Non-unitary multicriticality and $\mathcal{PT}$ symmetry breaking

Máté Lencsés

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M. Lencsés, A. Miscioscia, G. Mussardo, G. Takács
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Non-Hermitian physics, $\mathcal{PT}$ symmetry and its breaking

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REVIEW ARTICLE

Non-Hermitian physics

Yuto Ashida[a,b,c*], Zongping Gong[a,d] and Masahito Ueda[a,b,e]

Non-Hermitian physics, $\mathcal{PT}$ symmetry and its breaking

Hermitian wave physics. In particular, we discuss rich and unique phenomena found therein, such as unidirectional invisibility, enhanced sensitivity, topological energy transfer, coherent perfect absorption, single-mode lasing, and robust biological transport. We then explain in detail how non-Hermitian operators emerge as an effective description of open quantum systems on the basis of the Feshbach projection approach and the quantum trajectory approach. We discuss their applications to physical systems relevant to a variety of fields, including atomic, molecular and optical physics, mesoscopic physics, and nuclear physics with emphasis on prominent phenomena and subjects in quantum regimes, such as quantum resonances, superradiance, the continuous quantum Zeno effect, quantum critical phenomena, Dirac spectra in quantum chromodynamics, and nonunitary conformal field theories. Finally, we introduce the notion of band topology in complex spectra of non-Hermitian systems and

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nature physics

REVIEW ARTICLES

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Non-Hermitian physics and PT symmetry

Ramy El-Ganainy[1], Konstantinos G. Makris[2], Mercedeh Khajavikhan[3], Ziad H. Musslimani[4], Stefan Rotter[5] and Demetrios N. Christodoulides[3]*

Non-Hermitian physics, $\mathcal{PT}$ symmetry and its breaking

systems operating near exceptional points.

The peculiar behaviour of complex eigenvalues around an exceptional point manifests itself in the threshold and spectral response of lasing systems[57,58]. When operating a PT-symmetric 'photonic molecule' (Fig. 3a) as a laser, the exceptional points lead to an unexpected pump-induced suppression and revival of lasing[59] - an effect that was experimentally demonstrated in coupled quantum cascade lasers as well as in a pair of silica microcavities with Raman gain[60,61]. Exceptional points were also found to promote single-

Yang–Lee: experiment

Selected for a Viewpoint in *Physics*

PRL **114,** 010601 (2015) PHYSICAL REVIEW LETTERS week ending 9 JANUARY 2015

Experimental Observation of Lee-Yang Zeros

Xinhua Peng,[1,*] Hui Zhou,[1] Bo-Bo Wei,[2] Jiangyu Cui,[1] Jiangfeng Du,[1,†] and Ren-Bao Liu[2,‡]

[1]*Hefei National Laboratory for Physical Sciences at Microscale, Department of Modern Physics, and Synergetic Innovation Center of Quantum Information & Quantum Physics, University of Science and Technology of China, Hefei 230026, China*

[2]*Department of Physics, Centre for Quantum Coherence, and Institute of Theoretical Physics, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong, China*

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Lee-Yang zeros are points on the complex plane of physical parameters where the partition function of a system vanishes and hence the free energy diverges. Lee-Yang zeros are ubiquitous in many-body systems and fully characterize their thermodynamics. Notwithstanding their fundamental importance, Lee-Yang zeros have never been observed in experiments, due to the intrinsic difficulty that they would occur only at complex values of physical parameters, which are generally regarded as unphysical. Here we report the first observation of Lee-Yang zeros, by measuring quantum coherence of a probe spin coupled to an Ising-type spin bath. The quantum evolution of the probe spin introduces a complex phase factor and therefore effectively realizes an imaginary magnetic field. From the measured Lee-Yang zeros, we reconstructed the free energy of the spin bath and determined its phase transition temperature. This experiment opens up new opportunities of studying thermodynamics in the complex plane.

Yang–Lee: higher dimension

PHYSICAL REVIEW B **106**, 054402 (2022)

Lee-Yang theory of the two-dimensional quantum Ising model

Pascal M. Vecsei, Jose L. Lado, and Christian Flindt
Department of Applied Physics, Aalto University, 00076 Aalto, Finland

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Determining the phase diagram of interacting quantum many-body systems is an important task for a wide range of problems such as the understanding and design of quantum materials. For classical equilibrium systems, the Lee-Yang formalism provides a rigorous foundation of phase transitions, and these ideas have also been extended to the quantum realm. Here, we develop a Lee-Yang theory of quantum phase transitions that can include thermal fluctuations caused by a finite temperature, and it thereby provides a link between the classical Lee-Yang formalism and recent theories of phase transitions at zero temperature. Our methodology exploits analytic properties of the moment generating function of the order parameter in systems of finite size, and it can be implemented in combination with tensor-network calculations. Specifically, the onset of a symmetry-broken phase is signaled by the zeros of the moment generating function approaching the origin in the complex plane of a counting field that couples to the order parameter. Moreover, the zeros can be obtained by measuring or calculating the high cumulants of the order parameter. We determine the phase diagram of the two-dimensional quantum Ising model and thereby demonstrate the potential of our method to predict the critical behavior of two-dimensional quantum systems at finite temperatures.

DOI: 10.1103/PhysRevB.106.054402

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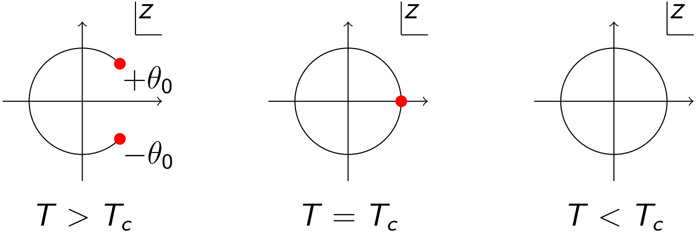
IM in magnetic field: the Yang–Lee singularity

- 2D Ising model (IM) in external magnetic field:

$$H = -J \sum_{\langle i,j \rangle} s_i s_j + h \sum_i s_i; \qquad s_i = \pm 1$$

- Let $z = e^{-\beta h} = e^{i\theta}$, then the zeros of the partition function $Z(z)$ are located on the unit circle (Lee, Yang, 1952).



IM in magnetic field: Yang–Lee singularity

- $T > T_c$: density of zeros is anomalous (Kortmann, Griffith, 1971):

$$\eta(\theta) \overset{\theta\to\theta_0}{\sim} |\theta - \theta_0|^{\alpha}$$

- Fisher (1978):

$$\mathcal{L}_{YL} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + (h - ih_c)\varphi + i\gamma\varphi^3 + \dots$$

- Cardy (1985): Non-unitary minimal conformal field theory $\mathcal{M}(2,5)$, $c_{eff} = 2/5$

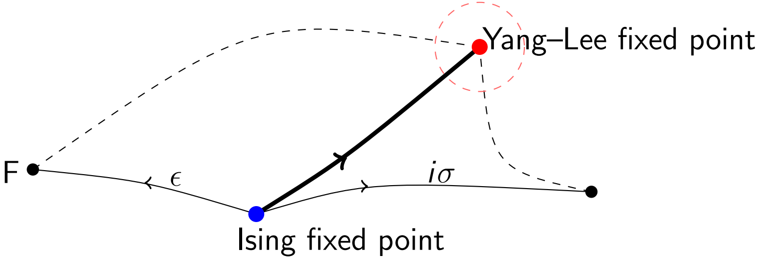
$$\{\mathbb{I}, \phi\}; \qquad \phi \times \phi = \mathbb{I} + \phi$$

IM in magnetic field: Yang–Lee singularity

- von Gehlen (1991): 1D quantum Ising in imaginary longitudinal field, exact diagonalization
- Scaling field theory around the Ising fixed point ("Ising spectroscopy", Zamolodchikov et. al. 2001, 2006, 2011, 2013, 2022, 2023):

$$\mathcal{A} = \mathcal{A}_{CFT} + \lambda \int d^2x\varepsilon + ih_{\|} \int d^2x\sigma$$

- $\mathcal{PT}$ symmetry: $\sigma \to -\sigma$, $i \to -i$
- Spontaneous breaking: complex spectrum



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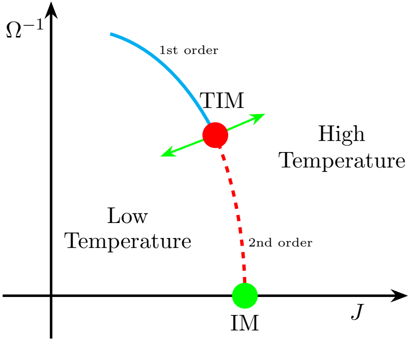
Tricritical Ising Model

- Blume–Capel model

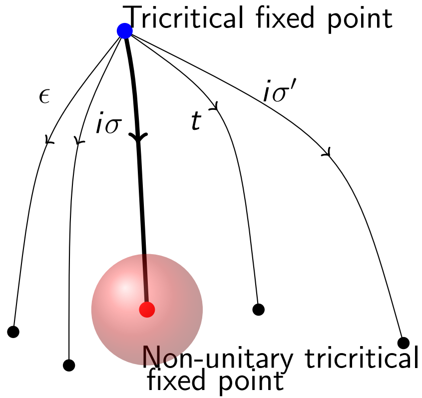
$$H_{BC} = -J\sum_{\langle i,j\rangle}^{N} s_i s_j t_i t_j - \Omega\sum_{i=1}^{N} t_i - H\sum_{i=1}^{N} s_i t_i$$
$$-H_3\sum_{\langle i,j\rangle}^{N}(s_i t_i t_j + s_j t_j t_i) - K\sum_{\langle i,j\rangle}^{N} t_i t_j \qquad s_i = \pm 1; t_i = 0, 1$$

- Tricritical point (tricritical Ising model, TIM): $H = H_3 = K = 0$, phase diagram:



TIM: Scaling limit

- $\mathcal{M}(4,5)$ unitary minimal model
- 2 nontrivial relavant even fields: ϵ,t
- 2 odd fields: σ,σ'
- $\mathcal{PT}$: $\epsilon, t \to \epsilon, t$, $\sigma, \sigma' \to -\sigma, -\sigma'$



Imaginary magnetic field

- von Gehlen 1994 from the lattice version: 1d quantum Blume–Capel chain
- 2nd order Lee–Yang line $\mathcal{M}(2,5)$ ending in a tricritical point $\mathcal{M}(2,7)$
- In agreement with c_{eff} theorem (Castro-Alvaredo et al. 2017):

$$c_{eff}^{IR} < c_{eff}^{UV} = \frac{7}{10}$$

- What happens in the scaling field theory?

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Truncated space approach: finite size spectrum of a QFT

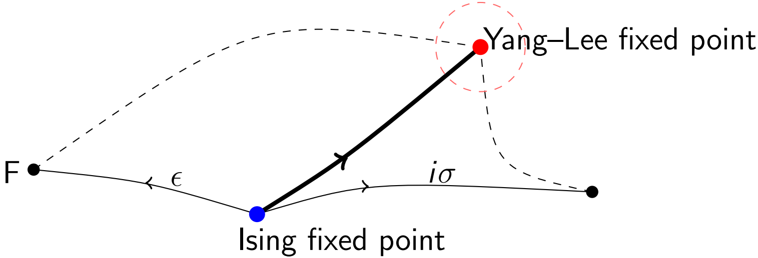
- Yurov, Zamolodchikov 1990 (around Yang–Lee fixed point)
- The Hamiltonian in the scaling limit:

$$H = H_0 + V$$

- Finite volume: discrete spectrum
- Diagonalize H using $\mathcal{H}_0$ as a basis (matrix elements of V can be calculated)
- Truncation: keep states with energy below a cut-off $E < \Lambda$
- Hope for convergence
- The more relevent the perturbation, the more convergent it is.

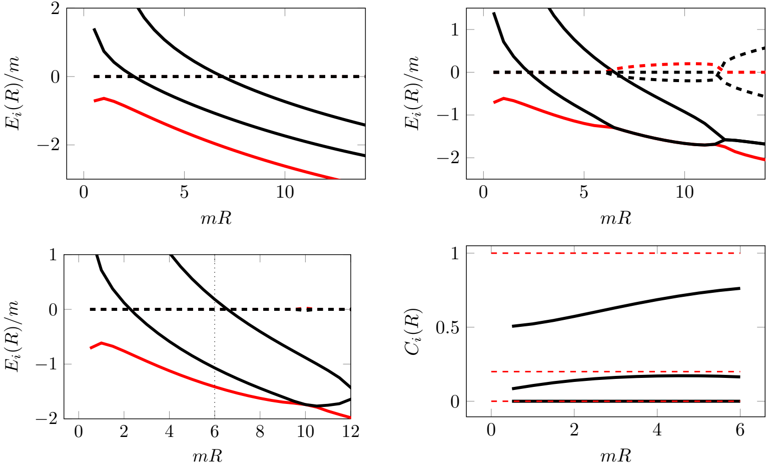
Ising model

- $$H = H_{CFT} + \lambda \int_0^R dx \epsilon + ih_{\parallel} \int_0^R dx \sigma$$
- We tune the imaginary magnetic field $ih_{\parallel}$ with fixed λ, in finite volume R, with PBC.
- Look for $E_i \propto \frac{1}{R}$
- See also Fonseca, Zamolodchikov 2001, Xu, Zamolodchikov 2022,2023



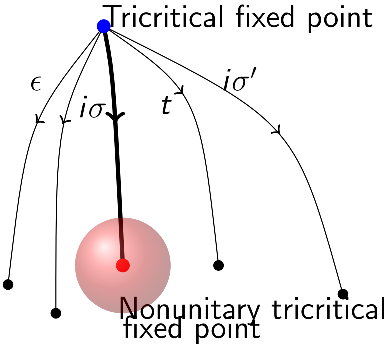
Results from truncated space: Ising model

We tune the imaginary magnetic field $ih_{\|}$ with fixed λ, in finite volume R, with PBC.



Tricritical Ising model

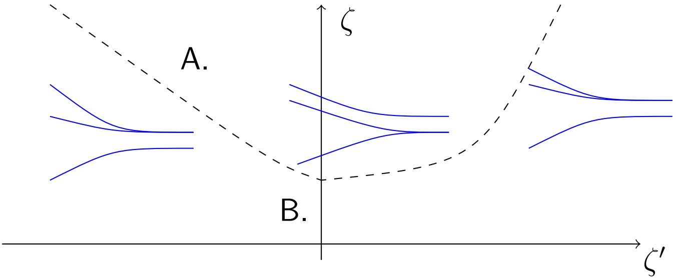
- $$H = H_{CFT} + \mu \int_0^R \epsilon(x,y)dx + ih \int_0^R \sigma(x,y)dx + ih' \int_0^R \sigma'(x,y)dx + v \int_0^R t(x,y)dx$$
- We tune both imaginary magnetic fields with fixed $\mu \neq 0$; $v = 0$ to get $\propto 1/R$ spectrum



Results from truncated space: Tricritical Ising model

- We tune both imaginary magnetic fields with fixed $\mu \neq 0$; $v = 0$
- ϵ generates a mass scale: $\mu = \frac{10^{-4}}{2\pi} {M'}^{9/5}$; the dimensionless couplings:

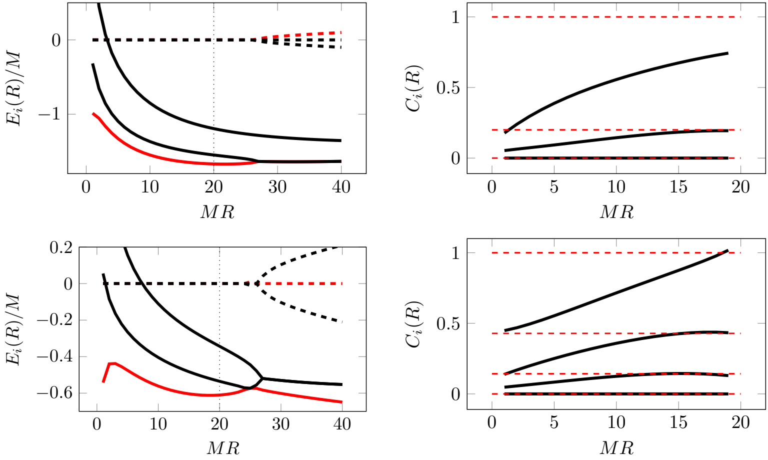
$$\zeta = \frac{h}{{M'}^{77/72}} \,, \qquad \zeta' = \frac{h'}{{M'}^{5/8}} \,, \tag{1}$$



$v \neq 0$ is required!

Results from truncated space: Tricritical Ising model

Critical YL: $\mathcal{M}(2,5)$; Tricritical YL: $\mathcal{M}(2,7)$ (von Gehlen)



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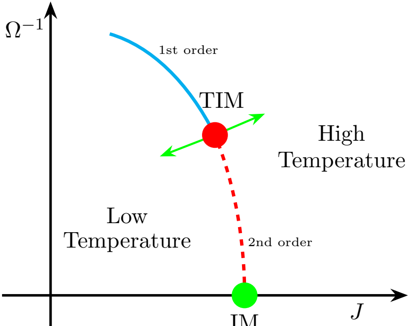
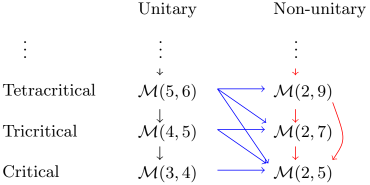
Unitary multicriticality and $\mathbf{Z}_2$ breaking

- Ising like models, critical surfaces: $\mathbf{Z}_2$ symmetry breaking
- Multicritical points: unitary minimal series: $\mathcal{M}(p, p+1)$ (A invariant): Ising $\mathcal{M}(3,4)$, tricritical Ising $\mathcal{M}(4,5)$, tetracritical $\mathcal{M}(5,6)$ etc. (Zamolodchikov 1986)
- General phase diagram: $\mathcal{M}(p-1, p)$ critical surface, boundary: codimension 1 $\mathcal{M}(p, p+1)$
- Series of massless flows $\mathcal{M}(p, p+1) \to \mathcal{M}(p-1, p)$ (Zamolodchikov 1991)



Non-unitary multicriticality and $\mathcal{PT}$ breaking

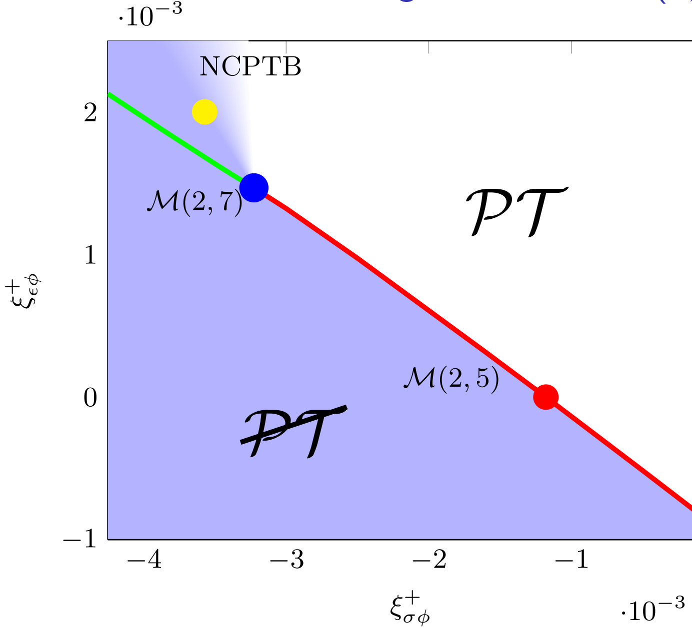
- We conjecture, that in general, the n-th multicritical Yang–Lee model is $\mathcal{M}(2, 2n+3)$
- They are related to $\mathcal{PT}$ symmetry breaking: $[H, \mathcal{PT}] = 0$.
  - Real spectrum: unbroken
  - Complex conjugate pairs in the spectrum: broken
- Analogy with unitary multicriticality: critical $\mathcal{M}(2, 2n+1)$ surfaces with codimension 1 boundary $\mathcal{M}(2, 2n+3)$
- Series of massless flows $\mathcal{M}(2, 2n+3) \to \mathcal{M}(2, 2n+1)$



Existence of Non-unitary massless flows: truncation

- We considered perturbations of $\mathcal{M}(2,7)$ and $\mathcal{M}(2,9)$ using truncated conformal space
- We found $\mathcal{PT}$ breaking around $\mathcal{M}(2,7)$ and massless spectrum compatible with $\mathcal{M}(2,5)$
- We found critical $\mathcal{PT}$ breaking along a line of $\mathcal{M}(2,5)$ with a tricritical endpoint, $\mathcal{M}(2,7)$
- Non-critical $\mathcal{PT}$ breaking: 1st and 2nd excited state forms complex conjugate pair

Phase diagram around $\mathcal{M}(2,9)$



Ginzburg–Landau for non-unitary multicriticality

- Zamolodchikov, GL for the n-th unitary multicritical model:

$$\mathcal{L}_{\mathcal{M}(2+n,3+n)} = \frac{1}{2}(\partial\varphi)^2 + g\varphi^{2(n+1)}$$

- Proposal for the n-th non-unitary multicritical model:

$$\boxed{\mathcal{L}_{\mathcal{M}(2,2n+3)} = \frac{1}{2}(\partial\varphi)^2 + ig_1\varphi + \sum_{k=0}^{n-2} g_{k+2}\varphi^2\,(i\varphi)^k + \varphi^2\,(i\varphi)^n}$$

- The flows considered:

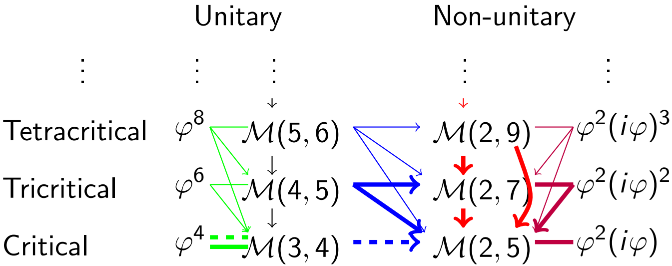
$$\mathcal{M}(2+n,3+n) \to \mathcal{M}(2,2m+3) \qquad \varphi^{2(n+1)} \to \varphi^2(i\varphi)^m \quad m \leq n$$

$$\mathcal{M}(2,2n+3) \to \mathcal{M}(2,2m+3) \qquad \varphi^2(i\varphi)^n \to \varphi^2(i\varphi)^m \quad m < n$$

Verification: Truncated massive boson

- Verification using the massive boson basis
- Mass pertrubation: gives back Bogoliubov
- Ising fixed point in φ^4: done by Rychkov, Vitale 2014, 2016
- We demonstrated the Chang duality: Ising fixed point with negative mass term
- $\mathcal{M}(2,5)$ in $\varphi^2(i\varphi)$
- $\mathcal{M}(2,5)$ critical line ending in the $\mathcal{M}(2,7)$ tricritical point
- The proposal is verified numerically in the first two cases

Summary


Unitary
Non-unitary
Tetracritical
Tricritical
Critical
φ^8
φ^6
φ^4
$\mathcal{M}(5,6)$
$\mathcal{M}(4,5)$
$\mathcal{M}(3,4)$
$\mathcal{M}(2,9)$
$\mathcal{M}(2,7)$
$\mathcal{M}(2,5)$
$\varphi^2(i\varphi)^3$
$\varphi^2(i\varphi)^2$
$\varphi^2(i\varphi)$

Outlook

- Order parameter, operator identification?
- Multi field models, supersymmetry? (see Klebanov et al. 2023)
- Entanglement properties in non-unitary CFT? (In progress...)
- Multicritical Ising spectroscopy, analytic structure of the free energy