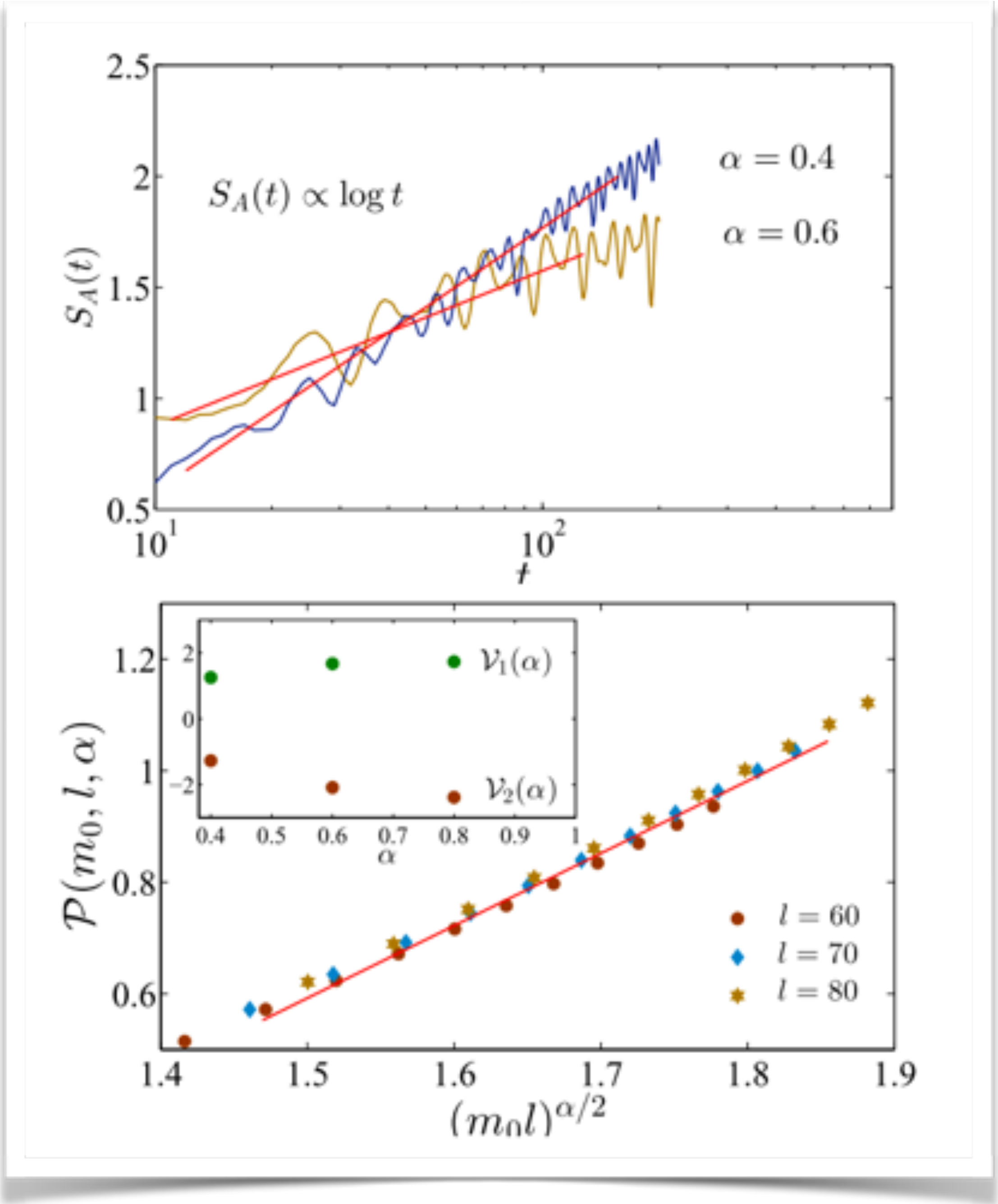
there is no maximum group velocity for quasi-particles !

Entanglement dynamics in short and long-range harmonic oscillators

M. Ghasemi Nezhadhaghighi[1] and M. A. Rajabpour[2,3,*]

arXiv:1408.3744

$$0 < \alpha < 1$$



$$S_A(t) = -\frac{c^g(\alpha)}{3}\log m_0 + \begin{cases} \kappa_2 t^2 & t \ll 1 \\ \mathcal{P}(m_0, l, \alpha)\log t & t \gg 1 \end{cases}$$

$$\mathcal{P}(m_0, l, \alpha) = \Big[\mathcal{V}_1(\alpha)(m_0 l)^{\alpha/2} + \mathcal{V}_2(\alpha)\Big]$$

Disordered systems

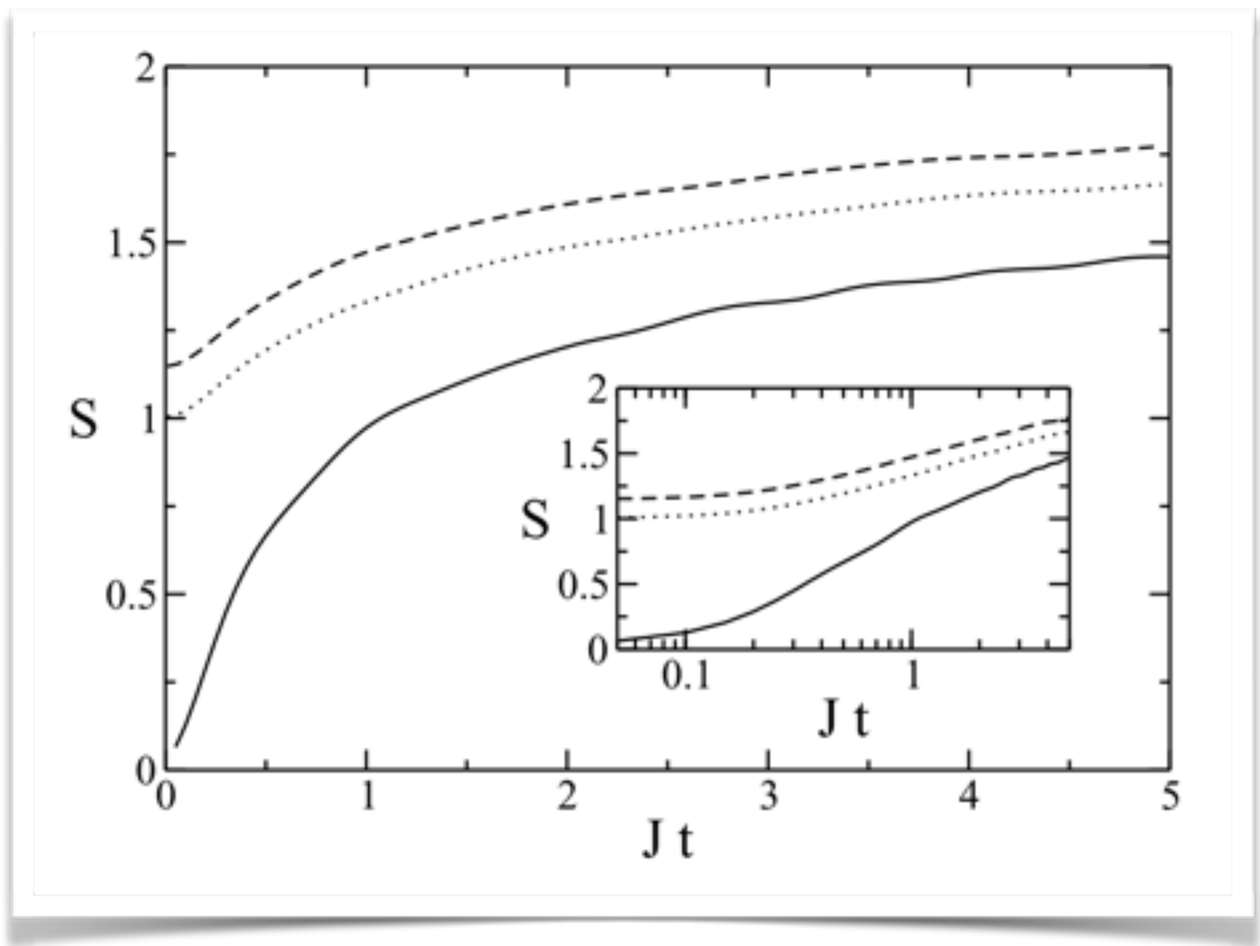
Entanglement entropy dynamics of Heisenberg chains

Gabriele De Chiara[1], Simone Montangero[1,2], Pasquale Calabrese[3] and Rosario Fazio[1,4]

J. Stat. Mech. (2006) P03001

$$H = \sum_{i=1}^{N-1} J_i(\sigma_x^i \sigma_x^{i+1} + \sigma_y^i \sigma_y^{i+1} + \Delta \sigma_z^i \sigma_z^{i+1})$$

random J



$$S_\ell \sim \kappa \ln Jt$$

$$S_\ell^{(\infty,0)} = -\frac{1}{2}\ln\left(\frac{1}{t} + a(\ell)\right) + b(\ell)$$

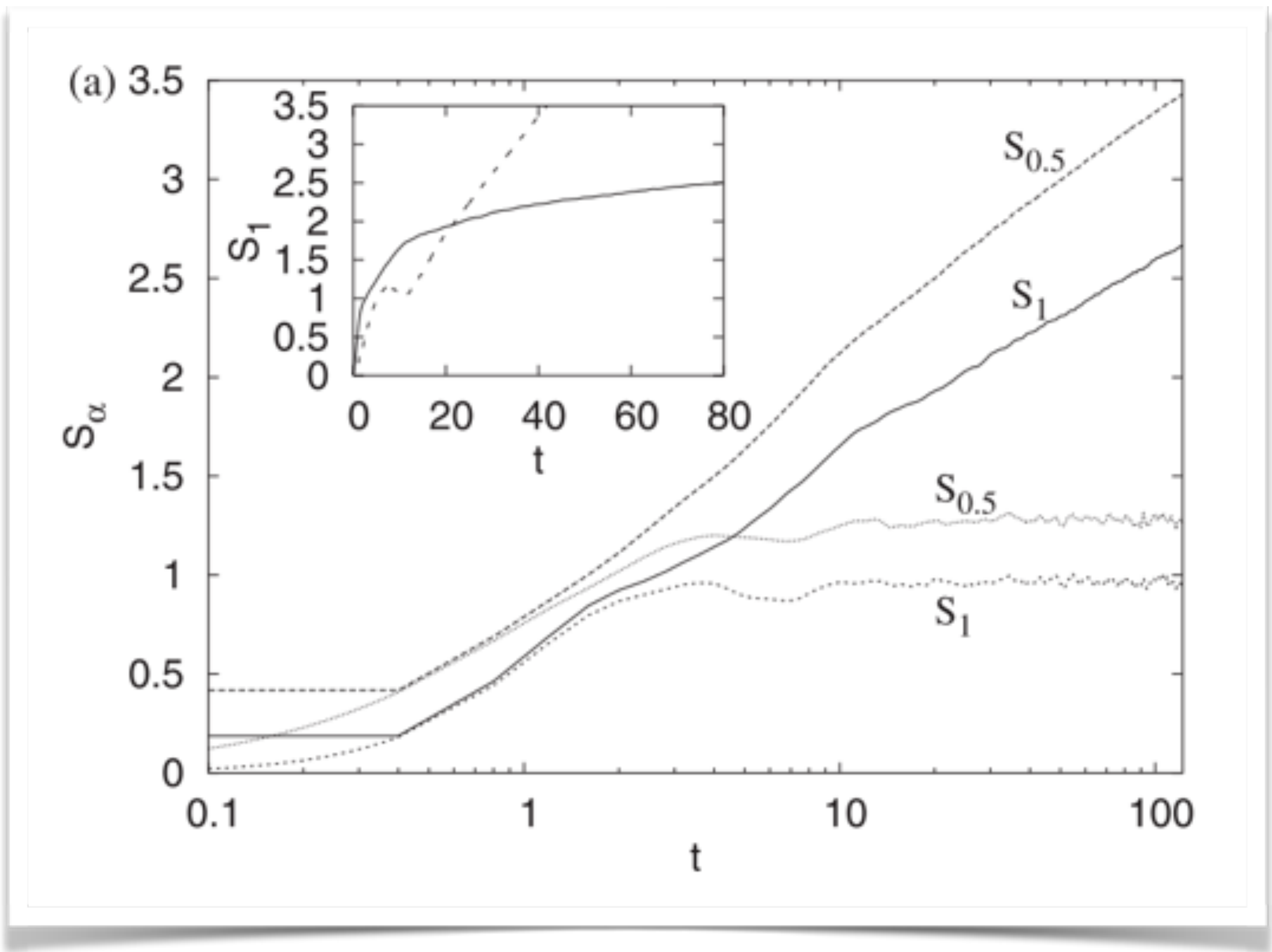
PHYSICAL REVIEW B **77**, 064426 (2008)

Many-body localization in the Heisenberg XXZ magnet in a random field

Marko Žnidarič,[1] Tomaž Prosen,[1] and Peter Prelovšek[1,2]

$$H = \sum_{j=1}^{n-1} (s_j^x s_{j+1}^x + s_j^y s_{j+1}^y + \Delta s_j^z s_{j+1}^z) + \sum_{j=1}^{n} h_j s_j^z$$

random magnetic field



PHYSICAL REVIEW B **85**, 094417 (2012)

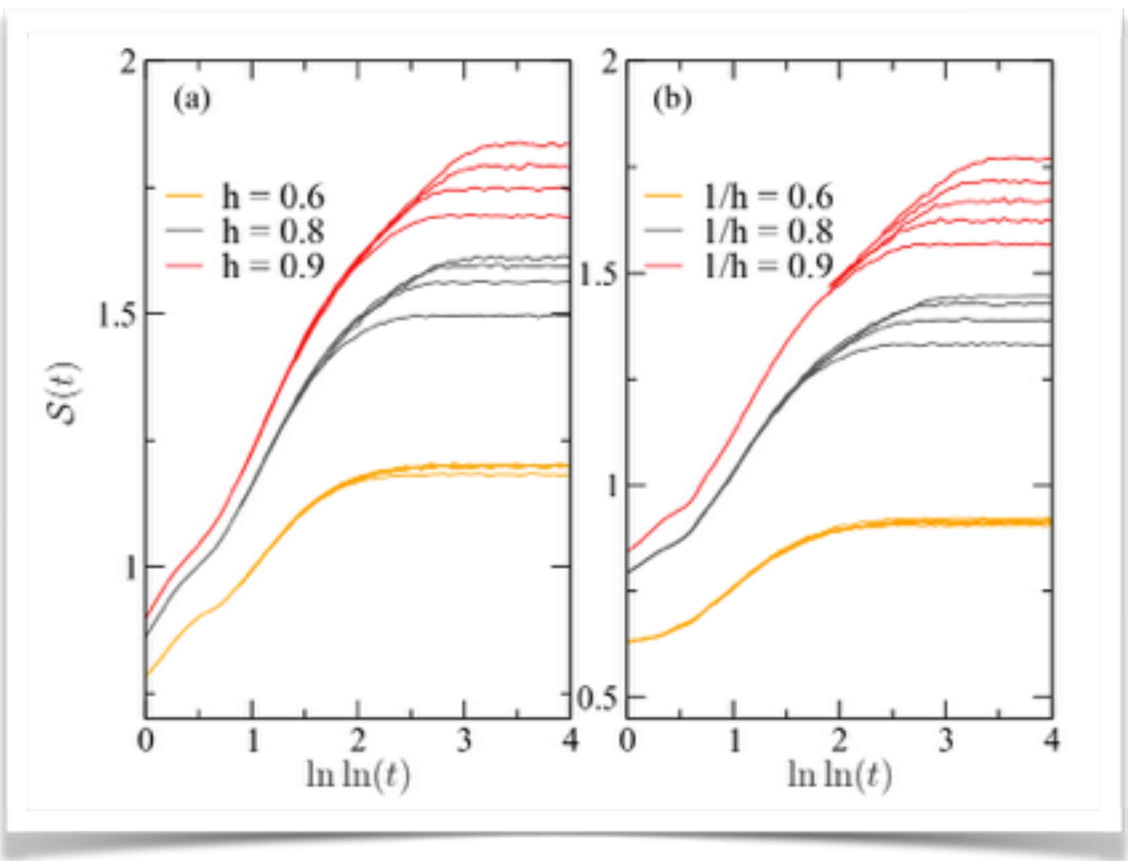
Entanglement entropy dynamics of disordered quantum spin chains

Ferenc Iglói,[1,2,*] Zsolt Szatmári,[2,†] and Yu-Cheng Lin[3,‡]

$$\mathcal{H} = -\sum_{i=1}^{L} J_i \sigma_i^x \sigma_{i+1}^x - \sum_{i=1}^{L} h_i \sigma_i^z$$

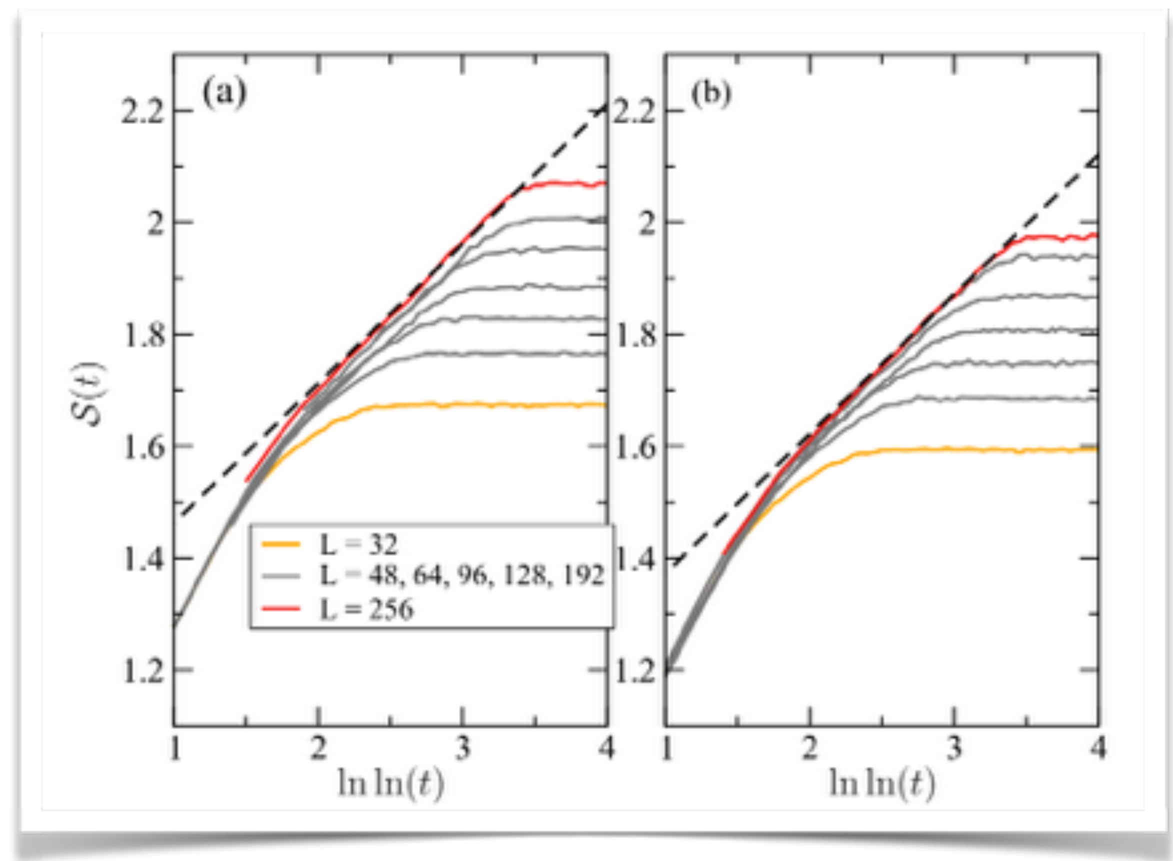
couplings $\{J_i\}$ for a given sample remains unaltered, whereas the width of the transverse-field distribution is changed from $h_0\,(t<0)$ to $h\,(t \geqslant 0)$

Quench to noncritical states



$$\hat{\mathcal{S}}(h_0,h) - \hat{\mathcal{S}}_L(h_0,h) \approx \exp[-L/\xi(h)].$$

Quench to the critical point



$$\mathcal{S}(t) = s + a \ln\ln t$$

$$\hat{\mathcal{S}}_L(h_0,1) = s(h_0) + b \ln L$$

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Entanglement entropy dynamics of disordered quantum spin chains

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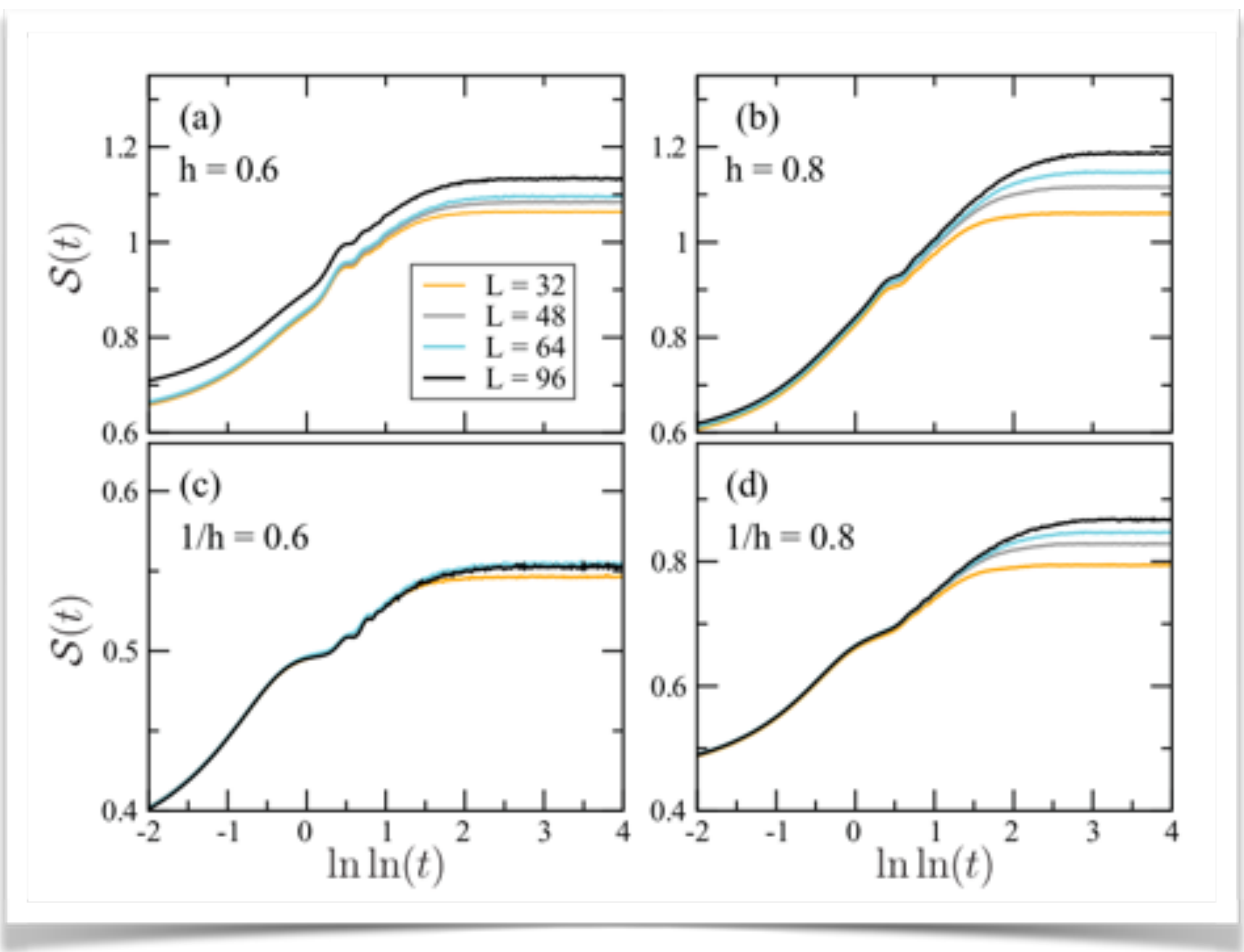
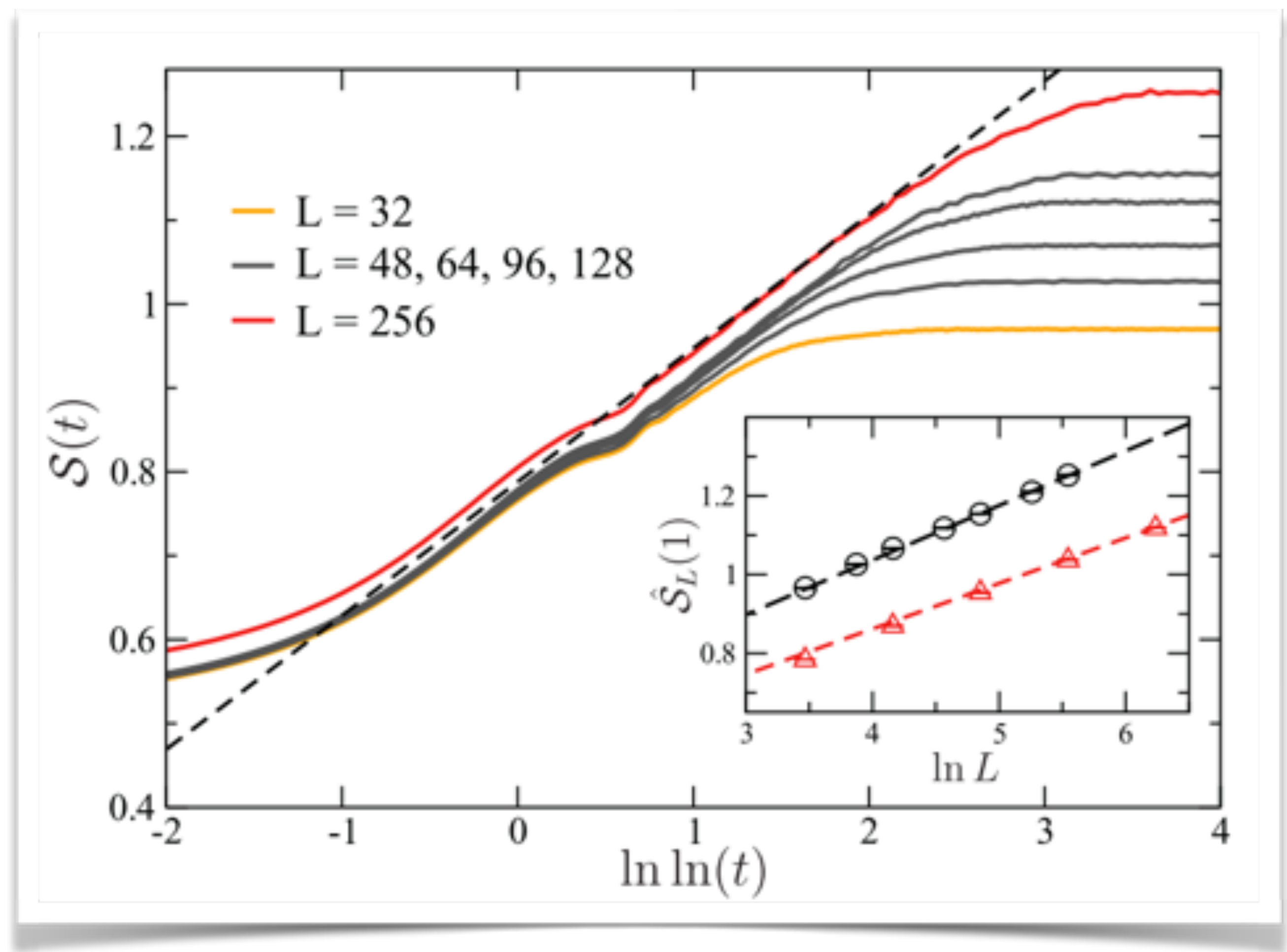
LOCAL QUENCH

Quench to noncritical states



Quench to the critical point



PRL **109,** 017202 (2012) PHYSICAL REVIEW LETTERS week ending 6 JULY 2012

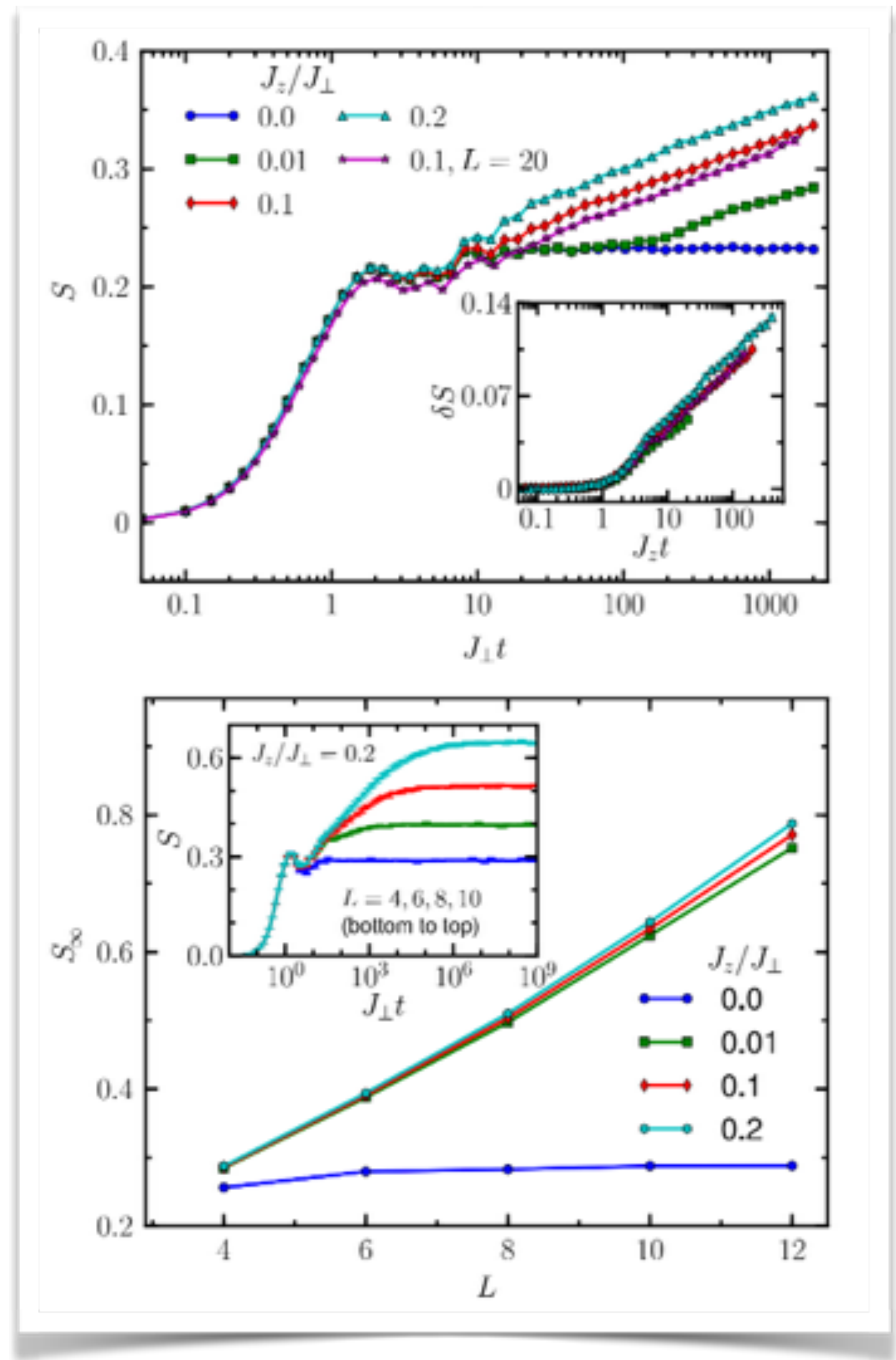
Unbounded Growth of Entanglement in Models of Many-Body Localization

Jens H. Bardarson,[1,2] Frank Pollmann,[3] and Joel E. Moore[1,2]

$$H_0 = J_\perp \sum_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + \sum_i h_i S_i^z$$

random magnetic field

$$H_{\text{int}} = J_z \sum_i S_i^z S_{i+1}^z$$



Theoretical explanations in

PRL **110,** 067204 (2013) PHYSICAL REVIEW LETTERS week ending 8 FEBRUARY 2013

Many-Body Localization in One Dimension as a Dynamical Renormalization Group Fixed Point

Ronen Vosk[1] and Ehud Altman[1,2]

PRL **110,** 260601 (2013) PHYSICAL REVIEW LETTERS week ending 28 JUNE 2013

Universal Slow Growth of Entanglement in Interacting Strongly Disordered Systems

Maksym Serbyn,[1] Z. Papić,[2] and Dmitry A. Abanin[3,4]

$$\log t_{\text{sat}} \sim L$$

Thanks for your attention!