Observables and initial conditions from exact rotational hydro solutions

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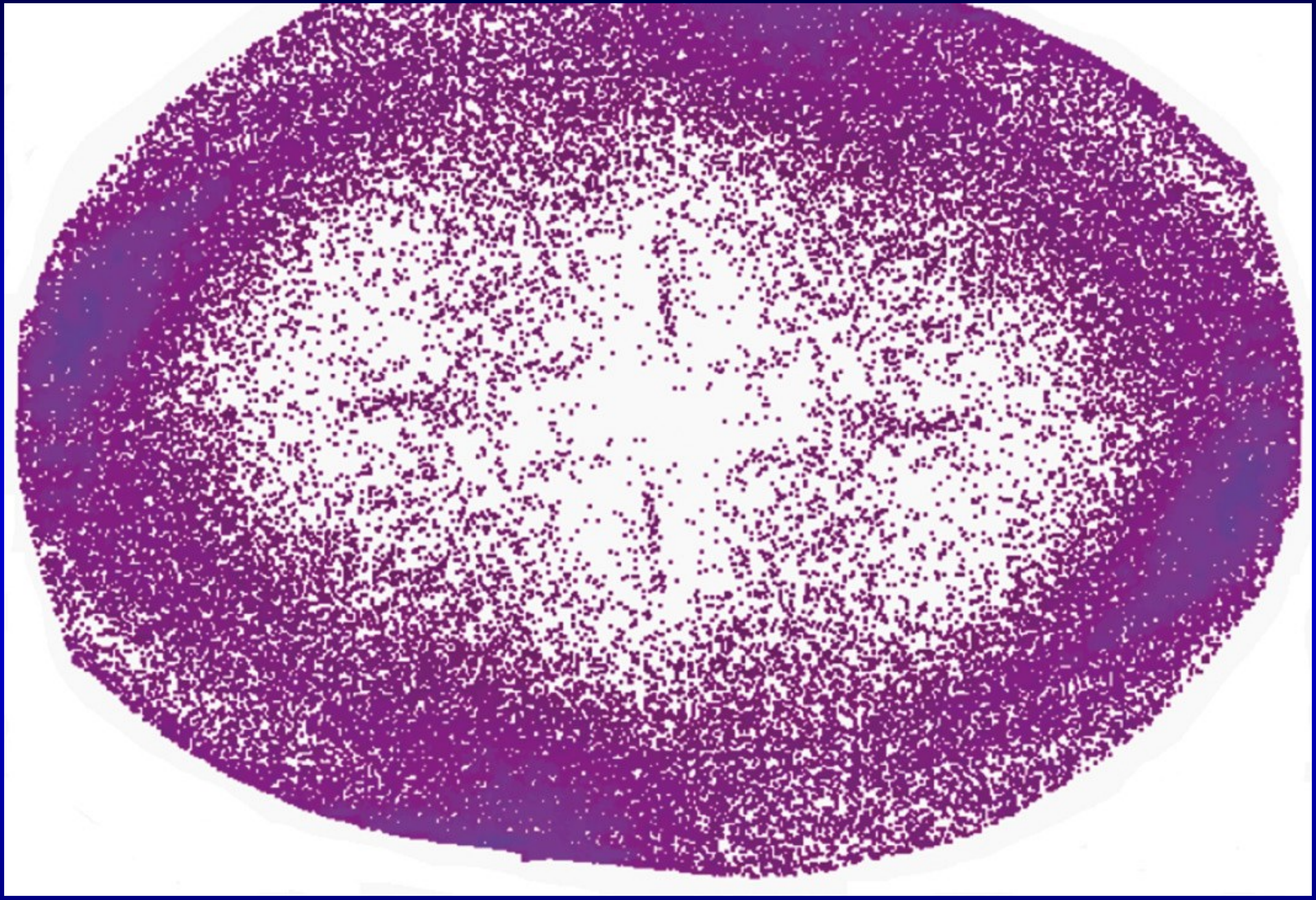
New exact rotating hydro solutions
Two different family of equations of state
Summary: new rotating solutions
Single particle spectra
Elliptic and higher order flows
Oscillations of HBT radii
Summary: effects on observables

arXiv:1309.4390v2 PRC 89, 044901 (2014)
+ manuscript in preparation

Motivation: initial angular momentum



Observation of conserved quantities: important
Example from L. Cifarelli, L.P. Csernai, H. Stöcker,
EPN 43/22 (2012) p. 91

Hydrodynamics: basic equations

$$\begin{aligned} \partial_t n + \nabla(n\mathbf{v}) &= 0, \\ (\partial_t + \mathbf{v}\nabla)\mathbf{v} &= -\frac{\nabla p}{mn}, \\ \partial_t \varepsilon + \nabla(\varepsilon \mathbf{v}) + p\nabla \mathbf{v} &= 0, \end{aligned}$$

Basic equations of non-rel hydrodynamics:
Euler equation needs to be modified for lattice QCD EoS

Use basic thermodynamical relations
for lack of conserved charge (baryon free region)

$$\begin{aligned} \varepsilon + p &= \mu n + T\sigma, \\ \mathrm{d}\varepsilon &= \mu \mathrm{d}n + T\mathrm{d}\sigma, \\ \mathrm{d}p &= n\mathrm{d}\mu + \sigma \mathrm{d}T, \end{aligned}$$

$$\begin{aligned} \varepsilon + p &= T\sigma, \\ \varepsilon &= \kappa(T)p. \end{aligned}$$

$$(\varepsilon + p)(\partial_t + \mathbf{v}\nabla)\mathbf{v} = -\nabla p,$$

Rewrite for v, T and (n, or σ)

$$
\begin{aligned}
\partial_t n + \nabla(n\mathbf{v}) &= 0, \\
\left[\frac{\mathrm{d}}{\mathrm{d}T}(\kappa T)\right](\partial_t + \mathbf{v}\nabla)T + T\nabla\mathbf{v} &= 0, \\
nm(\partial_t + \mathbf{v}\nabla)\mathbf{v} &= -\nabla p, \\
p &= nT.
\end{aligned}
$$

lattice QCD EoS: modification of the dynamical equations:

$$
\begin{aligned}
\partial_t \sigma + \nabla\sigma\mathbf{v} &= 0, \\
(1+\kappa)\left[\frac{\mathrm{d}}{\mathrm{d}T}\frac{\kappa T}{1+\kappa}\right](\partial_t + \mathbf{v}\nabla)T + T\nabla\mathbf{v} &= 0, \\
T\sigma(\partial_t + \mathbf{v}\nabla)\mathbf{v} &= -\nabla p, \\
p &= \frac{T\sigma}{\kappa+1}.
\end{aligned}
$$

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Ansatz for rotation and scaling

$$\mathbf{v} = \mathbf{v}_H + \mathbf{v}_R,$$

$$\mathbf{v}_H = \left(\frac{\dot X}{X} r_x, \frac{\dot Y}{Y} r_y, \frac{\dot Z}{Z} r_z\right),$$

$$\mathbf{v}_R = \omega \times \mathbf{r} = (-\omega r_y, \omega r_x, 0),$$

The case without rotation:
known self-similar solution
T. Cs, hep-ph/00111139
S. V. Akkelin et al,
hep-ph/0012127, etc.

Let's try to add rotation!

$$\mathcal{D} \equiv \partial_t + (\mathbf{v}\nabla),$$

$$s_R = \frac{r_x^2 + r_y^2}{R^2},$$

$$s_Z = \frac{r_z^2}{Z^2}.$$

$$\mathcal{D} s_R = \mathcal{D} s_Z = 0.$$

$$s = s_R + s_Z,$$

First good news: scaling variable remains good
→ a hope to find ellipsoidal rotating solutions!

Common properties of solutions

$$\mathbf{v} = \mathbf{v}_H + \mathbf{v}_R,$$
$$\mathbf{v}_H = \left(\frac{\dot{X}}{X} r_x, \frac{\dot{Y}}{Y} r_y, \frac{\dot{Z}}{Z} r_z\right),$$
$$\mathbf{v}_R = \omega \times \mathbf{r} = (-\omega r_y, \omega r_x, 0),$$
$$\omega = \omega_0 \frac{R_0^2}{R^2},$$
$$R = X = Y \neq Z,$$
$$s = s_T + s_Z = \frac{r_x^2 + r_y^2}{R^2} + \frac{r_Z^2}{Z^2},$$
$$V = XYZ = R^2 Z.$$

From self-similarity

Ellipsoidal ->spheroidal

time dependence of
R and ω coupled

Conservation law!

For details, see arXiv:1309.4390

Solutions for conserved particle n

Similar to irrotational case:

Family of self-similar solutions, T profile free

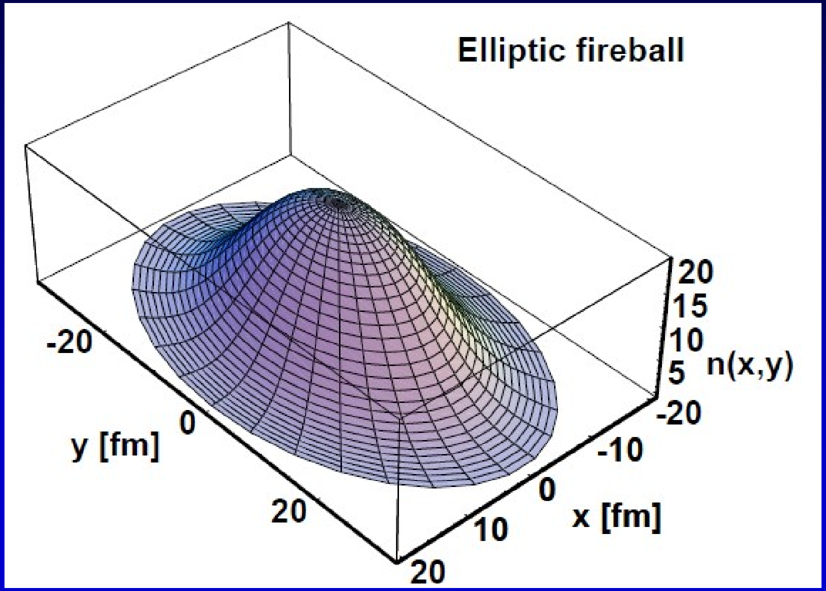
T. Cs, hep-ph/00111139

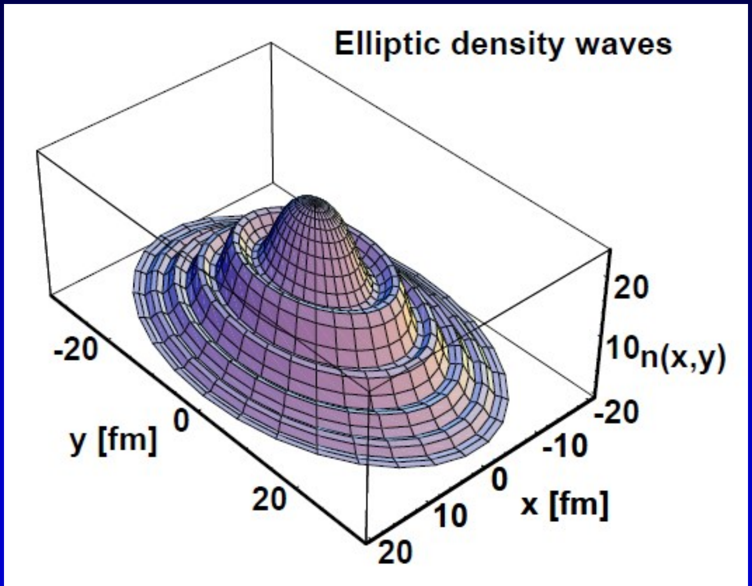
Rotation leads to increased transverse acceleration!

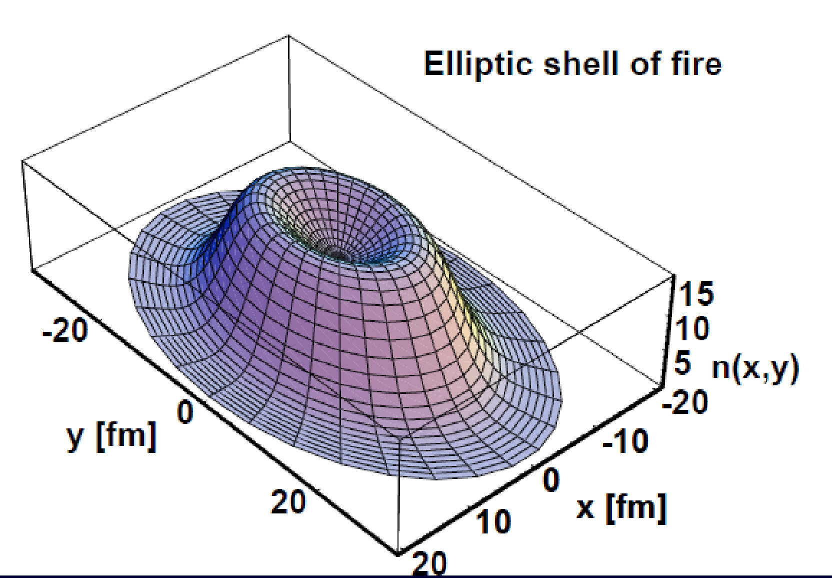
$$
\begin{aligned}
n &= n_0 \frac{V_0}{V} \nu(s), \\
T &= T_0 \left(\frac{V_0}{V}\right)^{1/\kappa} \mathcal{T}(s), \\
\nu(s) &= \frac{1}{\mathcal{T}(s)} \exp\left(-\frac{1}{2}\int_0^s \frac{\mathrm{d}u}{\mathcal{T}(u)}\right), \\
R\ddot{R} - R^2\omega^2 &= Z\ddot{Z} = \frac{T_0}{m}\left(\frac{V_0}{V}\right)^{1/\kappa},
\end{aligned}
$$

For details, see arXiv:1309.4390

Role of temperature profiles







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1B: conserved n, T dependent e/p

Works only if T is T(t) only

But more general EoS

Similar to

T. Cs. et al,

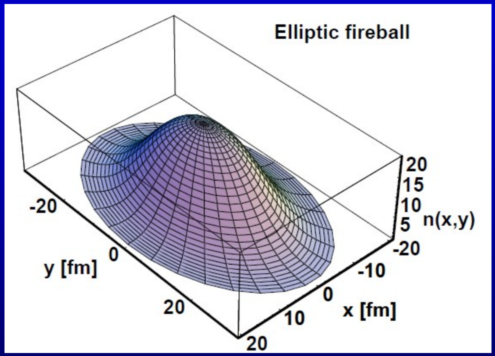
hep-ph/0108067

$$n = n_0 \frac{V_0}{V} \exp(-s/2),$$

$$T \equiv T(t),$$

$$R\ddot{R} - R^2\omega^2 = Z\ddot{Z} = \frac{T}{m},$$

$$\frac{d(\kappa T)}{dT}\frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0.$$



Rotation: increases transverse acceleration

Solutions for lattice QCD type EoS

$$\sigma = \sigma_0 \frac{V_0}{V} \mathcal{S}(s),$$

$$T = T_0 \left(\frac{V_0}{V}\right)^{1/\kappa} \mathcal{T}(s),$$

$$\mathcal{S}(s) = \frac{1}{\mathcal{T}(s)} \exp\left(-s/2\right),$$

$$R\ddot{R} - R^2\omega^2 = Z\ddot{Z} = \frac{1}{1+\kappa}.$$

$$\sigma = \sigma_0 \frac{V_0}{V} \exp(-s/2),$$

$$T \equiv T(t),$$

$$R\ddot{R} - R^2\omega^2 = Z\ddot{Z} = \frac{1}{1+\kappa(T)},$$

$$\frac{T}{1+\kappa}\left[\frac{d\kappa}{dT} + \kappa\right]\frac{\dot{T}}{T} + \frac{\dot{V}}{V} = 0.$$

Two different class of solutions: p/e = const(T) or p/e = f(T)
Notes: increased acceleration (vs conserved n)
Observables calculable if $\sigma \sim$ n(final) (Landau)

First integrals: Hamiltonian motion

$$\begin{aligned} H_{1A} &= \frac{P_x^2 + P_y^2 + P_z^2}{2m} + \kappa T_0 \left(\frac{X_0 Y_0 Z_0}{XYZ}\right)^{1/\kappa} + \frac{2m\omega_0^2 R_0^4}{X^2+Y^2}, \\ X_0 &= Y_0 = R_0, \\ \dot{X}_0 &= \dot{Y}_0 = \dot{R}_0. \end{aligned}$$

$$\begin{aligned} H_{2A} &= \frac{P_x^2 + P_y^2 + P_z^2}{2m} - \frac{m}{1+\kappa} \ln\left(\frac{XYZ}{X_0 Y_0 Z_0}\right) + \frac{2m\omega_0^2 R_0^4}{X^2+Y^2}, \\ X_0 &= Y_0 = R_0, \\ \dot{X}_0 &= \dot{Y}_0 = \dot{R}_0. \end{aligned}$$

$$E_R = \frac{2m\omega_0^2 R_0^4}{X^2+Y^2} = 2mR^2\omega^2, \quad I_z = \Theta\omega = mR^2\omega_0 \frac{R_0^2}{R^2} = m\omega_0 R_0^2$$

1A: n is conserved
2A: n is not conserveed

Angular momentum conserved
Energy in rotation → 0

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Summary so far: rotating solutions

New and rotating
exact solutions of fireball hydro

Also for lattice QCD family of EoS
Now analyzed in detail
(after 53 years)

Important observation:

Rotation leads to stronger radial expansion
lQCD EoS leads to stronger radial expansion

Perhaps connected to large radial flows
in RHIC and LHC data

Observables:
Next slides

arXiv:1309.4390 (v2)

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Observables from rotating solutions

$$
\begin{aligned}
n(t,\mathbf{r}') &= n_0 \frac{V_0}{V} \exp\left(-\frac{{r'_x}^2}{2X^2} - \frac{{r'_y}^2}{2Y^2} - \frac{{r'_z}^2}{2Z^2} \right) , \\
\mathbf{v}'(t,\mathbf{r}') &= \left(\frac{\dot X}{X} r'_x, \frac{\dot Y}{Y} r'_y, \frac{\dot Z}{Z} r'_z \right) ,
\end{aligned}
$$

Note: (r’, k’) in fireball frame
rotated wrt lab frame (r,k)

$$
\begin{aligned}
\ddot X X - X^2\omega^2 = \ddot Y Y = \ddot Z Z - Z^2\omega^2 &= \frac{T}{m} , \\
\dot T \frac{d}{dT}(\kappa T) + T \left(\frac{\dot X}{X} + \frac{\dot Y}{Y} + \frac{\dot Z}{Z} \right) &= 0 , , \\
X &= Z \equiv R \\
\dot X &= \dot Z \equiv \dot R
\end{aligned}
$$

Note: in this talk, rotation in the
(X,Z) impact parameter plane !

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Role of EoS on acceleration

$$\ddot{X}X - X^2\omega^2 = \ddot{Y}Y = \ddot{Z}Z - Z^2\omega^2 \quad = \quad \frac{1}{1+\kappa} \, .$$

$$\ddot{X}X - X^2\omega^2 = \ddot{Y}Y = \ddot{Z}Z - Z^2\omega^2 \quad = \quad \frac{T}{m} \, ,$$

Lattice QCD type Eos is explosive

$$\begin{aligned}
\theta(t) &= \theta_0 + \int \mathrm{d}t \, \omega(t), \\
\omega &= \omega_0 \frac{R_0^2}{R^2}, \\
R &= X = Z \neq Y, \\
s &= s_T + s_Z = \frac{r_x^2 + r_z^2}{R^2} + \frac{r_y^2}{Y^2}, \\
V &= XYZ = R^2 Y.
\end{aligned}$$

Tilt angle θ integrates rotation in (X,Z) plane: sensitive to EoS!

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Single particle spectra

$$
\begin{aligned}
E\frac{d^3n}{d\mathbf{k}'} &\propto E\exp\left(-\frac{k_x'^2}{2mT_x'}-\frac{k_y'^2}{2mT_y'}-\frac{k_z'^2}{2mT_z'}\right),\\
T_x' &= T_f + m(\dot{X}_f^2 + \omega_f^2 Z_f^2)\\
T_y' &= T_f + m\dot{Y}_f^2\\
T_z' &= T_f + m(\dot{Z}_f^2 + \omega_f^2 X_f^2),
\end{aligned}
$$

In the rest frame of the fireball:

Rotation increases effective temperatures both in the longitudinal and impact parameter direction

in addition to Hubble flows

Directed, elliptic and other flows

From the single particle spectra
→ flow coefficents v_n

$$\frac{d^3n}{dk_z k_t dk_t d\phi} = \frac{d^2n}{2\pi dk_z k_t dk_t}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n\phi)\right].$$

$$\begin{aligned} v_1 &= 0, \\ v_2 &= \frac{I_1(w)}{I_0(w)}, \\ v_3 &= 0, \ldots \\ v_{2n} &= \frac{I_n(w)}{I_0(w)}, \end{aligned}$$

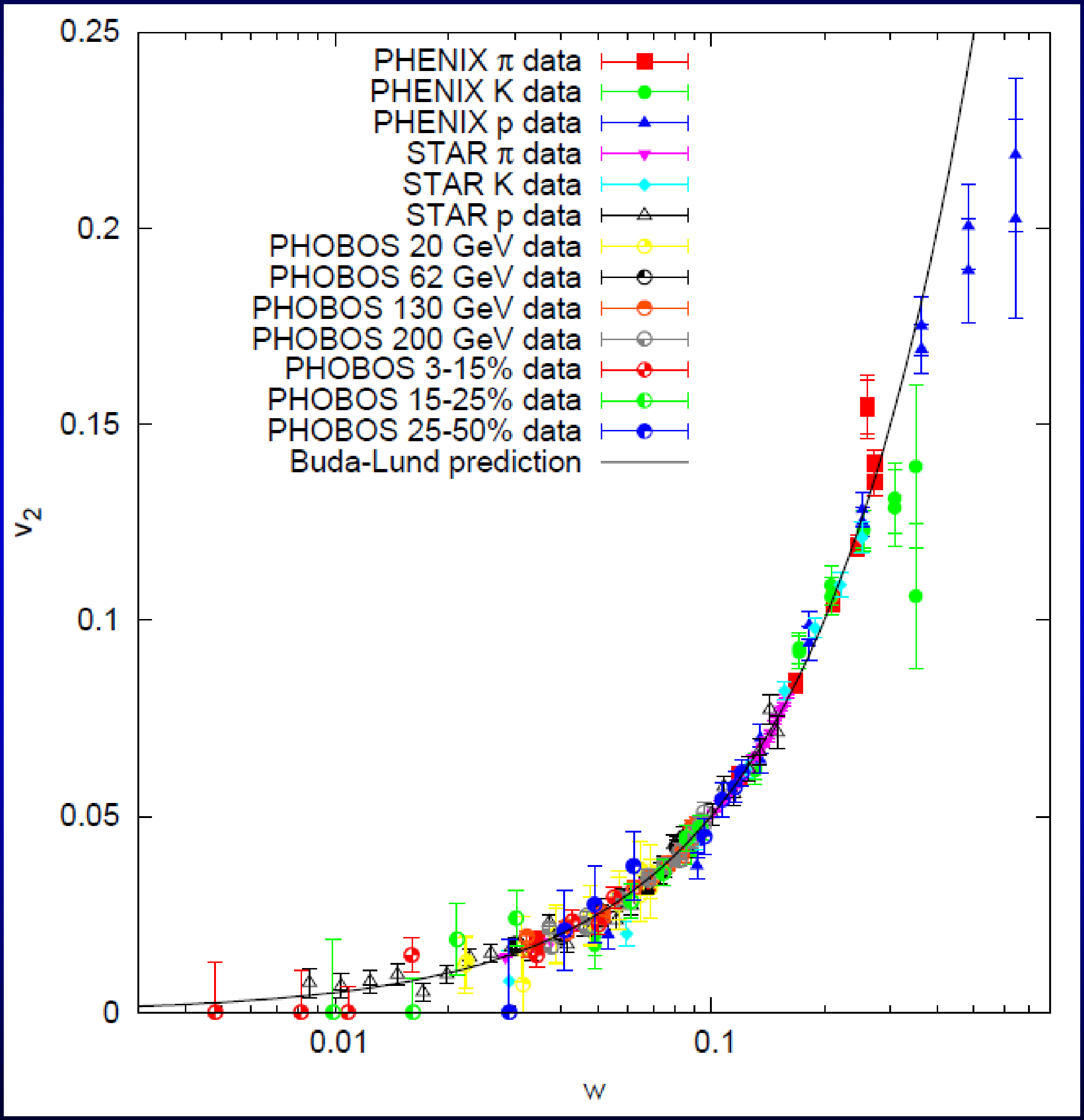
Note:
model is fully analytic

As of now, only X = Z = R(t)
spheroidal solutions are found

→ vanishing odd order flows

See next talk for fluctuations

Reminder: Universal w scaling of v_2



$$v_2 = \frac{I_1(w)}{I_0(w)},$$

Rotation does not change v2 scaling, but it modifies radial flow

Black line: Buda-Lund prediction from 2003 nucl-th/0310040

Comparision with data: nucl-th/0512078

Note: v2 data depend on particle type, centrality, colliding energy, rapidity, pt

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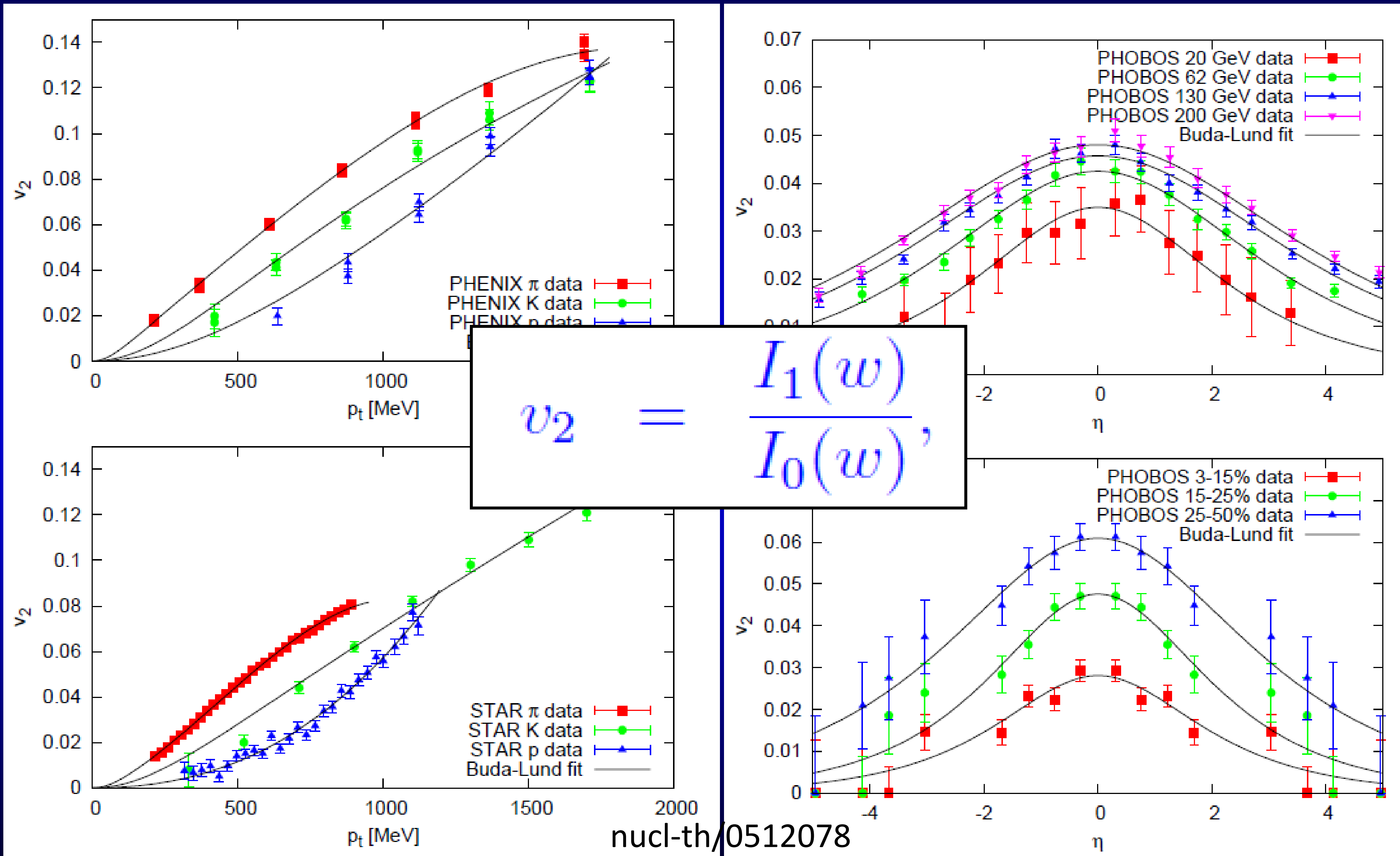
Details of universal w scaling of v_2



$$v_2 = \frac{I_1(w)}{I_0(w)},$$

nucl-th/0512078

HBT radii for rotating spheroids

Diagonal Gaussians in natural frame

$$
\begin{aligned}
C(\mathbf{K}',\mathbf{q}') &= 1+\lambda \exp\left(-q_x'^2 R_x'^2 - q_y'^2 R_y'^2 - q_z'^2 R_z'^2\right),\\
\mathbf{K}' &= \mathbf{K}'_{12} = 0.5(\mathbf{k}'_1+\mathbf{k}'_2),
\end{aligned}
$$

$$
\begin{aligned}
R_x'^{-2} &= X_f^{-2}\left(1+\frac{m}{T_f}(\dot X_f^2 + Z_f^2\omega_f^2)\right),\\
R_y'^{-2} &= Y_f^{-2}\left(1+\frac{m}{T_f}\dot Y_f^2\right),\\
R_z'^{-2} &= Z_f^{-2}\left(1+\frac{m}{T_f}(\dot Z_f^2 + X_f^2\omega_f^2)\right).
\end{aligned}
$$

New terms with blue

→ Rotation decreases HBT radii similarly to Hubble flow.

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HBT radii in the lab frame

$$C_2(\mathbf{K},\mathbf{q}) = 1 + \lambda \exp\left(- \sum_{i,j=\mathrm{s,o,l}} q_i q_j R_{ij}^2 \right),$$

$$R_\mathrm{s}^2 = R_y'^2 \cos^2\phi + R_x^2 \sin^2\phi\,,$$

$$R_\mathrm{o}^2 = R_x^2 \cos^2\phi + R_y'^2 \sin^2\phi + \beta_t^2 \Delta t^2,$$

HBT radii without flow

$$R_\mathrm{l}^2 = R_z'^2 \cos^2\theta + R_x'^2 \sin^2\theta + \beta_\mathrm{l}^2 \Delta t^2,$$

$$R_\mathrm{ol}^2 = (R_x'^2 - R_z'^2)\cos\theta \sin\theta \cos\phi + \beta_t \beta_\mathrm{l} \Delta t^2,$$

$$R_\mathrm{os}^2 = (R_x^2 - R_y'^2)\cos\phi \sin\phi\,,$$

$$R_\mathrm{sl}^2 = (R_x'^2 - R_z'^2)\cos\theta \sin\theta \sin\phi\,,$$

$$R_x^2 = R_x'^2 \cos^2\theta + R_z'^2 \sin^2\theta$$

But spheriodal symmetry of the solution
Makes several cross terms vanish → need for ellipsoidal solutions

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HBT radii in the lab frame

$$C_2(\mathbf{K},\mathbf{q}) = 1 + \lambda \exp\left(-\sum_{i,j=\mathrm{s,o,l}} q_i q_j R_{ij}^2\right),$$

$$R_\mathrm{s}^2 = R_y'^2 \cos^2 \ldots$$

$$R_\mathrm{o}^2 = R_x^2 \cos^2 \ldots$$

$$R_\mathrm{l}^2 = R_z'^2 \cos^2\theta + \ldots$$

$$R_\mathrm{ol}^2 = (R_x'^2 - R_z'^2) \ldots$$

$$R_\mathrm{os}^2 = (R_x^2 - R_y'^2) \ldots$$

$$R_\mathrm{sl}^2 = (R_x'^2 - R_z'^2) \ldots$$

$$R_x^2 = R_x'^2 \cos^2\theta + \ldots$$

HBT radii without flow

$$C_2(\mathbf{K},\mathbf{q}) = 1 + \lambda \exp\left(-\sum_{i,j=\mathrm{s,o,l}} q_i q_j R_{ij}^2\right),$$

$$R_\mathrm{s}^2 = R_y'^2 \cos^2\phi + R_x^2 \sin^2\phi,$$

$$R_\mathrm{o}^2 = R_x^2 \cos^2\phi + R_y'^2 \sin^2\phi + \beta_t^2 \Delta t^2,$$

$$R_\mathrm{l}^2 = R_z'^2 \cos^2\theta + R_x'^2 \sin^2\theta + \beta_\mathrm{l}^2 \Delta t^2,$$

$$R_\mathrm{ol}^2 = 0 + \beta_t \beta_\mathrm{l} \Delta t^2,$$

$$R_\mathrm{os}^2 = (R_x^2 - R_y'^2) \cos\phi \sin\phi,$$

$$R_\mathrm{sl}^2 = 0.$$

But spheriodal symmetry of the solution
Makes several cross terms vanish → need for ellipsoidal solutions

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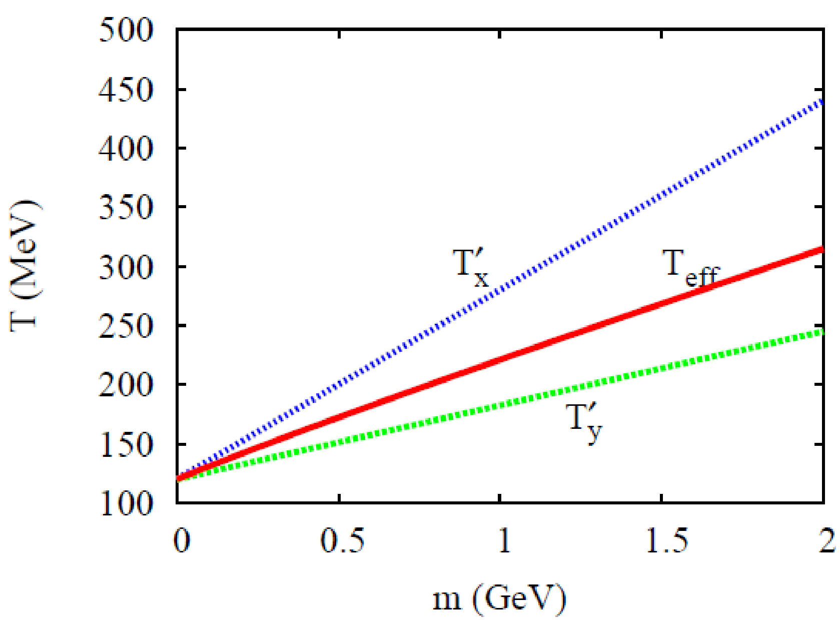
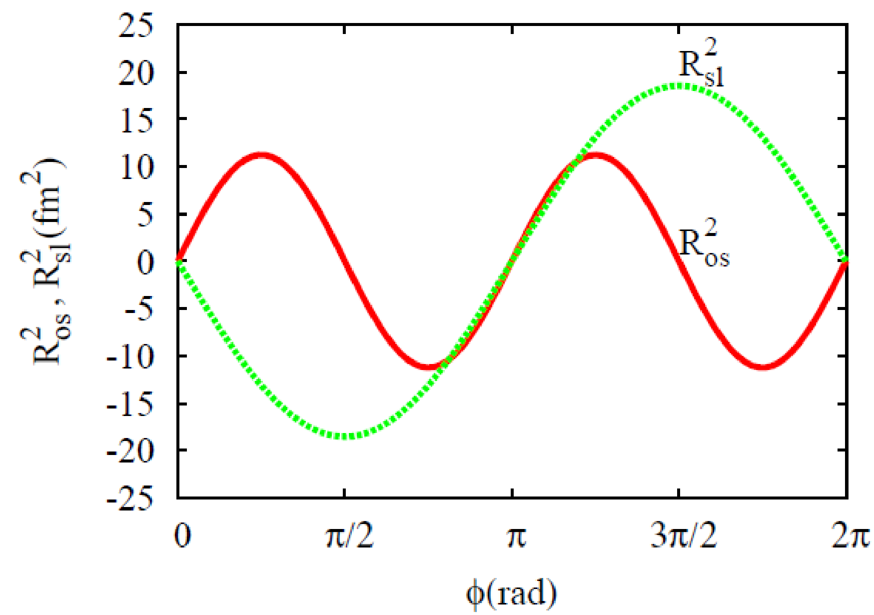
Qualitatively rotation and flow similar





But need more time dependent calculations

and less academic studies (relativistic solutions)

Summary

Observables calculated

Effects of rotation and flow
Combine and have same mass dependence

Spectra: slope increases
v_2: universal w scaling remains valid
HBT radii:
Decrease with mass intensifies

Even for spherical expansions:
v_2 from rotation.

Picture: vulcano
How to detect the rotation?
Next step: penetrating probes

Summary

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Combine and have same mass dependence

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Picture: vulcano
How to detect the rotation?
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